Adaptive Time Delay Estimation with Noise Suppression for Sinusoidal Signals

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Abstract - A least mean square algorithm is devised for time delay estimation between noisy sinusoidal signals received at two spatially separated sensors. Two adaptive finite impulse response (FIR) filters whose coefficients are samples of a sine function are used for delay modeling as well as noise suppression. The convergence behavior and variance of the estimated delay are also derived. Computer simulations are presented to validate the theoretical derivations of the proposed estimator for static and linearly varying delays.

I. Introduction

The problem of estimating the propagation delay between two noisy versions of the same signal received at spatially separated sensors has attracted much attention in the literature [1]-[2]. It is widely used in passive sonar where the bearing of a moving target can be determined from the time delay measurements by triangulation [3]. Other applications include determination of the center of earthquakes, navigation, speed sensing [4] and speech enhancement [5].

The discrete-time sensor outputs can be modeled as

$$\begin{aligned} r_1(k) &= s(k) + n_1(k) \\ r_2(k) &= s(k - D) + n_2(k) \end{aligned} \quad (1)$$

where $s(k)$ is the signal of interest, $n_1(k)$ and $n_2(k)$ are uncorrelated zero-mean noises which are statistically independent of $s(k)$ and D is the differential delay to be determined. For simplicity but without loss of generality, it is assumed that the sampling period is 1 second.

When the time difference of arrival is nonstationary due to either relative source/receiver motion or time-varying characteristics of the transmission medium, adaptive tracking of D is necessary. However, most of the existing adaptive delay estimators [6]-[12] assume that $s(k)$ is a stochastic process and thus it is not appropriate for use in situations where the source signal is deterministic.

In this paper, we develop an adaptive delay estimation algorithm for a moving source that emits a constant

tone in radar and certain types of underwater acoustic systems [13]-[14]. The source signal is expressed as

$$s(k) = \alpha \cos(\omega_0 k + \phi) \quad (2)$$

where α and ϕ represent the unknown tone amplitude and phase, respectively, and $\omega_0 \in (0, \pi)$ is the known radian frequency. Section II derives the FIR filter that can generate the time shifted version of a pure sinusoid and its delay modeling error is investigated. It is proved that the devised filter can increase the signal-to-noise ratio (SNR) as well. An adaptive delay estimator for sinusoid (ADES) is then developed in Section III. In particular, learning behavior and mean square delay error of the algorithm for both static and linearly varying delays are analyzed. Simulation results are presented in Section IV to corroborate the theoretical analyses and to evaluate the delay estimation performance of the proposed approach. Finally, conclusions are drawn in Section V.

II. Modeling of Delay for Sinusoid

Using inverse discrete-time Fourier transform and noting that $s(k)$ is a pure sinusoid, the FIR filter coefficients $\{h_l\}$ that can time shift $s(k)$ by a delay D are derived as

$$\begin{aligned} h_l &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega(l-D)} \cdot \pi(\delta(\omega + \omega_0) + \delta(\omega - \omega_0)) d\omega \\ &= \cos(\omega_0(l-D)), \quad l = \dots, -1, 0, 1, \dots \end{aligned} \quad (3)$$

where $\delta(\cdot)$ is the Dirac delta function. From the convolution theorem and with proper scaling of h_l , $s(k-D)$ can thus be expressed as

$$s(k-D) = \lim_{L \rightarrow \infty} \frac{2}{L} \sum_{l=0}^{L-1} s(k-l) \cos(\omega_0(l-D)) \quad (4)$$

Apparently, L should be a finite integer which is chosen sufficiently large in order to reduce the delay modeling error. In fact, the finite length filter can perfectly model the time shift when $L = \pi/\omega_0, 2\pi/\omega_0, \dots$, etc.

Furthermore, if $\omega_0 L$ is an integral multiple of π , the filter output of $r_1(k)$ is of the form

$$\begin{aligned}\hat{r}_1(k) &= \frac{2}{L} \sum_{l=0}^{L-1} r_1(k-l) \cos(\omega_0(l-\Delta)) \\ &= s(k-\Delta) + \frac{2}{L} \sum_{l=0}^{L-1} n_1(k-l) \cos(\omega_0(l-\Delta))\end{aligned}\quad (5)$$

where Δ is a real number which represents the desired time shift. Obviously, the power of the signal component remains unchanged while it can be easily shown that the noise power reduces to $2\sigma_n^2/L$, which means that the filter also increases the SNR by $L/2$ times.

Let $\hat{s}(k-D)$ be the finite filter length representation of (4). Making use of trigonometric identities, the normalized delay modeling error, ϵ , is given by

$$\begin{aligned}\epsilon &\triangleq \frac{\hat{s}(k-D) - s(k-D)}{\alpha} \\ &= \frac{1}{L} \sum_{l=0}^{L-1} \cos(\omega_0(2l-k-D) + \phi)\end{aligned}\quad (6)$$

It can be observed that ϵ is a function of L , ω_0 , D and ϕ and it is easy to see that its magnitude decreases as L increases. Furthermore, $\epsilon = 0$ when $\omega_0 L$ is an integral multiple of π .

