Accurate Three-Step Algorithm for Joint Source Position and Propagation Speed Estimation

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Abstract : A popular strategy for source localization is to utilize the measured differences in arrival times of the source signal at multiple pairs of receivers. Most of the time-difference-of-arrival (TDOA) based algorithms in the literature assume that the signal transmission speed is known which is valid for in-air propagation. However, for in-solid scenarios such as seismic and tangible acoustic interface applications, the signal propagation speed is unknown. In this paper, we exploit the ideas in the two-step weighted least squares method [1] to design a three-step algorithm for joint source position and propagation speed estimation. Simulation results are included to contrast the proposed estimator with the linear least squares scheme as well as Cramér-Rao lower bound.

1 Introduction

Passive source localization using time-difference-of-arrival (TDOA) information from an array of spatially separated sensors is an important problem in signal processing. In the TDOA method, the differences in arrival times of the source signal at multiple pairs of sensors are measured. Each TDOA measurement defines a hyperbolic locus on which the source must lie and the position is given by the intersection of two or more hyperbolas for noise-free two-dimensional localization. Although there are numerous TDOA-based positioning algorithms in the literature, such as [1]-[6], most of them assume that the signal transmission speed is known which is valid for in-air propagation. However, for in-solid scenarios such as seismic [7] and tangible interface for human-computer interaction [8] applications, the signal propagation speed is unknown, and we need to find it together with the source position for accurate estimation. In this paper, an efficient three-step algorithm for joint source position and propagation speed estimation is derived by applying the ideas of [1], namely, employment of weighted least squares (WLS) and exploitation of the relationship between parameter estimates. Our major contributions are to address the positioning problem with unknown propagation speed and exploit the nonlinear relationship between the source position and speed parameters in the third step of the developed algorithm.

The rest of the paper is organized as follows. The proposed three-step method is developed in Section 2. It is proved that the first step solution is in fact equal to the least squares (LS) algorithm of [6]. Simulation results are included in Section 3 to evaluate the estimation performance of the three-step algorithm by comparing with the LS method and Cramér-Rao lower bound (CRLB). Finally, conclusions are drawn in Section 4.

2 Joint Source Position and Propagation Speed Estimation Algorithm

In this Section, we develop a three-step algorithm to jointly estimate source location and propagation speed using TDOA measurements from M sensors. The discrete-time signal received at the ith sensor can be expressed as

$$r_i(k) = s(k - D_i) + q_i(k) \qquad i = 1, 2, ..., M \tag{1}$$

where $s(k)$ is the signal radiating from the source, and D_i and $q_i(k)$ are the time-of-arrival and additive noise, respectively, at the ith sensor.

Let (x, y) and (x_i, y_i), $i = 1, 2, \cdots, M$, be the unknown source location and known position of the ith sensor, respectively. Denote $d_{i,1}$ and $D_{i,1} = D_i - D_1$ as the range difference and TDOA with respect to the first sensor, respectively, then we have the following relationship:

$$d_{i,1} = cD_{i,1} = \sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}, \qquad i = 2, 3, ..., M \tag{2}$$

where c is the unknown propagation speed. Our task is to find (x, y) and c with the use of $\{D_{i,1}\}$ and $\{(x_i, y_i)\}$. Following the idea of [3]-[4], we rearrange and square (2) to yield

$$2(x_i - x_1)x + 2(y_i - y_1)y - 2D_{i,1}u + D_{i,1}^2 v = x_i^2 - x_1^2 + y_i^2 - y_1^2, \qquad i = 2, 3, ..., M \tag{3}$$

where $u = c\sqrt{(x_1 - x)^2 + (y_1 - y)^2}$ and $v = c^2$ are introduced to make a linear representation. In matrix form, we have

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} \tag{4}$$

where

$$\mathbf{A} = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) & D_{2,1}^2 & -2D_{2,1} \\ \vdots & \vdots & \vdots & \vdots \\ 2(x_M - x_1) & 2(y_M - y_1) & D_{M,1}^2 & -2D_{M,1} \end{bmatrix}$$

$$\boldsymbol{\theta} = \begin{bmatrix} x \\ y \\ v \\ u \end{bmatrix}$$

and

$$\mathbf{b} = \begin{bmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \vdots \\ x_M^2 - x_1^2 + y_M^2 - y_1^2 \end{bmatrix}$$

In the presence of $\{q_i(k)\}$, the TDOA measurements are noisy which can be modelled as

$$D_{i,1} = D_{i,1}^0 + n_{i,1}, \qquad i = 2, 3, ..., M \tag{5}$$

where $\{D_{i,1}^0\}$ denote the noise-free TDOA's while each TDOA estimation error $n_{i,1}$ is characterized by $r_i(k)$ and $r_1(k)$.

