A Comparative Study of Using Random Switching Schemes for DC/DC Converters

K. K. Tse, Student Member, IEEE, Henry S. H. Chung, Member, IEEE, S. Y. R. Hui, Senior Member, IEEE, and H. C. So, Member, IEEE

Department of Electronic Engineering
City University of Hong Kong
Tat Chee Avenue, Kowloon
Hong Kong

Abstract – This paper addresses a comparative investigation on the use of random modulation schemes for dc/dc converters. The modulation schemes under the investigation include randomized pulse position modulation, randomized pulse width modulation, and randomized carrier frequency modulation with fixed and variable duty cycle. The study emphasizes the suitability and applicability of each scheme in dc/dc converters, the randomness level on the effectiveness of spreading the dominating frequencies in constant frequency pulse width modulation scheme, and the low-frequency power spectral density (PSD) of each scheme. Validity of analyzes is confirmed experimentally by using a dc/dc buck converter, where the PSD of the output for each scheme will be presented.

I. INTRODUCTION

With the introduction of the international electromagnetic compatibility (EMC) directive, there is an increasing awareness of EMC issues that highlights the electromagnetic interference (EMI) problem of switched-mode power supplies (SMPS) [1, 2]. The control of SMPS is generally associated with the use of pulse-width-modulation (PWM) technique. During the last decade random-switching technique, which is originated from statistical communication theory [3], is applied to power electronics converters. Certain parameters, which are named as stochastic variables, in the PWM modulator are subject to randomization. The methodology is not merely a way to comply with the EMC regulations, but also provides a flexible and practical approach to solving acoustic noise problems in inverter-based motor drive [4].

In dc/dc converters, the output voltage is generally controlled by varying the duty cycle of the main switch. The duty ratio and switching frequency are kept constant in steady state operation, so that the harmonic powers of the input current and output voltage are concentrated on the multiples of the switching frequency. With random switching scheme the harmonic power concentration in the frequency domain is relaxed, the peak level of the power spectral density (PSD) becomes less than that of classical PWM. Discrete harmonics are significantly reduced and the harmonic power is spread over as continuous noise spectrum of insignificant magnitude [5]. Spreading of harmonic energy present at one frequency across other frequencies may be carried out in various ways. Many randomization schemes and their synthesis have been addressed in many articles [5-7]. They can be categorized into four modulation schemes, including random-pulse-position-modulation (RPPM), random-pulse-width-modulation (RPWM), and random-carrier-frequency-modulation with fixed-duty-cycle (RCFMFD) and with variable-duty-cycle (RCFMVD), respectively. However, the suitability and applicability of the use of random modulation schemes in dc/dc converters have not been addressed. It will be shown that random switching schemes introduce undesirable continuous noise spectrum within the pass-band of the low-pass filter in dc/dc converters, which require tight output voltage regulation.

This paper presents a comparative investigation on the use of random modulation schemes in dc/dc converters. Several aspects will be discussed, including the effects of the stochastic variable randomness level on the PSD of the above schemes, low-frequency characterizations, and practicability of implementation. They are quantitatively investigated with uniform probability density distribution on the stochastic variable. The theoretical predictions are compared to experimental measurements by using a dc/dc buck converter.

II. RANDOM SWITCHING SCHEMES

A. Characteristics of random modulation schemes

RPPM is similar to the classical PWM scheme with constant switching frequency. However, the position of the gate pulse is randomized within each switching period, instead of commencing at the start of each cycle. RPWM allows the pulse width to vary, but the average pulse width is equal to the required duty cycle. RCFMFD exhibits randomized switching period and constant duty cycle, while RCFMVD exhibits randomized switching period and constant pulse width. With the aid of Fig. 1, the characteristics of the pulse $g(t)$ in each scheme are summarized in Table I. $g(t)$ has two discrete levels of $A_1$ and $A_2$, which are applicable to study the behaviors of classical dc/dc converters. $T_k$ is the duration of the $k$th cycle. $\alpha_k$ is the duration of the gate pulse in the $k$th cycle. $\varepsilon_k$ is the delay time of the gate pulse. $d_k$, which is equal to $\alpha_k / T_k$, is the duty cycle of the switch in the $k$th cycle.
\[ g(t) \]

![Switching signal with randomized modulation scheme.](image)

