# Comparison of DDE and ETDGE for Time-Varying Delay Estimation

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*Abstract* : The performances of DDE and ETDGE, which are two recently proposed methods for direct estimation of time delay between signals received at two spatially separated sensors, are compared. Although both algorithms are computationally efficient, it is shown that the ETDGE generally outperforms the DDE for tracking nonstationary delays with different source signals.

*Introduction* : The problem of estimating and tracking the time delay between signals received at two spatially separated sensors arises in many application fields such as sonar, radar and seismology [1]. Let the two sensor outputs be represented by

$$\begin{aligned} x(k) &= s(k) + n_1(k) \\ y(k) &= s(k-D) + n_2(k) \end{aligned} \tag{1}$$

where $s(k)$ is the unknown source signal, $n_1(k)$ and $n_2(k)$ are the uncorrelated white Gaussian noises which are statistically independent of $s(k)$, and $D$ is the differential delay to be determined. Without loss of generality, it is assumed that the signal and noise spectra are bandlimited between $-0.5$ Hz and 0.5 Hz while the sampling period is 1 second.

Based on the property that a time-shifted version of a bandlimited signal can be expressed as the convolution of a sinc function and the signal itself, Chan *et al.* [2] have introduced a parameter estimation approach to model the time delay as an FIR filter, $W(z) = \sum_{i=-P}^{P} w_i z^{-i}$, in one of the receiver channels. Once the filter coefficients are estimated, the time difference of arrival is found by interpolating their values. For time-varying delay estimation, this approach can be made adaptive by adjusting the filter weights according to Widrow's least mean square (LMS) algorithm [3]. Recently, two LMS-style algorithms, the direct delay estimator (DDE) [4] and the explicit time delay and gain estimator (ETDGE) [5], have been proposed to provide direct delay measurements and their computational complexities are much less than that of [3] because they do not involve interpolation of filter weights. Basically, the DDE uses the largest filter weight and one of its adjacent coefficients to compute the delay estimate while the filter coefficients are expressed as a function of the delay estimate and a gain factor in the ETDGE. In this Letter, we will compare the DDE and ETDGE in terms of delay convergence rates and variances as well as computational requirement.

*Performances of DDE and ETDGE*: In the DDE, the filter weights $\{w_i(k)\}$, $i = -P, -P+1, \cdots, P$, are adapted iteratively to minimize the mean square output error $E\{e^2(k)\}$ subject to $\sum_{i=-P}^{P} w_i^2(k) = 1$ as follows [4],

$$w_i(k+1) = \left(1 + \mu_d \hat{\sigma}_n^2(k)\right) w_i(k) + \mu_d e(k) x(k-i) \tag{2}$$

where $e(k) = y(k) - \sum_{i=-P}^{P} w_i(k)x(k-i)$ and $\hat{\sigma}_n^2(k) = \beta\hat{\sigma}_n^2(k-1) + (1-\beta)(x^2(k) - y(k)\sum_{i=-P}^{P} w_i(k)x(k-i))$. The positive scalar $\mu_d$ controls the convergence rate and stability of the algorithm while $\hat{\sigma}_n^2(k)$ represents the estimate of $\sigma_n^2$ which is the power of $n_1(k)$ or $n_2(k)$ and $\beta \in (0, 1]$ is a smoothing factor. Denote the peak weight at time $k$ by $w_L(k)$, the delay estimate of the DDE, $\hat{D}_d(k)$, is calculated as

$$\hat{D}_d(k) = L + (A-L)\frac{w_A(k)}{w_L(k) + w_A(k)} \tag{3}$$

where $w_A(k) = \max\{w_{L-1}(k), w_{L+1}(k)\}$. Assuming that $s(k)$ is a white process with variance $\sigma_s^2$ and taking expectation on (2), we get

$$E\{w_i(k)\} = \text{sinc}(i-D) + (w_i(0) - \text{sinc}(i-D))(1-\mu_d\sigma_s^2)^k \tag{4}$$

