On Adaptive Cancellation of Multiple Interfering Sinusoids

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September 7, 1998

Indexing terms : adaptive signal processing, interference suppression

Abstract : The adaptive sinusoidal interference canceller (ASIC) is a least-mean-square (LMS)-style method for eliminating a sine wave of known frequency from an observed signal. In this Letter, the ASIC is generalized to cancel multiple interfering sinusoids. Performance of the algorithm, particularly when the interference frequencies are not exactly known, is analyzed. It is also shown that the ASIC can provide significant signal-to-noise ratio (SNR) improvement.

Introduction : Elimination of sinusoidal interferences from a received time series has found applications in many areas such as communication, control and biomedical engineering [1]-[2]. A typical example is to remove unwanted 50 or 60 Hz interference due to magnetic induction and displacement current from power line in electrocardiography [2]. Based on adaptive noise canceling approach [3], the ASIC [4] has been proposed recently for eliminating a single pure sinusoid, assuming that its frequency is given. The idea is to use a synthetic tone, which is a function of the explicit amplitude and phase measurements provided by an LMS-style algorithm, to remove the interfering signal. Basically, this Letter is a follow-up of [4] that generalizes the ASIC so that it can be applied to situations when there are multiple interfering sinusoids, such as harmonic noise cancellation in power line communications [1]. Performance measures of the algorithm, viz convergence behaviors and mean square errors of the system parameters as well as SNR improvement ratio, are analyzed. It is shown that the method can cancel out the interfering sinusoids satisfactorily even if their frequencies are not exactly known, for example, when there is frequency drifting in transmission power lines [5].

The Generalized ASIC : In the presence of sinusoidal interferences with frequencies which are not exactly known, the received signal, $r(kT_s)$, is expressed as

$$r(kT_s) = s(kT_s) + \sum_{i=1}^M \alpha_i \cos((\omega_i + \Delta\omega_i)kT_s + \phi_i) \quad (1)$$

where $s(kT_s)$ is the stationary source signal, T_s is the sampling period, and $(\omega_i + \Delta\omega_i)$, α_i and ϕ_i , $i = 1, 2, \dots, M$, represent the frequencies, amplitudes and phases of the interfering sinusoids, respectively. It is assumed that M and ω_i are known while $\Delta\omega_i$, α_i and ϕ_i are unknown constants with $|\Delta\omega_i/\omega_i| < 1$ for $i = 1, 2, \dots, M$. The task is to estimate $s(kT_s)$ from the corrupted signal $r(kT_s)$. For simplicity, we drop T_s in the following analysis.

By extending the ASIC to multiple sinusoidal interference cancellation, we construct the recovered signal, $\hat{s}(k)$, which is of the form

$$\hat{s}(k) = r(k) - \sum_{i=1}^M \hat{\alpha}_i(k) \cos(\omega_i k + \hat{\phi}_i(k)) \quad (2)$$

where $\hat{\alpha}_i(k)$ and $\hat{\phi}_i(k)$, $i = 1, 2, \dots, M$, are the amplitude and phase parameters, respectively. It is observed from (2) that $s(k)$ can be extracted perfectly when $\hat{\alpha}_i(k) = \alpha_i$ and $\hat{\phi}_i(k) = \phi_i + \Delta\omega_i k$. These desired values of $\hat{\alpha}_i(k)$ and $\hat{\phi}_i(k)$ can be acquired by minimizing the mean square value of $\hat{s}(k)$, $E\{\hat{s}^2(k)\}$, which is derived as

$$E\{\hat{s}^2(k)\} = \sigma_s^2 + \frac{1}{2} \sum_{i=1}^M (\hat{\alpha}_i^2(k) + \alpha_i^2) - \sum_{i=1}^M \hat{\alpha}_i(k) \alpha_i \cos(\hat{\phi}_i(k) - \phi_i - \Delta\omega_i k) \quad (3)$$

where σ_s^2 denotes the power of $s(k)$. In the generalized ASIC, $\hat{\alpha}_i(k)$ and $\hat{\phi}_i(k)$ are adapted on a sample-by-sample basis to minimize $E\{\hat{s}^2(k)\}$ according to the LMS-style algorithm [4]

$$\hat{\alpha}_i(k+1) = \hat{\alpha}_i(k) + \mu_{\alpha_i} \hat{s}(k) \cos(\omega_i k + \hat{\phi}_i(k)) \quad (4)$$

and

$$\hat{\phi}_i(k+1) = \hat{\phi}_i(k) - \mu_{\phi_i} \hat{s}(k) \sin(\omega_i k + \hat{\phi}_i(k)) \quad (5)$$

The quantities μ_{α_i} and μ_{ϕ_i} , $i = 1, 2, \dots, M$, are positive scalars that control convergence rate and ensure system stability of the algorithm. To reduce computation, values of the sine and cosine functions are retrieved from a pre-stored table [4]. For each sampling interval, $5M$ multiplications, $3M$ additions and $2M$ look-up operations are required to update the parameter estimates.