**TABLE I CHARACTERISTICS OF DIFFERENT RANDOM SWITCHING SCHEMES**

<table>
<thead>
<tr>
<th>Switching schemes</th>
<th>( T_i )</th>
<th>( \alpha_i )</th>
<th>( \varepsilon_i )</th>
<th>( d_i = \alpha_i / T_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard PWM</td>
<td>F</td>
<td>F</td>
<td>Z</td>
<td>F</td>
</tr>
<tr>
<td>RPMM</td>
<td>F</td>
<td>F</td>
<td>R</td>
<td>F</td>
</tr>
<tr>
<td>RPMM</td>
<td>F</td>
<td>R</td>
<td>Z</td>
<td>R</td>
</tr>
<tr>
<td>RCFMFD</td>
<td>R</td>
<td>R</td>
<td>Z</td>
<td>F</td>
</tr>
<tr>
<td>RCFMVD</td>
<td>R</td>
<td>F</td>
<td>Z</td>
<td>R</td>
</tr>
</tbody>
</table>

'F' - Fixed. 'R' - Randomized. 'Z' - Zero.

**B. Definitions of the randomness level**

In order to investigate the effectiveness of the stochastic variable randomness level on spreading harmonic power, a randomness level \( \mathcal{R} \) for each scheme is defined.

For RPMM,

\[ \mathcal{R}_{RPMM} = \frac{\varepsilon_2 - \varepsilon_1}{T_S} \]  

(1)

where \( \varepsilon_1, \varepsilon_2 \) are the minimum and maximum limits of the pulse position in each cycle. \( \varepsilon_1 \) is generally equal to zero. \( T_S \) is the nominal switching period.

For RPWM,

\[ \mathcal{R}_{RPWM} = \frac{\alpha_2 - \alpha_1}{T_S} = \frac{d_2 - d_1}{T_S} \]  

(2)

where \( \alpha_1, \alpha_2 \) are the minimum and maximum possible values of the duty cycle in classical PWM scheme.

For RCFMFD and RCFMVD,

\[ \mathcal{R}_{RCFMFD} = \mathcal{R}_{RCFMVD} = \frac{T_2 - T_1}{T_S} \]  

(3)

In these two modulation schemes, \( T_1 \) varies between a minimum possible value \( T_1 \) and maximum possible value \( T_2 \).

**C. Power Spectral Density**

The PSD \( S_p(f, \mathcal{R}) \) of the waveform in Fig. 1 with RPPM and RPWM can be shown to be equal to

\[
S_p(f, \mathcal{R}) = f_s \left[ E\left[ |G(f)|^2 \right] - |E[G(f)]|^2 \right] + f_s |E[G(f)]|^2 \sum_{k=0}^{\infty} \delta(f-kf_s).
\]  

(4)

where \( f_s \) is the nominal switching frequency and \( G(f) \) is the Fourier transform of a cycle of \( g(t) \) with randomness level \( \mathcal{R} \). \( E[\cdot] \) is an expectation operator taking over the whole ensemble. For RPMM,

\[
E[|G(f)|^2] = \frac{2}{(2\pi f)^2} \left[ (A_1 - A_2)^2 \left[ 1 - \cos(2\pi f \alpha) \right] + A_2^2 \left[ 1 - \cos(2\pi f T_S) \right] + A_2 (A_1 - A_2) \left[ \sin(2\pi f DT_S) - \sin(2\pi f (\alpha + \mathcal{R}_{RPWM} T_S)) \right] + \sin(2\pi f \mathcal{R}_{RPWM} T_S) - 1 \right] + \sin(2\pi f (1 - \mathcal{R}_{RPWM}) T_S) \left[ \sin(2\pi f T_S) - \sin(2\pi f (1 - D) T_S) \right],
\]  

(5)

For RPWM,

\[
E[|G(f)|^2] = \frac{1}{(2\pi f)^2} \left[ \frac{A_1 - A_2}{2\pi f \mathcal{R}_{RPWM} T_S} \left( e^{-j2\pi f DT_S} - 1 \right) \right] + j A_2 (e^{-j2\pi f T_S} - 1).
\]  

(6)

For RPWM,

\[
E[|G(f)|^2] = \frac{2}{(2\pi f)^2} \left[ A_1^2 + A_2^2 - A_1 A_2 \cos(2\pi f T_S) + A_2 (\sin(2\pi f DT_S) - \sin(2\pi f D T_S)) + A_2 (\sin(2\pi f (1 - D) T_S) - \sin(2\pi f (1 - D) T_S)) \right],
\]  

(7)

For RPWM,

\[
E[|G(f)|^2] = \frac{1}{(2\pi f)^2} \left[ \frac{A_2 - A_1}{2\pi f \mathcal{R}_{RPWM} T_S} \left( e^{-j2\pi f DT_S} - 1 \right) \right] + j A_2 (e^{-j2\pi f T_S} - A_1).
\]  

(8)

Thus, the PSD of RPPM and RPWM with randomness level \( \mathcal{R}_{RPMM} \) and \( \mathcal{R}_{RPWM} \) can be obtained by substituting (5) and (6), and (7) and (8) into (4), respectively.