Based on (4), the standard LS estimate of $\boldsymbol{\theta}$, denoted by $\hat{\boldsymbol{\theta}}_1$, is simply:

$$\hat{\boldsymbol{\theta}}_1 = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \tag{6}$$

where ^{T} and $^{-1}$ denote the transpose operator and matrix inverse, respectively. It is noteworthy that (6) is in fact identical to the solution of [6], although we work on the hyperbolic equations from TDOA measurements while circular equations from time-of-arrival information are considered in the latter.

For more accurate estimation, we propose to utilize the ideas of [1], namely, employing WLS and exploiting the relationship between x, y, u, and v, in the following two steps. For sufficiently small noise conditions, the measured error vector in (5), denoted by $\boldsymbol{\varepsilon}$, can be approximated as [1]:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \mathbf{b} - \mathbf{A}\boldsymbol{\theta} \\ &\approx 2[(u + vD_{2,1}^0)n_{2,1} \quad (u + vD_{3,1}^0)n_{3,1} \;...\; (u + vD_{M,1}^0)n_{M,1}]^T \end{aligned} \tag{7}$$

The covariance matrix for $\boldsymbol{\varepsilon}$, denoted by $\mathbf{\Phi_2}$, is then

$$\mathbf{\Phi_2} = E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\} \approx 4\mathbf{B_2}\mathbf{Q}\mathbf{B_2} \tag{8}$$

where E denotes expectation operator, $\mathbf{B_2} = \text{diag}(u + vD_{2,1}^0,\ u + vD_{3,1}^0,\ ...,\ u + vD_{M,1}^0)$ and $\mathbf{Q}$ is the covariance matrix for $\{n_{i,1}\}$ which can be determined using the power spectra of $s(k)$ and $\{q_i(k)\}$ [1],[9]. For simplicity, we assume that the source signal and noises in (1) are white processes and the signal-to-noise ratios at all $\{r_i(k)\}$ are identical. In doing so, $\mathbf{Q}$ will be proportional to

$$\begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}$$

and we will substitute this matrix for $\mathbf{Q}$ in our study. With the use of (8), the WLS estimate of $\boldsymbol{\theta}$, denoted by $\hat{\boldsymbol{\theta}}_2$, is [10]:

$$\hat{\boldsymbol{\theta}}_2 = (\mathbf{A}^T\mathbf{\Phi_2}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{\Phi_2}^{-1}\mathbf{b} \tag{9}$$

Note that the technique of WLS has already been utilized in localization [1]-[2], although $\mathbf{A}$ and $\mathbf{\Phi_2}$ in (9) are of different forms because we have the unknown parameter of speed as well.

To compute $\mathbf{\Phi_2}$ in practice, $\{D_{i,1}^0\}$ in $\mathbf{B_2}$ are replaced by $\{D_{i,1}\}$ in (5), and v and u are approximated by

$$\hat{v} = \left(\frac{\mathbf{D}^T\mathbf{d}}{\mathbf{D}^T\mathbf{D}}\right)^2 \tag{10}$$

and

$$\hat{u} = \sqrt{\hat{v}(([\hat{\boldsymbol{\theta}}_1]_1 - x_1)^2 + ([\hat{\boldsymbol{\theta}}_1]_2 - y_1)^2)} \tag{11}$$

