Adaptive TDOA Estimation in the Presence of Impulsive Noise

H. C. So

Department of Electronic Engineering, City University of Hong Kong

Tat Chee Avenue, Kowloon, Hong Kong

Email : hcs@ee.cityu.edu.hk

February 25, 2004

Indexing terms : adaptive algorithm, time difference of arrival, stable non-Gaussian process

Abstract : A fractional lower-order moment based adaptive algorithm for estimating time difference of arrival between signals received at two spatially separated sensors in the presence of impulsive noise modeled as a stable process is proposed. The method is computationally efficient and is a generalization of the simplified explicit time delay estimator. Convergence of the algorithm is analyzed and simulation results are included to evaluate its estimation performance for both static and nonstationary delays.

Introduction : The estimation of propagation delay in a common signal arriving at two spatially separated sensors is a problem of considerable interest in many areas such as radar, underwater acoustics and biomedical engineering [1]. In this Letter, we consider the time delay estimation problem in the presence of impulsive noise which is modeled as a stable non-Gaussian random process with characteristic exponent $\alpha \in (0, 2)$ [2]. Under impulsive environments, conventional methods based on the assumption of finite noise variance such as generalized cross correlation [3] and parameter estimation approach [4] fail to provide reliable delay estimates because the variance of the stable distribution does not exist. Using the theory of α -stable distributions, Ma and Nikias [5] have developed several robust algorithms to tackle the problem for $0 < \alpha \leq 2$. However, these methods assume that the delay to be estimated is static and has a value equals to an integral multiple of the sampling interval. By extending the idea of [5] and generalizing the simplified explicit time delay estimator (SETDE) [6], a computationally efficient adaptive algorithm is proposed which can track time-varying delays of any real values under stable noise condition.

The Proposed Method : Given the two received discrete-time sensor outputs,

$$\begin{aligned} r_1(k) &= s(k) + n_1(k) \\ r_2(k) &= As(k - D) + n_2(k) \end{aligned} \tag{1}$$

where $s(k)$ is the signal of interest, $n_1(k)$ and $n_2(k)$ are independent symmetric α -stable (S α S) noise processes with zero location parameter and $0 < \alpha < 2$ which are independent of $s(k)$ and $A \in (0, 1]$ is the attenuation factor between the sensors. The aim is to estimate the time difference of arrival (TDOA) D from $r_1(k)$ and $r_2(k)$. Without loss of generality, we assume that the signal and the noise spectra are bandlimited between -0.5 Hz and 0.5 Hz while the sampling period is 1 second.

Based on the SETDE [6] which iteratively maximizes the mean product of $r_2(k)$ and

the time-shifted version of $r_1(k)$ and extending the concept of the fractional lower order covariance approach [5], a new adaptive algorithm for TDOA estimation in the presence of stable noise is proposed as follows,

$$\hat{D}(k+1) = \hat{D}(k) - \mu r_2(k)^{<a>} \left(\sum_{i=-P}^P f(i - \hat{D}(k)) r_1(k-i) \right)^{} \quad (2)$$

where

$$u^{<v>} = \text{sign}(u)|u|^v \quad \text{and} \quad f(w) = \frac{\cos(\pi w) - \text{sinc}(w)}{w}, \quad \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

and $\hat{D}(k)$ denotes the estimate of D which is updated on a sample-by-sample basis according to a least-mean-square (LMS) style method. The positive quantity μ as well as the non-negative constants a and b are parameters that control convergence rate and stability of the algorithm while P should be chosen large enough to reduce the delay modeling error [4]. Notice that when $a = b = 1$, it becomes the SETDE algorithm. With the use of the cosine and sinc look-up tables [6], the algorithm requires $(4P + 2)$ additions, $(4P + 4)$ multiplications, two look-ups and two power operations for each sampling interval. Therefore, the proposed method is quite computationally simple and allows real-time implementation.

For ease of analysis, we assume that $a = b$ and $s(k)$ is a white process which is uncorrelated with $\hat{D}(k)$. The convergence behavior of the delay estimate is obtained by taking the expected value of (2):

$$\begin{aligned} E\{\hat{D}(k+1)\} &= E\{\hat{D}(k)\} - \mu E\left\{ \left(A s(k-D) \sum_{i=-P}^P f(i - \hat{D}(k)) s(k-i) \right)^{<a>} \right\} \\ &\approx E\{\hat{D}(k)\} - \mu A^a E\{|s(k)|^{2a}\} E\left\{ \left(\sum_{i=-P}^P \text{sinc}(i - D) f(i - \hat{D}(k)) \right)^{<a>} \right\} \\ &\approx E\{\hat{D}(k)\} - \mu A^a E\{|s(k)|^{2a}\} f(D - E\{\hat{D}(k)\})^{<a>} \end{aligned} \quad (3)$$

where $E\{|s(k)|^{2a}\}$ represents the $2a$ th-order moment of $s(k)$ which can be directly computed from [2]. Although a closed form expression for $E\{\hat{D}(k)\}$ is not available, the learning trajectory of the delay estimate can be easily acquired using (3) by brute force. In doing so, the upper bound of μ can be deduced as well.