For RCFMVD and RCFMFD, the PSD can be shown to be equal to
\[ S_p(f, \mathcal{R}_{RCFM}) = \frac{1}{E[T_s]} [E[|G(f)|^2]] + 2 \text{Re} \left\{ \frac{E[G(f)e^{j2\pi f T_s}]}{1 - E[e^{j2\pi f T_s}]} \right\} \]

where \( G^*(f) \) is the complex conjugate of \( G(f) \), and the expected terms are expressed as follows.

For RCFMFD,

\[ E[|G(f)|^2] = \frac{2}{(2\pi f)^3 \mathcal{R}_{RCFMFD} T_s} \{(A_2 - A_1) \]
\[ \frac{A_1}{D} \left[ \sin(2\pi f DT_2) - 2 \sin(2\pi f DT_1) \right] - \frac{A_2}{1 - D} \left[ \sin(2\pi f (1-D) T_2) - \sin(2\pi f (1-D) T_1) \right] \]
\[ - A_1 A_2 \sin(2\pi f T_2) - \sin(2\pi f T_1) + 2\pi f \mathcal{R}_{RCFMFD} T_s (A_1^2 + A_2^2 - A_1 A_2) \}
\]

\[ E[G(f)e^{j2\pi f T_s}] = \frac{1}{(2\pi f)^2 \mathcal{R}_{RCFMFD} T_s} \{ \frac{A_1 - A_2}{1 - D} [e^{j2\pi f (1-D) T_1} - e^{j2\pi f (1-D) T_2}] - A_1 [e^{j2\pi f T_2} - e^{j2\pi f T_1}] \}
\]

Thus, the PSD of RCFMFD and RCFMVD with randomness level \( \mathcal{R}_{RCFMFD} \) and \( \mathcal{R}_{RCFMVD} \) can be obtained by substituting (10) - (13), and (14) - (17) into (9), respectively.

Comparing (4) and (9), the PSD of RPPM and RPWM contains discrete harmonics at the multiples of \( f_s \), while the one in RCFMFD and RCFMVD shows continuous spectrum.

D. Estimation of output noise ripple

A dc/dc converter is considered as a low-pass filter supplying from different type of input sources, which are dependent on the circuit configuration. With the aid of Table II, the most common type of transformed dc/dc converters is illustrated in Fig. 2. For the sake of simplicity, converters operating in continuous conduction mode and relatively constant inductor current are considered.
TABLE II TRANSFER CHARACTERISTICS OF VARIOUS DC/DC CONVERTERS

<table>
<thead>
<tr>
<th>Converters</th>
<th>$H(f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>$v_{out}(f) = \frac{1 + 2\pi f \tau_c C}{LC \left[ 1 + 2\pi f \left( \frac{\tau_c}{R} - \frac{\tau_c}{C} \right) R \right]}$</td>
</tr>
<tr>
<td>Boost</td>
<td>$\frac{v_{out}}{i_D}(f) = \frac{R(1 + j2\pi f \tau_c C)}{1 + j2\pi f (\tau_c + R) C}$</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>$\frac{v_{out}}{i_D}(f) = \frac{R(1 + j2\pi f \tau_c C)}{1 + j2\pi f (\tau_c + R) C}$</td>
</tr>
</tbody>
</table>

Since the $S_p(f, \mathcal{R})$ of RPPM and RPWM consists of continuous spectral component $[S_p^c(f, \mathcal{R})]$ and discrete spectral components $[S_p^d(f, \mathcal{R})]$, they can be expressed as,

$$S_p^c(f, \mathcal{R}) = f_s \left( E \left[ |G(f, \mathcal{R})|^2 \right] - \frac{1}{2} \left| E \left[ G(f, \mathcal{R}) \right]^2 \right| \right)$$  

$$S_p^d(f, \mathcal{R}) = f_s^2 \frac{1}{2} \left| E \left[ G(f, \mathcal{R}) \right]^2 \right|.$$  