By taking the expected value of (5), we obtain the learning characteristics of $\hat{\phi}_i(k)$ as

$$E\{\hat{\phi}_i(k)\} = \phi_i + \Delta\omega_i k - \frac{2\Delta\omega_i}{\mu_{\phi_i} \alpha_i} + \left(\hat{\phi}_i(0) - \phi_i + \frac{2\Delta\omega_i}{\mu_{\phi_i} \alpha_i} \right) \left(1 - \frac{\mu_{\phi_i} \alpha_i}{2} \right)^k \quad (6)$$

where $\hat{\phi}_i(0)$, $i = 1, 2, \dots, M$, represent the initial values of $\hat{\phi}_i(k)$. Provided that $4/\alpha_i > \mu_{\phi_i} > 0$, each $\hat{\phi}_i(k)$ will converge to $\phi_i + \Delta\omega_i k - 2\Delta\omega_i/(\mu_{\phi_i}\alpha_i)$ with a time constant of $2/(\mu_{\phi_i}\alpha_i)$. When the transient behaviors of $\hat{\phi}_i(k)$ are ignored and the condition $4 > \mu_{\alpha_i} > 0$ is satisfied for each i , the learning trajectory of $\hat{\alpha}_i(k)$ can be calculated as

$$E\{\hat{\alpha}_i(k)\} = \alpha_i \cos\left(\frac{2\Delta\omega_i}{\mu_{\phi_i}\alpha_i}\right) + \left(\hat{\alpha}_i(0) - \alpha_i \cos\left(\frac{2\Delta\omega_i}{\mu_{\phi_i}\alpha_i}\right)\right) \left(1 - \frac{\mu_{\alpha_i}}{2}\right)^k \quad (7)$$

where $\hat{\alpha}_i(0)$, $i = 1, 2, \dots, M$, are the initial estimates of α_i . As $k \rightarrow \infty$, $E\{\hat{\alpha}_i(k)\} \rightarrow \alpha_i \cos(2\Delta\omega_i/(\mu_{\phi_i}\alpha_i))$. From (6) and (7), it is seen that for $\Delta\omega_i \neq 0$, $\hat{\phi}_i(k)$ and $\hat{\alpha}_i(k)$ will not converge exactly to the desired values of $\phi_i + \Delta\omega_i k$ and α_i , but with biases of $2\Delta\omega_i/(\mu_{\phi_i}\alpha_i)$ and $\alpha_i(1 - \cos(2\Delta\omega_i/(\mu_{\phi_i}\alpha_i)))$, respectively. Nevertheless, the biases can be significantly reduced when μ_{ϕ_i} are chosen much larger than $\Delta\omega_i/\alpha_i$. It is noteworthy that when $\Delta\omega_i = 0$, the ASIC provides unbiased estimates of ϕ_i and α_i . The mean square errors of $\hat{\alpha}_i(k)$ and $\hat{\phi}_i(k)$, denoted by $\text{mse}(\hat{\alpha}_i)$ and $\text{mse}(\hat{\phi}_i)$, $i = 1, 2, \dots, M$, can be shown to be

$$\text{mse}(\hat{\alpha}_i) \triangleq \lim_{k \rightarrow \infty} E\{(\hat{\alpha}_i(k) - \alpha_i)^2\} \approx \frac{\mu_{\alpha_i}\sigma_s^2}{2} + \alpha_i^2 \left(1 - \cos\left(\frac{2\Delta\omega_i}{\mu_{\phi_i}\alpha_i}\right)\right)^2 \quad (8)$$

and

$$\text{mse}(\hat{\phi}_i) \triangleq \lim_{k \rightarrow \infty} E\{(\hat{\phi}_i(k) - \phi_i - \Delta\omega_i k)^2\} \approx \frac{\mu_{\phi_i}\sigma_s^2}{2\alpha_i} + \frac{4\Delta\omega_i^2}{\mu_{\phi_i}^2\alpha_i^2} \quad (9)$$

Another important performance of the ASIC is its SNR improvement ratio, denoted by SNR_{ir} . It is defined as the ratio of the SNR at the restored signal, $\hat{s}(k)$, to the SNR at the received signal, $s(k)$, and it is given by

$$\text{SNR}_{ir} \approx \frac{\sum_{i=1}^M \text{mse}(\hat{\alpha}_i) + \sum_{i=1}^M \alpha_i^2 \cos^2\left(\frac{2\Delta\omega_i}{\mu_{\phi_i}\alpha_i}\right) \text{mse}(\hat{\phi}_i)}{\sum_{i=1}^M \alpha_i^2} \quad (10)$$