In order to ensure the variance of the delay estimate is finite, the last term of (2) should be bounded in the mean square sense. That is,

$$E \left\{ \left(r_2(k)^{<a>} \left(\sum_{i=-P}^P f(i - \hat{D}(k)) r_1(k - i) \right)^{} \right)^2 \right\} \\ = E \left\{ |r_2(k)|^{2a} \cdot \left| \sum_{i=-P}^P f(i - \hat{D}(k)) r_1(k - i) \right|^{2b} \right\} < \infty \quad (4)$$

It can be revealed that the inequality holds if $a < \alpha/2$ and $b < \alpha/2$ and these choices of a and b also agree with [5].

Simulation Results : Computer simulations had been conducted to evaluate the delay estimation performance of the proposed method in the presence of stable non-Gaussian noise. The source signal $s(k)$ was a zero-mean white Gaussian process with variance 10 while the impulsive noises $n_1(k)$ and $n_2(k)$ were independent SαS processes with zero location parameter, $\alpha = 1.7$ and unity dispersion. The attenuation A was assigned to one. To provide a delay resolution of approximately 0.1% of the sampling interval, the size of the cosine and sinc tables were 1024 and 513 x 31 respectively, whilst P was chosen to 15 in order to maintain an acceptable truncation error. Without loss of generality, the initial delay estimate was selected to 0 and the values of both a and b were set to be equal. The results provided were averages of 100 independent runs.

The learning trajectory of the delay estimate for a static delay with $D = 0.9$ s is shown in Figure 1. In this test, $\mu = 2 \times 10^{-4}$ while a was set to 0.8. It is seen that the delay estimate converged to the desired value at approximately the 2000th iteration.

The convergence time was slightly longer than the theoretical calculation because of the approximations made in deriving (3). Figure 2 plots the steady state mean square delay errors for different values of a when the convergence speeds of the delay estimates were kept identical to the previous test by properly adjusting μ . We observe that when $a = 0.6$, the algorithm gave the smallest delay variance of 5.1×10^{-4} , although comparable mean square errors were provided by the adjacent a . As a rule of thumb, a should be chosen close to $(\alpha - 1)$ and equal to 0 for $\alpha \geq 1$ and $\alpha < 1$, respectively, in order to attain optimum delay estimation performance.

Figure 3 illustrates the ability of the proposed method to estimate time-varying delays. In this trial, the delay was a sinusoidal function with $D(k) = \sin(0.0006\pi k)$ while μ was increased to 8×10^{-4} for a faster convergence speed and $a = 0.8$. It is observed that the algorithm tracked the delay satisfactorily with a time lag of approximately 0.1s.

Conclusions : A computationally efficient algorithm which provides explicit delay measurements in the presence of stable non-Gaussian noise is proposed. Choices of the parameters that guarantee convergence of the algorithm are discussed and the learning trajectory of the delay estimate is derived. Computer simulations are presented to demonstrate its capability of estimating nonstationary delays.

References

- [1] CARTER, G.C.: 'Coherence and Time Delay Estimation: An Applied Tutorial for Research, Development, Test, and Evaluation Engineers,' (IEEE Press, 1993).
- [2] NIKIAS, C.L. and MA, X.: 'Signal Processing with Alpha-Stable Distributions and Applications,' (John Wiley & Sons, 1995).
- [3] KNAPP, C.H. and CARTER, G.C.: 'The generalized correlation method for estimation of time delay,' *IEEE Trans. Acoust., Speech, Signal Processing*, 1976, **24**, (4), pp. 320-327.
- [4] CHAN, Y.T., RILEY, J.M. and PLANT, J.B: 'Modeling of time-delay and its application to estimation of nonstationary delays,' *IEEE Trans. Acoust., Speech, Signal Processing*, 1981, **29** (3), pp.577-581.
- [5] MA, X. and NIKIAS, C.L.: 'Joint estimation of time delay and frequency delay in impulsive noise using fractional lower order statistics,' *IEEE Trans. Signal Processing*, 1996, **44** (11), pp.2669-2687.
- [6] SO, H.C.: 'Fast algorithm for nonstationary delay estimation,' *IEE Electronics Letters*, 1998, **34**, (8), pp.722-723.

Figure 1: Delay estimate for a static delay

Figure 2: Steady state mean square delay error versus a

Figure 3: Delay estimate for a sinusoidally varying delay