where $[\hat{\boldsymbol{\theta}}_1]_1$ and $[\hat{\boldsymbol{\theta}}_1]_2$ represent the first and second elements of $\hat{\boldsymbol{\theta}}_1$, which are the LS estimates of x and y, respectively. The $\mathbf{D} = [D_{2,1}, D_{3,1}, ..., D_{M,1}]^T$ and $\mathbf{d} = [d_{2,1}, d_{3,1}, ..., d_{M,1}]^T$ are the TDOA vector and range difference vector, respectively, where $d_{i,1}$ is computed as $d_{i,1} = \sqrt{(x_i - [\hat{\boldsymbol{\theta}}_1]_1)^2 + (y_i - [\hat{\boldsymbol{\theta}}_1]_2)^2} - \sqrt{(x_1 - [\hat{\boldsymbol{\theta}}_1]_1)^2 + (y_1 - [\hat{\boldsymbol{\theta}}_1]_2)^2}$. It is noteworthy that choosing the above initial estimates of u and v is based on the fact that the location estimate in (6) is more accurate than the speed estimate [5]. The covariance matrix for the WLS estimate in (9) is [10]:

$$\mathrm{cov}(\hat{\boldsymbol{\theta}}_2) = (\mathbf{A}^T \mathbf{\Phi_2}^{-1} \mathbf{A})^{-1} \tag{12}$$

The relationship between x, y, u, and v has not been exploited so far. In the third step, we utilize their relationship:

$$u^2 = v((x - x_1)^2 + (y - y_1)^2) \tag{13}$$

We first define the matrices:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ [\hat{\boldsymbol{\theta}}_2]_3 & [\hat{\boldsymbol{\theta}}_2]_3 & 0 \end{bmatrix}$$

$$\boldsymbol{\vartheta} = \begin{bmatrix} (x - x_1)^2 \\ (y - y_1)^2 \\ v \end{bmatrix}$$

and

$$\mathbf{p} = \begin{bmatrix} ([\hat{\boldsymbol{\theta}}_2]_1 - x_1)^2 \\ ([\hat{\boldsymbol{\theta}}_2]_2 - y_1)^2 \\ [\hat{\boldsymbol{\theta}}_2]_3 \\ [\hat{\boldsymbol{\theta}}_2]_4^2 \end{bmatrix}$$

The $\hat{\boldsymbol{\theta}}_2$ can be written in terms of $\boldsymbol{\theta}$ as:

$$[\hat{\boldsymbol{\theta}}_2]_i = [\boldsymbol{\theta}]_i + e_i, \qquad i = 1, 2, 3, 4 \tag{14}$$

where $\{e_i\}$ are the estimation errors in (9). Now we have

$$\mathbf{H}\boldsymbol{\vartheta} \approx \mathbf{p} \tag{15}$$

Similar to (7), the error vector for (15) can be approximated as

$$\begin{aligned} \boldsymbol{\xi} &= \mathbf{p} - \mathbf{H}\boldsymbol{\vartheta} \\ &\approx [2(x - x_1)e_1,\ 2(y - y_1)e_2,\ e_3,\ 2ue_4 - re_3]^T \end{aligned} \tag{16}$$

where $r = (x - x_1)^2 + (y - y_1)^2$. Following [1], the covariance matrix for $\boldsymbol{\xi}$, denoted by $\mathbf{\Phi_3}$, is

$$\mathbf{\Phi_3} = E\{\boldsymbol{\xi}\boldsymbol{\xi}^T\} \approx \mathbf{B_3}\mathrm{cov}(\hat{\boldsymbol{\theta}}_2)\mathbf{B_3} + [\mathbf{0} \quad \mathbf{G}] \tag{17}$$

where $\mathbf{B_3} = \mathrm{diag}(2(x - x_1),\ 2(y - y_1),\ 1,\ 2u)$, $\mathbf{0}$ is a 4×3 zero matrix and $\mathbf{G} = [-2r(x - x_1)E\{e_1 e_3\},\ -2r(y - y_1)E\{e_2 e_3\},\ -rE\{e_3^2\},\ r^2 E\{e_3^2\} - 4urE\{e_3 e_4\}]^T$ with $E\{e_i e_j\}$

corresponding to the (i,j) entry of $\text{cov}(\hat{\boldsymbol{\theta}}_2)$ in (12). In practice, the values of x, y, and u are approximated by the estimates of (9). The WLS estimate of $\boldsymbol{\vartheta}$, denoted by $\hat{\boldsymbol{\vartheta}}$, is then:

$$\hat{\boldsymbol{\vartheta}} = (\mathbf{H}^T \mathbf{\Phi_3}^{-1}\mathbf{H})^{-1}\mathbf{H}^T \mathbf{\Phi_3}^{-1}\mathbf{p} \tag{18}$$