Simulation Results : Computer simulations were carried out to verify the performance of the ASIC algorithm for eliminating sinusoidal interferences. The sampling interval T_s was 5×10^{-4} second while all initial values of the amplitude and phase parameters were arbitrarily set to zero. The number of interfering sinusoids was 2 with frequencies $\omega_1 = 100\pi$ rad/s and $\omega_2 = 200\pi$ rad/s. The unknown amplitudes and phases were given by $\alpha_1 = 20$, $\alpha_2 = 15$, $\phi_1 = 1.0$ and $\phi_2 = 0.7$, while $\Delta\omega_i/\omega_i = 0.05\%$. The source signal was a zero-mean white Gaussian process with $\sigma_s^2 = 1$. The step sizes μ_{α_i} were set to 2.0×10^{-2} for all i while $\mu_{\phi_1} = 1.0 \times 10^{-3}$ and $\mu_{\phi_2} = 1.2 \times 10^{-3}$. All simulation results provided were averages of 200 independent runs. Figure 1 shows the learning characteristics of $\hat{\alpha}_i(k)$ and $\hat{\phi}_i(k)$, $i = 1, 2$, and we see that all parameters converged at approximately the 400 iteration. The trajectories of the phase parameters agreed well with the theoretical derivation as given by (6) and the steady-state phase lags between the desired functions $\phi_i + \Delta\omega_i k$ were close to the predicted values of $2\Delta\omega_i/(\mu_{\phi_i}\alpha_i)$. While $\hat{\alpha}_1(k)$ and $\hat{\alpha}_2(k)$ reached the optimum values of 20 and 15 upon convergence and their learning trajectories converged at slower rates than the predicted curves because of inaccurate phase estimates during transients. The mean square errors of the parameters and the SNR improvement ratio are tabulated together with the case of $\Delta\omega_i = 0$ in Table 1. It can be seen that the measured values of these performance measures confirmed with those derived from (8)-(10), and the ASIC reduced the sinusoidal noises significantly in both cases.

Conclusions : The ASIC, which is an LMS-style adaptive algorithm, has been generalized for multiple sinusoidal interference cancellation. Theoretical analysis of the method is derived and confirmed by computer simulations. It is demonstrated that the ASIC can effectively remove interfering sinusoids and provide large SNR improvement ratios, even when the interference frequencies are not exactly known.

References

- [1] CLARK, D : 'Powerline communications: finally ready for prime time,' *IEEE Internet Computing*, 1998, **2**, (1), pp.10-11
- [2] PEI, S.C. and TSENG, C.C. : 'IIR multiple notch filter design based on allpass filter,' *IEEE Trans. Circuits Syst., II: Analog and Digital Signal Processing*, 1997, **44**, (2), pp.133-136
- [3] WIDROW, B. et al. : 'Adaptive noise cancelling: principles and applications,' *Proc. IEEE*, 1975, **63**, (12), pp.1692-1716
- [4] SO, H.C. : 'Adaptive algorithm for sinusoidal interference cancellation,' *Electron. Lett.*, 1997, **33** pp.1910-1912
- [5] O'NEAL, J.B. : 'Overview of power distribution line carrier communications systems,' *Proc. IEEE Global Telecommun.*, 1983, pp.478-482

Performance Measure	$\Delta\omega_i = 5 \times 10^{-4}\omega_i$		$\Delta\omega_i = 0$	
	Theoretical	Measured	Theoretical	Measured
$\text{mse}(\hat{\alpha}_1)$	1.00×10^{-2}	1.20×10^{-2}	1.00×10^{-2}	1.01×10^{-2}
$\text{mse}(\hat{\phi}_1)$	8.67×10^{-5}	8.93×10^{-5}	2.50×10^{-5}	2.48×10^{-5}
$\text{mse}(\hat{\alpha}_2)$	1.00×10^{-2}	1.20×10^{-2}	1.00×10^{-2}	1.04×10^{-2}
$\text{mse}(\hat{\phi}_2)$	3.45×10^{-4}	3.52×10^{-4}	4.00×10^{-5}	3.94×10^{-5}
SNR_{ir}	36.75 dB	35.03 dB	42.05 dB	41.98 dB

Table 1: Performance measures of the ASIC for $M = 2$

(a)

(b)

Figure 1 Learning characteristics of (a) amplitude parameters and (b) phase parameters