where $\{w_i(0)\}$ are the initial values of the filter weights. Substituting (4) into (3), the learning trajectory of $\hat{D}_d(k)$ can be obtained. For $\mu_d\sigma_s^2 << 1$, the delay variance of the DDE, $\text{var}(\hat{D}_d)$, is given by

$$\text{var}(\hat{D}_d) \approx \frac{\mu_d\sigma_s^2(2(1-2|D_i|+2D_i^2)(1+\text{SNR}) + ((1-|D_i|)^2 w_A^{o2} + D_i^2 w_L^{o2}))}{2(w_L^o + w_A^o)^2\text{SNR}^2} \tag{5}$$

where $D_i = (L-D) \in (-0.5, 0.5)$, $w_i^o = \text{sinc}(i-D)$, $i = A, L$, and $\text{SNR} = \sigma_s^2/\sigma_n^2$. Fig. 1 shows the values of $\text{var}(\hat{D}_d)$ versus $|D_i|$ at different SNRs with $\mu_d\sigma_s^2 = 0.004$. It can be seen that $\text{var}(\hat{D}_d)$ decreases monotonically with increasing $|D_i|$ or SNR. For each sampling interval, $(6P+7)$ additions and $(6P+10)$ multiplications are required for the DDE algorithm.

In the ETDGE, the filter weights $\{w_i(k)\}$ are expressed as $\{\hat{\alpha}(k)\text{sinc}(i-\hat{D}_e(k))\}$ where $\hat{D}_e(k)$ represents the delay estimate while $\hat{\alpha}(k)$ is a variable gain for optimal filtering. The ETDGE algorithm is given by [5]

$$\hat{D}_e(k+1) = \hat{D}_e(k) - \mu_e e(k) \sum_{i=-P}^{P} f(i - \hat{D}_e(k))x(k-i) \tag{6}$$

$$\hat{\alpha}(k+1) = \hat{\alpha}(k) + \mu_e e(k) \sum_{i=-P}^{P} \text{sinc}(i - \hat{D}_e(k))x(k-i) \tag{7}$$

where $e(k) = y(k) - \hat{\alpha}(k)\sum_{i=-P}^{P}\text{sinc}(i-\hat{D}_e(k))x(k-i)$. The function $f$ is defined as $f(v) = (\cos(\pi v) - \text{sinc}(v))/v$ and $\mu_e$ represents the convergence factor for the algorithm. Taking the expected value of (6) yields [5]

$$E\{\hat{D}_e(k)\} = D + (\hat{D}_e(0) - D)(1 - \frac{\mu_e \sigma_s^2 \pi^2}{3})^k \tag{8}$$

where $\hat{D}_e(0)$ denotes the initial delay estimate. Moreover, the delay variance of the ETDGE, $\text{var}(\hat{D}_e)$, is given by [5]

$$\text{var}(\hat{D}_e) \approx \frac{\mu_e \sigma_s^2 (1 + 2\text{SNR})}{2\text{SNR}^2} \tag{9}$$

At every sampling point, $(6P+4)$ additions, $(6P+8)$ multiplications and two table lookup operations are required to compute $\hat{\alpha}(k)$ and $\hat{D}_e(k)$ and thus the computational complexities of the DDE and ETDGE are quite similar. Furthermore, the minimum $E\{e^2(k)\}$ attained by both algorithms are identical.

*Results & Discussions*: Simulation tests had been conducted to compare the performances of the DDE and ETDGE for nonstationary delay estimation. The source signal $s(k)$ had unity power and SNR was set to 5 dB. The parameter $P$ was selected to be 15 in both methods. For the DDE, $\beta = 0.995$, $\hat{\sigma}_n^2(-1) = 0$ and $w_i(0) = \text{sinc}(i)$, $i = -P, -P+1, \cdots, P$, which corresponded to $\hat{D}_d(0) = 0$, were assigned. Whilst the initial values of $\hat{\alpha}(k)$ and