III. Adaptive Delay Estimator for Sinusoid

Based on (4) and the noise reduction property of the delay modeling filter, the ADES is developed to compensate the time difference between $r_1(k)$ and $r_2(k)$ and its block diagram is shown in Figure 1. Two adaptive filters $H_1(z)$ and $H_2(z)$ are employed to delay $r_1(k)$ by $D/2$ and advance $r_2(k)$ by $D/2$, respectively, as well as to suppress the noise in both received signals. It is noteworthy that this configuration is identical to [9], which shows that the split structure can achieve a faster convergence speed as well as a better ability to track nonstationary delays, when comparing with the standard configuration [5] with only one FIR filter in one of the receiver channels. The filter coefficients of $H_1(z)$ and $H_2(z)$ are given by $\{2/L \cos(\omega_0(l - \hat{D}(k)/2))\}$ and $\{2/L \cos(\omega_0(l + \hat{D}(k)/2))\}$, $0 \leq l \leq L-1$, which are a function of the delay estimate, $\hat{D}(k)$, only. The output error function $e(k)$ is computed from

$$\begin{aligned}e(k) &= \frac{2}{L} \sum_{l=0}^{L-1} r_2(k-l) \cos\left(\omega_0\left(l + \frac{\hat{D}(k)}{2}\right)\right) \\ &\quad - \frac{2}{L} \sum_{l=0}^{L-1} r_1(k-l) \cos\left(\omega_0\left(l - \frac{\hat{D}(k)}{2}\right)\right)\end{aligned}$$

(7)

By differentiating $e^2(k)$ with respect to $\hat{D}(k)$, a stochastic gradient estimate which is similar to that in the Widrow's LMS algorithm [15] is obtained. The estimated delay is adapted iteratively to minimize the mean square output error, $E\{e^2(k)\}$, according to

$$\begin{aligned}\hat{D}(k+1) &= \hat{D}(k) - \frac{\mu}{2\omega_0} \frac{\partial e^2(k)}{\partial \hat{D}(k)} \\ &= \hat{D}(k) \\ &\quad + \frac{\mu}{L} e(k) \sum_{l=0}^{L-1} r_2(k-l) \sin\left(\omega_0\left(l + \frac{\hat{D}(k)}{2}\right)\right) \\ &\quad + \frac{\mu}{L} e(k) \sum_{l=0}^{L-1} r_1(k-l) \sin\left(\omega_0\left(l - \frac{\hat{D}(k)}{2}\right)\right)\end{aligned}\quad (8)$$

where μ is a positive scalar that controls convergence rate and ensures system stability of the algorithm. To reduce computation, values of the sine and cosine functions are retrieved from a pre-stored sine vector. At each sampling interval, the algorithm requires $(4L+5)$ multiplications, $(4L-1)$ additions and $4L$ look-up operations.

Let $\omega_0 L/\pi$ be an integer and it can be achieved by properly choosing the sampling interval as ω_0 is known *a priori*. Taking the expected value of (8) yields

$$\begin{aligned}E\{\hat{D}(k+1)\} &= E\{\hat{D}(k)\} + \mu\sigma_s^2 E\{\sin(\omega_0(D - \hat{D}(k)))\} \\ &\approx E\{\hat{D}(k)\} + \mu\sigma_s^2 \omega_0 (D - E\{\hat{D}(k)\}) \\ &= D + (1 - \mu\sigma_s^2 \omega_0)^{k+1} (\hat{D}(0) - D)\end{aligned}\quad (9)$$

where $\sigma_s^2 = \alpha^2/2$ and $\hat{D}(0)$ represent the signal power and the initial delay estimate, respectively. Provided that $2/(\sigma_s^2 \omega_0) > \mu > 0$, $\hat{D}(k)$ will converge to D in the mean sense with a time constant of $1/(\mu\sigma_s^2 \omega_0)$.

Assuming that $n_1(k)$ and $n_2(k)$ are Gaussian processes, the steady state variance of the delay estimate, denoted by $\text{var}(\hat{D})$ is derived using the following steps. Subtracting D from both sides of (8), squaring both sides, taking expectation, considering $k \rightarrow \infty$ and then after simplifications gives

$$\begin{aligned}\text{var}(\hat{D}) &\triangleq \lim_{k \rightarrow \infty} E\{(\hat{D}(k) - D)^2\} \\ &\approx \frac{\mu\sigma_s^2}{\omega_0 \text{SNR}} + \frac{0.7\mu\sigma_s^2}{\omega_0 \text{SNR}^2 L}\end{aligned}\quad (10)$$

where $\text{SNR} = \sigma_s^2/\sigma_n^2$. It can be seen that $\text{var}(\hat{D})$ is proportional and inversely proportional to μ and ω_0 ,

respectively, and its value decreases with increasing L for $\text{SNR} \ll 1$. Moreover, the delay variance has zero value in the absence of noise.

