Modified LMS Algorithm for Unbiased Impulse Response Estimation in Nonstationary Noise

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Abstract : In the presence of input interference, the Wiener solution for impulse response estimation is biased. In this Letter, it is proved that bias removal can be achieved by proper scaling the optimal filter coefficients and a modified least mean squares (LMS) algorithm is then developed for accurate system identification in noise. Simulation results show that the proposed method outperforms two total least squares (TLS) based adaptive algorithms under nonstationary interference condition.

Introduction : Estimation of the impulse response of an unknown system from its measured input and output has found applications in many fields such as control and signal processing [1]. If measurement noise exists only in the output, the least mean squares (LMS) algorithm [2], which is based on the minimum mean squared error (MMSE) criterion, can provide accurate estimates of the system parameters. However, when both input and output contain additive noise, which is the more realistic case, the LMS solution is biased. The total least squares (TLS) approach [3] seems to be an appropriate choice for this problem because it considers both stimulation error and response error. Recently, several TLS-based computationally efficient algorithms [4]-[5] have been proposed for adaptive impulse response estimation in noise. In this Letter, we first analyze the MMSE estimates for system identification in the presence of input and output interference. It is shown that an unbiased solution can be obtained by weighting the Wiener filter weights with a correction factor and the LMS algorithm is modified for its adaptive realization. Simulation results illustrate that the proposed method gives more accurate impulse response estimates than those of [4] and [5] in the presence of time-varying noise.

Proposed Method : Consider the system identification configuration in Figure 1. It is assumed that the unknown system $H(z)$ is of finite impulse response (FIR) and has the form $H(z) = \sum_{i=0}^{N-1} h_i z^{-i}$ where N is known *a priori*. The signals $s(k)$ and $d(k)$ represent the noise-free input and output of $H(z)$, respectively, while $n_i(k)$ and $n_o(k)$ are the input and output measurement noise. Given the corrupted input and output, viz $x(k)$ and $r(k)$, the aim is to find $H(z)$ using the filter $W(z) = \sum_{i=0}^{N-1} w_i z^{-i}$. For ease of analysis, we assume that $s(k)$, $n_i(k)$ and $n_o(k)$ are independent white processes with variances σ_s^2 , $\sigma_{n_i}^2$ and $\sigma_{n_o}^2$, respectively. For time-invariant filter coefficients w_i , the error function $e(k)$ is

given by

$$e(k) = r(k) - \sum_{i=0}^{N-1} w_i x(k-i) \quad (1)$$

The mean squared value of $e(k)$, $E\{e^2(k)\}$, can be shown to be

$$E\{e^2(k)\} = \sigma_s^2 \sum_{i=0}^{N-1} (h_i - w_i)^2 + \sigma_{n_o}^2 + \sigma_{n_i}^2 \sum_{i=0}^{N-1} w_i^2 \quad (2)$$

Differentiating $E\{e^2(k)\}$ with respect to each w_i and then equating the resultant expressions to zero, we obtain the Wiener filter weights as follows,

$$w_i^o = \alpha h_i, \quad i = 0, 1, \dots, N-1 \quad (3)$$

where

$$\alpha = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{n_i}^2} \quad (4)$$

It is clear that $0 < \alpha \leq 1$ and the Wiener solution equals the unknown impulse response only if $\sigma_{n_i}^2 = 0$. In the presence of input interference, w_i^o are biased estimates of h_i and the bias increases with the power of $n_i(k)$. Notice that the output noise $n_o(k)$ has no effect on the biasedness of the system parameter estimates. We also observe that the desired filter coefficients can be acquired by dividing w_i^o by α if it is known. It is found that the parameter α can be calculated as

$$\alpha = \frac{\sum_{i=0}^{N-1-\tau} w_{i+\tau}^o w_i^o \cdot E\{x^2(k)\}}{E\{r(k)r(k-\tau)\}} \quad (5)$$

where $\tau \in (0, N)$ represents a time-shifted parameter. If $n_o(k) = 0$, we can even choose $\tau = 0$. In practice, τ should be selected to maximize $\sum_{i=0}^{N-1-\tau} w_{i+\tau}^o w_i^o$ in order to minimize the estimation error.

Based on (3)-(5), we propose the following modified LMS algorithm to estimate $H(z)$ adaptively:

$$w_i(k+1) = w_i(k) + \mu e(k)x(k-i), \quad i = 0, 1, \dots, N-1 \quad (6)$$

$$\hat{R}_{xx}(k) = \beta x^2(k) + (1-\beta)\hat{R}_{xx}(k-1) \quad (7)$$

$$\hat{R}_{rr}(k) = \beta r(k)r(k-\tau) + (1-\beta)\hat{R}_{rr}(k-1) \quad (8)$$

$$\hat{w}_i(k) = w_i(k) \cdot \frac{\hat{R}_{rr}(k)}{\sum_{i=0}^{N-1-\tau} w_{i+\tau}(k)w_i(k) \cdot \hat{R}_{xx}(k)}, \quad i = 0, 1, \dots, N-1 \quad (9)$$