$\hat{D}_e(k)$ were set to 1 and 0, respectively, in the ETDGE. The results provided were averages of 400 independent runs. Fig.2 shows the learning curves of the delay estimates and delay variances of the DDE and ETDGE for step changes in delay when $s(k)$ was a white process. The step sizes, $\mu_d$ and $\mu_e$, were chosen to be $4.00 \times 10^{-3}$ and $1.47 \times 10^{-3}$, respectively, so that $\text{var}(\hat{D}_d) \approx \text{var}(\hat{D}_e)$ at $|D_i| = 0.5$s. It can be seen that both methods had identical convergence speeds and they tracked all these step changes accurately in approximately one thousand iterations. The convergence dynamics of $\hat{D}_d(k)$ also agreed with the predicted trajectory which was calculated using (3) and (4). We also observe that the delay variances of the DDE depended significantly on $D$ and $\text{var}(\hat{D}_d)$ equaled $5.12 \times 10^{-4}s^2$, $1.70 \times 10^{-3}s^2$ and $1.02 \times 10^{-3}s^2$ for delay values of 0.5s, 1.0s and 1.15s, respectively, while $\text{var}(\hat{D}_e)$ was close to the theoretical value of $5.38 \times 10^{-4}s^2$ in all cases. This implies that the DDE is inferior to the ETDGE except when $|D_i| = 0.5$s, they have identical performance. The above test was repeated for $s(k) = 0.5s(k-1) + q(k)$ where $q(k)$ was white and the results are shown in Fig.3. Again, the DDE and ETDGE tracked the time-varying delay correctly in a similar manner but their convergence rates were less than those of Fig.2 and they needed about two thousand iterations to reach the desired delay. The decrease in convergence speeds can be explained by examining (2) and (6) for this autoregressive source signal. Furthermore, we see $\text{var}(\hat{D}_d) \approx \text{var}(\hat{D}_e) = 6.54 \times 10^{-4}s^2$ at $D = 0.5$s but $\text{var}(\hat{D}_d)$ were approximately equal to four and two times the values of $\text{var}(\hat{D}_e)$ for $D = 1.0$s and $D = 1.15$s, respectively.

*Conclusions*: Two adaptive time delay estimators, the DDE and ETDGE, which provide direct delay measurements, have been compared. Although their computational complexities are similar, the ETDGE estimation accuracy is generally better than the DDE, whose delay variance depends on the value of time delay, for different source signals.

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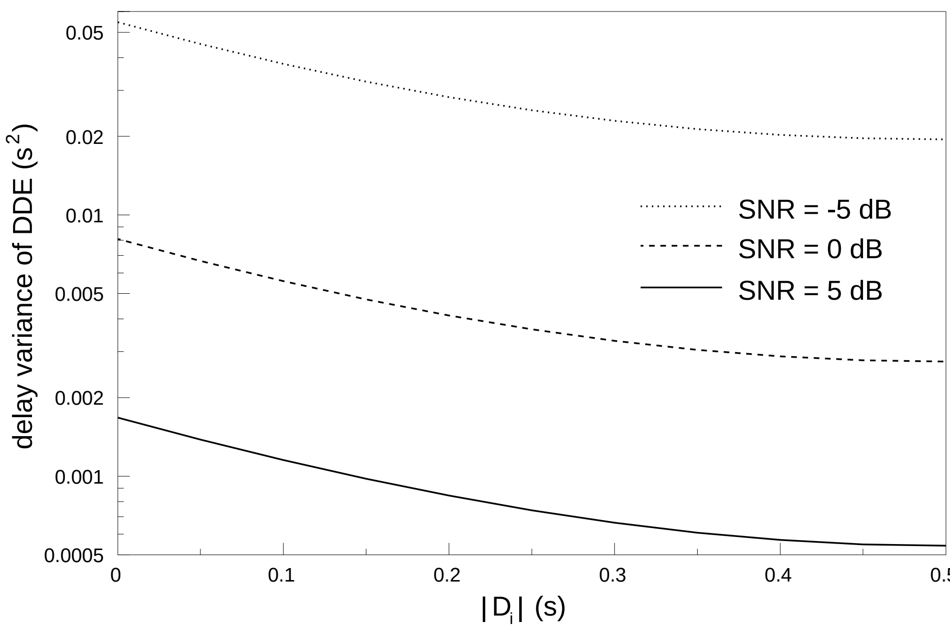