For nonstationary delays, we can still use (8) and (9) to obtain the mean delay estimates, although closed form expressions are generally not available. In particular, when the time delay is linearly varying, that is, $D(k) = D_0 + \lambda k$, where D_0 and λ represent the delay at $k = 0$ and Doppler time compression, respectively, the mean value of (8) can be approximated as

$$E\{\hat{D}(k+1)\} \approx E\{\hat{D}(k)\} + \mu\sigma_s^2\omega_0(D(k) - E\{\hat{D}(k)\}) \quad (11)$$

Solving (11) yields the tracking behavior for a linearly varying delay which is expressed as

$$E\{\hat{D}(k)\} = D(k) - \frac{\lambda}{\mu\sigma_s^2\omega_0} + (1 - \mu\sigma_s^2\omega_0)^k \cdot \left(\hat{D}(0) - D + \frac{\lambda}{\mu\sigma_s^2\omega_0} \right) \quad (12)$$

The second term of the right hand side represents the steady state time lag which is directly proportional to λ and inversely proportional to μ , σ_s^2 and ω_0 while the last term is a transient factor that converges to zero as k goes to infinity. In this case, the mean square delay error, $\text{mse}(\hat{D})$, is equal to the delay variance in (10) plus the square of time lag [16] and has the form

$$\text{mse}(\hat{D}) = \frac{\mu\sigma_s^2}{\omega_0\text{SNR}} + \frac{0.7\mu\sigma_s^2}{\omega_0\text{SNR}^2L} + \frac{\lambda^2}{\mu^2\sigma_s^4\omega_0^2} \quad (13)$$

Since the first and second terms increase with μ but the third term decreases with μ^2 , μ must be selected appropriately in order to achieve the best performance. As a rule of thumb, when noise is high, a smaller value of μ should be used. Otherwise, a larger value of μ is preferred particularly when the delay is changing rapidly with time.

IV. Simulation Results

Computer simulation had been conducted to evaluate the delay estimation performance of the proposed approach for sinusoidal signals in the presence of white Gaussian noise. The amplitude and phase of the sinusoid were $\sqrt{2}$ and 1.0, respectively, while ω_0 was assigned to be $0.25\pi\text{rad/s}$ which made $\omega_0 L/\pi$ an integer. Different SNRs were obtained by properly scaling of the noise sequences. The initial delay estimate was set to 0. The results provided were averages of 200 independent runs.

Figure 2 shows the trajectories for the delay estimates of the proposed estimator for a step change in D with

$L = 8$ and $L = 80$ in nonstationary noise. The step size parameter μ was selected to be 0.005. In the first 4000 iterations, $D = 2.5$ s and $\text{SNR} = 10$ dB, but afterwards, the delay and SNR were changed instantaneously to 4.5 s and -10 dB. As predicted by (9) that the convergence characteristics should be independent of L , the learning curves of the two cases were almost identical and agreed very well with the theoretical calculation after the 1000th iteration. The initial convergence rate was slightly slower than the theoretical calculation because the delay estimate was not close to D at the beginning of adaptation. We see that the delay estimates converged to the desired values of 2.5 s and 4.5 s at approximately the 1600th and 5300th iteration, respectively. Furthermore, the measured variances of $\hat{D}(k)$ with $L = 8$ were found to be $6.1 \times 10^{-4}\text{s}^2$ and $1.2 \times 10^{-1}\text{s}^2$ at $\text{SNR} = 10$ dB and $\text{SNR} = -10$ dB, respectively. While for $L = 80$, the corresponding variances were measured as $6.1 \times 10^{-4}\text{s}^2$ and $5.8 \times 10^{-2}\text{s}^2$. All of these results conformed to (10) and notice that a larger L gave smaller variances at low SNR.

Figures 3 illustrates the tracking performance of the ADES when the actual delay was a linearly time-varying function of the form $D(k) = 0.0005k$ at $\omega_0 = 0.25\pi\text{rad/s}$. The step size μ was increased to 0.05 for a faster convergence speed and $L = 8$ was used. We observe that the algorithm tracked the delay satisfactorily with a time lag of approximately $1.495 \times 10^{-2}\text{s}$ and this value agreed with (12). Furthermore, the measured mean square delay error had a value of $6.332 \times 10^{-3}\text{s}^2$ which was quite close to that derived from (13).

V. Conclusions

The ADES has been proposed for tracking the time difference of arrival of a noisy sinusoid received at two separated sensors. Adaptive FIR filters whose coefficients are samples of a sine function are employed to model the delay and to suppress the noise. Using an LMS-style algorithm, the delay estimate is adjusted explicitly on a sample-by-sample basis. Learning behavior and mean square delay error of the ADES for both static and linearly varying delays are derived. Numerical examples are included to validate the theoretical analysis and to demonstrate the effectiveness of the proposed method.

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Figure 1: System block diagram for adaptive time delay estimation

Figure 2: Delay estimate for a step change in delay in nonstationary noise

Figure 3: Delay estimate for linearly varying delay using $L = 8$ at $\text{SNR} = 10$ dB