The error signal $e(k)$ is computed from (1) where w_i are now replaced by $w_i(k)$. Equation (6) is the standard LMS algorithm and the mean value of each $w_i(k)$ will converge to w_i^o in steady state. The parameters $\hat{R}_{xx}(k)$ and $\hat{R}_{rr}(k)$ represent the estimates of $E\{x^2(k)\}$ and $E\{r(k)r(k-\tau)\}$, respectively, with $0 < \beta < 1$. The modified filter parameters $\hat{w}_i(k)$ are given by (9) where the instantaneous values of $w_i(k)$ are used to estimate $\sum_{i=0}^{N-1-\tau} w_{i+\tau}^o w_i^o$. As k tends to infinity, it is expected that $\hat{w}_i(k) \rightarrow h_i$, $i = 0, 1, \dots, N-1$, in the mean sense. For each iteration, the algorithm requires $(3N+10)$ multiplications and $(2N+2)$ additions and its computational complexity is comparable to [4] and [5].

Simulation Results : Computer simulations had been carried out to evaluate the impulse response estimation performance of the modified LMS algorithm in noise. Comparisons were also made with the TLS-based methods in [4] and [5], denoted by GAS-TLS and FBJ-TLS, respectively. The noise-free input $s(k)$ and the measurement noises, $n_i(k)$ and $n_o(k)$, were independent white Gaussian random variables. The power of $s(k)$ was fixed to be unity and different input and output signal-to-noise ratios, $\text{SNR}_i = \sigma_s^2/\sigma_{n_i}^2$ and $\text{SNR}_o = \sigma_s^2/\sigma_{n_o}^2$, were obtained by proper scaling of the respective noise sequences. The

unknown FIR system was characterized by $\mathbf{h} = [-0.3 \ -0.9 \ 0.8 \ -0.7 \ 0.6]$. All adaptive filter coefficients were set to zero initially except that the initial weight for $r(k)$ in the FBJ-TLS was -1 . The step size of the proposed method was chosen to be 0.001 while $\mu = 0.0008$ was used in the GAS-TLS and FBJ-TLS. By so doing, the convergence speeds of the impulse response estimates of the three algorithms were almost identical. The parameters τ and β were assigned to be 1 and 0.0015, respectively. All simulation results provided were averages of 400 independent runs.

Figure 1 compares the learning trajectories of the mean squared coefficient errors ρ [4] of the three algorithms. The SNR_i was fixed to be 5 dB while SNR_o was stepped changed from 10 dB to 0 dB at the 5000th iteration. It can be seen that at $\text{SNR}_o = 10$ dB, the steady state values of ρ for the modified LMS algorithm and GAS-TLS were approximately equal to -26 dB while that of the FBJ-TLS was -24 dB. When $\text{SNR}_o = 0$ dB, the proposed method outperformed significantly the TLS-based algorithms and there was about 9 dB improvement in estimation accuracy. Table 1 tabulates the values of their mean impulse response estimates for both cases of SNR_o . We observe that the proposed algorithm gave unbiased estimates in all noise conditions, while all the GAS-TLS and FBJ-TLS estimates were biased, although the bias was relatively small at $\text{SNR}_o = 10$ dB.

Conclusions :

The LMS algorithm has been modified for unbiased impulse response estimation in the presence of input interference. The idea is to weight the LMS coefficients by a correction factor whose value depends on the powers of the noise-free input and input noise. It is shown that the proposed method is superior to two TLS-based algorithms, GAS-TLS and FBJ-TLS, in different noise conditions.

References

- [1] SODERSTROM, T. and STOICA, P.: 'System Identification,' (Prentice-Hall, 1989)
- [2] WIDROW, B. and STEARNS, S.D.: 'Adaptive Signal Processing,' (Prentice-Hall, 1985)
- [3] GOLUB, G.H. and VAN LOAN, C.F.: 'An analysis of the total least squares problem,' *SIAM J. Numer. Anal.*, 1980, **17**, (6), pp. 883-893
- [4] GAO, K., AHMAD, M.O. and SWAMY, M.N.S.: 'A constrained anti-Hebbian learning algorithm for total least-squares estimation with applications to adaptive FIR and IIR filtering,' *IEEE Trans. Circuits Syst. II*, 1994, **41** (11), pp.718-729
- [5] FENG, D.Z., BAO, Z. and JIAO, L.C.: 'Total least mean squares algorithm,' *IEEE Trans. Signal Processing*, 1998, **46** (8), pp.2122-2130

Filter Coefficient		h_0	h_1	h_2	h_3	h_4
Actual Value		-0.3	-0.9	0.8	-0.7	0.6
Proposed Method	SNR _o = 10 dB	-0.3003	-0.9053	0.8036	-0.7035	0.6021
	SNR _o = 0 dB	-0.2981	-0.8972	0.7942	-0.6958	0.5985
GAS-TLS	SNR _o = 10 dB	-0.2798	-0.8453	0.7503	-0.6570	0.5625
	SNR _o = 0 dB	-0.3617	-1.0928	0.9694	-0.8499	0.7313
FBJ-TLS	SNR _o = 10 dB	-0.2806	-0.8473	0.7525	-0.6608	0.5655
	SNR _o = 0 dB	-0.3625	-1.0941	0.9721	-0.8556	0.7362

Table 1: Mean impulse response estimates of different methods

Figure 1: System block diagram for system identification

Figure 2: Mean squared coefficient errors of different methods