Class Exercises for Chapter 8

1. Consider the following frequency responses:

$$H_1(e^{j\omega}) = \frac{1}{1-0.1e^{-j\omega}}$$

and

$$H_2(e^{j\omega}) = \frac{1}{1-10e^{-j\omega}}$$

Discuss the causality of the systems which correspond to the two spectra.

2. Consider a causal linear time-invariant system with system function

$$H(z)=\frac{1-a^{-1}z^{-1}}{1-az^{-1}}, \qquad a \text{ is real}$$

(a) Write the difference equation that relates the input $x[n]$ and output $y[n]$ of this system.

(b) For what range of values of a is the system stable?

(c) Find the impulse response of the system.

(d) Is the system a finite impulse response (FIR) or infinite impulse response (IIR) filter?

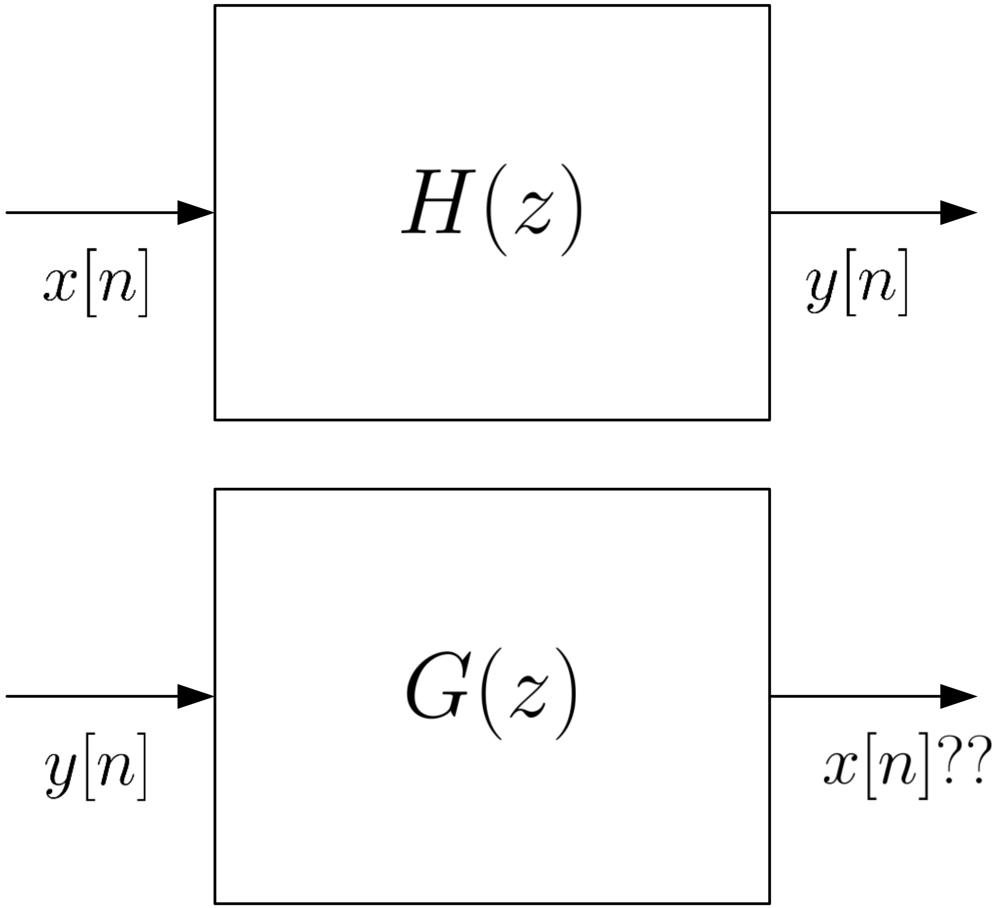
(e) Assume $|a|<1$. Show that the system is an all-pass system, i.e., the magnitude of the frequency response is a constant. Also, specify the value of this constant.

3. Consider a discrete-time signal $x[n]$ is passed through a system with transfer function $H(z)$ to produce an output $y[n]$.

Given $y[n]$, is it possible to get back $x[n]$?

This is referred to as an equalization or deconvolution problem which arises in many applications such as communications. For example, the transmitter sends out information $x[n]$. After passing through the transmission channel (telephone line, air, etc.), the receiver obtains $y[n]$ which is a filtered version of $x[n]$.

(Hint: consider a simple case of $H(z) = 1 - az^{-1}$)



Can we get back $x[n]$?

If yes, what is $G(z)$?

Under what conditions?