$\tilde{N}_o$, can be shown to be

$$\tilde{N}_o = \left( \frac{4f_s}{N} \sum_{k=1}^{N} S_p^c \left( \frac{2k f_s}{N}, \mathcal{R} \right) \right) \left[ H \left( \frac{2k f_s}{N} \right) \right]^{\frac{1}{2}}$$  

$$+ S_p^d \left( f_s, \mathcal{R} \right) \left[ H(f_s) \right]^{\frac{1}{2}}$$  

$N$ is number of frequency points over the range of $[0, 2f_s]$. For RC-FMFD and RCFMVD, only continuous component exists over the spectrum. Thus,

$$\tilde{N}_o = \left( \frac{4f_s}{N} \sum_{k=1}^{N} S_p^c \left( \frac{2k f_s}{N}, \mathcal{R} \right) \right) \left[ H \left( \frac{2k f_s}{N} \right) \right]^{\frac{1}{2}}$$  

\[24\]

III. EXPERIMENTAL VERIFICATIONS

A dc/dc buck converter is illustrated with a test setup in Fig. 3. The component value is $L = 1$ mH, $r_L = 0.25\Omega$, $C = 220$ µF, $r_C = 0.5\Omega$, and $R = 10\Omega$. The input voltage $V_{in}$ is 60V. The nominal switching frequency $f_s$ is 20kHz and duty cycle is 0.5. The voltage waveform of $D$ (i.e., $v_D$) is similar to the pulse train in Fig. 1 with $A_1 = V_{in}$ and $A_2 = 0$.  

The $H(f)$ of the output $LC$ filter is

$$H(f) = \frac{1 + 2\pi f \tau_c C}{LC \left[ 1 + \frac{\tau_c}{R} \right] \left( 1 + \frac{\tau_c}{R} \right) - \left( 2\pi f \right)^2 + \frac{1}{LC} \left( 1 + \frac{\tau_c}{R} \right)}$$  

\[25\]
Fig. 4 shows the variations of the PSD of $v_o$ with respect to the changes in $\Re$ under the four random switching schemes. $\Re$ is changed from zero to 0.2. When $\Re$ equals zero, the PSD is same as the PSD of classical PWM scheme. In general, the harmonic spectrum is gradually spread over as $\Re$ increases. Nevertheless, the PSD of the RPPM and RPWM schemes consists of two components, including a discrete component at the multiples of $f_s$ and a continuous component over the frequencies. The PSD of the RCFMFD and RCFMVD only contain continuous spectrum only, which is due to the fact that the switching frequency is randomized. Fig. 5 and 6 show the theoretical and experimental results of the PSD under the four schemes with $\Re = 0.2$. The measurements are obtained from a signal analyzer HP89410A with resolution bandwidth of 100Hz. There is an overall 30dB difference between the theoretical and experimental results. It is mainly due to the reason that the analyzer gives the comparative results of the actual spectral power with 1mW, as reference. The unit of the theoretical spectral power is in dB, while the measured one is in dBm. The relationship between these two units is demonstrated as follows. For instance, if the actual spectral power is $P$,

$$
P_{\text{dBm}} = 10 \log \frac{P}{0.001} = 10 \log P + 30 \text{dB} = P_{\text{dB}} + 30 \text{dB} \quad (26)$$

With the consideration of (26), the theoretical results show close agreement with the experimental ones. The low-frequency characterizations of $v_o$ within the pass-band of the LC filter under the four schemes are shown in Fig. 7. It can be observed that RCFMFD introduces the lowest low-frequency harmonic spectrum. It integrates the advantages of spreading harmonic power at the multiples of the switching frequency and not introducing significant disturbances at the low-frequency range. It is because the duty cycle in every switching cycle is constant even if the switching frequency is varied, providing a relatively stable output voltage. This factor can be observed from the behaviors of RPWM and RCFMVD schemes, where they introduce low-frequency variations because of the varying duty cycle. Although the average duty cycle in RPPM is constant, the actual ratio between the on time and off time of $S$ in every switching cycle is randomized. Thus, the low-frequency harmonic components are higher than that of RCFMFD. From the above observations, RCFMFD facilitates the usage in dc-dc converters, which require tight regulation.
Fig. 4 Theoretical PSD of the diode voltage with respect to the change in $\mathcal{R}$ from 0 to 0.2.

Fig. 5 Theoretical PSD of the diode voltage with $\mathcal{R} = 0.2$. 
Fig. 7 Low-frequency characterizations with different random switching scheme.

IV. CONCLUSIONS

This paper gives a comparative investigation on the effects of the randomness level on the PSD of four random modulation schemes that are applied to dc/dc converters. All schemes can gradually spread the discrete frequency component in classical PWM scheme to a continuous frequency spectrum. However, the continuous noise spectrum within the pass-band of the low-pass filter results in a noise-induced low-frequency ripple in the converter output voltage. Among all schemes, the RCFMFD gives the lowest low-frequency harmonic spectrum, which is considered as the best choice in applying to dc/dc converter.

REFERENCES