Fig.1 Delay variance of DDE versus $|D_i|$ at different SNRs

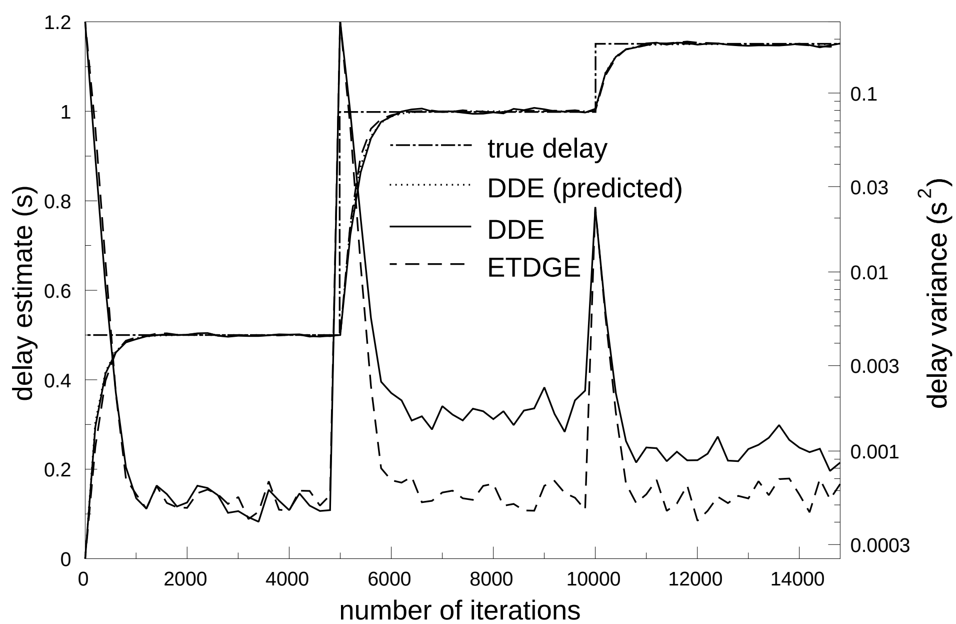


Fig.2 Delay estimates and variances of DDE and ETDGE for white $s(k)$

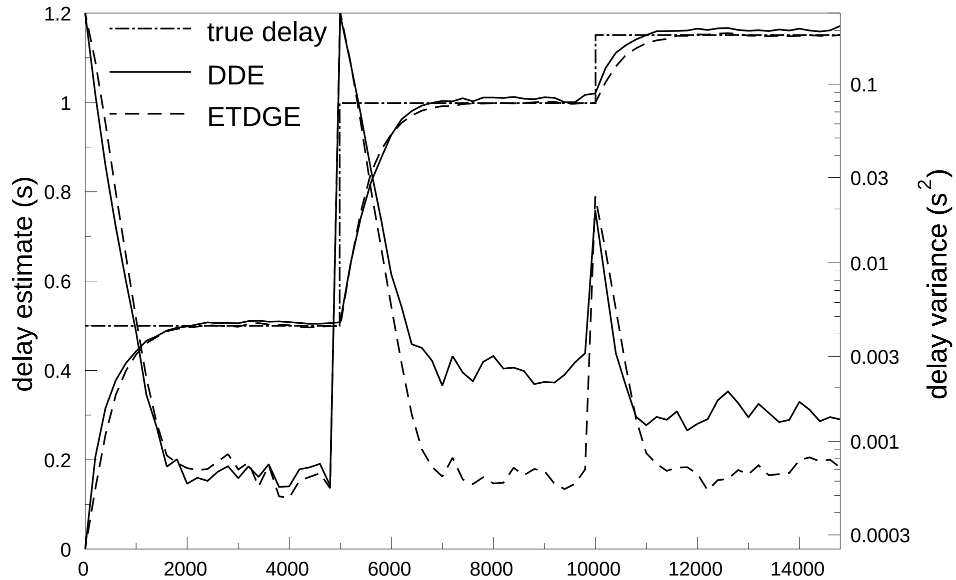


Fig.3 Delay estimates and variances of DDE and ETDGE for nonwhite $s(k)$