The final estimates of the location and speed are then computed as:

$$\begin{aligned} \hat{x} &= \pm\sqrt{[\hat{\boldsymbol{\vartheta}}]_1} + x_1 \\ \hat{y} &= \pm\sqrt{[\hat{\boldsymbol{\vartheta}}]_2} + y_1 \\ \hat{c} &= \sqrt{[\hat{\boldsymbol{\vartheta}}]_3} \end{aligned} \tag{19}$$

The values of $\hat{x}$ and $\hat{y}$ which are closest to the corresponding estimates in $\hat{\boldsymbol{\theta}}_2$ are chosen as the position estimate. Note that when the square root is a complex number, the imaginary component will be set to zero, and this happens when (x_1, y_1) is very close to (x, y).

The proposed 3-step algorithm for joint source position and propagation speed estimation is summarized as follows:

(i) Compute the LS solution using (6).
(ii) Compute the second step solution of (9) with the use of (10) and (11).
(iii) Compute the third step solution using (18) and (19) with the use of $\hat{\boldsymbol{\theta}}_2$.

3 Simulation Results

Computer simulations are conducted to evaluate the proposed three-step algorithm for source localization and speed estimation by comparing it with the LS solution of (6) or [6], as well as CRLB [5]. We consider a tangible acoustic interface application of interactive displays [11]. Five sensors are placed on a 1m×1m pane of glass with coordinates (0.5, 0.5)m, (0, 0)m, (1, 0)m, (1, 1)m, and (0, 1)m while the unknown source position is located at (0.2, 0.1)m. Note that the acoustic propagation speed in solid is dependent on the material of medium as well as the type of tactile interaction. Here we assume a 1-cm thick pane with a knuckle tap, the propagation speed is set to 1200ms^{-1} [11]. For this relatively high speed, the TDOA's become much smaller than the location coordinates. As a result, the values in the last two columns of $\mathbf{A}$ are significantly less than those of the first two columns, which will make $\mathbf{A}$ a badly scaled matrix and result in inaccuracy of the solution. To avoid this problem, we multiply both sides of (2) by 10^3. This operation scales up x, y, x_i, y_i and $D_{i,1}$ by 10^3. In doing so, the elements in the first, second and fourth columns will be multiplied by 10^3 while those of the third column will be multiplied by 10^6 and thus the condition number of $\mathbf{A}$ will be decreased. The solution of (x, y) in (19) is scaled down accordingly to obtain our solution. The same scaling operation is also applied to the LS method in [6]. The noise-free TDOA's are added by the correlated Gaussian noises with covariance matrix given by $\mathbf{Q}$ with diagonal elements equal $2\sigma^2$ and all other elements equal σ^2. All simulation results are averages of 5000 independent runs.

Figures 1 and 2 compare the positioning and speed estimation accuracy, respectively, of the proposed method with the LS approach and CRLB for different values of σ^2. The mean square error (MSE) is used as the performance measure. The MSE of the position estimate is defined as $E\left\{(x-\hat{x})^2 + (y-\hat{y})^2\right\}$ where $\hat{x}$ and $\hat{y}$ denote the estimates of x and y, respectively. On the other hand, the MSE of the speed estimate is defined as $E\left\{(c-\hat{c})^2\right\}$

where $\hat{c}$ is the estimate of c. From Figures 1 and 2, it is observed that the MSEs of the proposed method are very close to CRLB, and are smaller than those of the LS method, for both location and speed estimation.

4 Conclusion

A three-step algorithm for joint source localization and propagation speed estimation is developed with the use of time-difference-of-arrival (TDOA) measurements. Two intermediate variables are introduced to make a linear representation of the nonlinear TDOA equations. The least squares (LS) solution in the first step provides the initial estimates. The second step refines the estimation by employing weighted least squares (WLS) while the final step further improves the estimates by another WLS via utilizing the relationship between the source position, speed and intermediate variables. For sufficiently small noise conditions, it is shown that the accuracy of the proposed method approaches Cramér-Rao lower bound and outperforms the LS method of [6].

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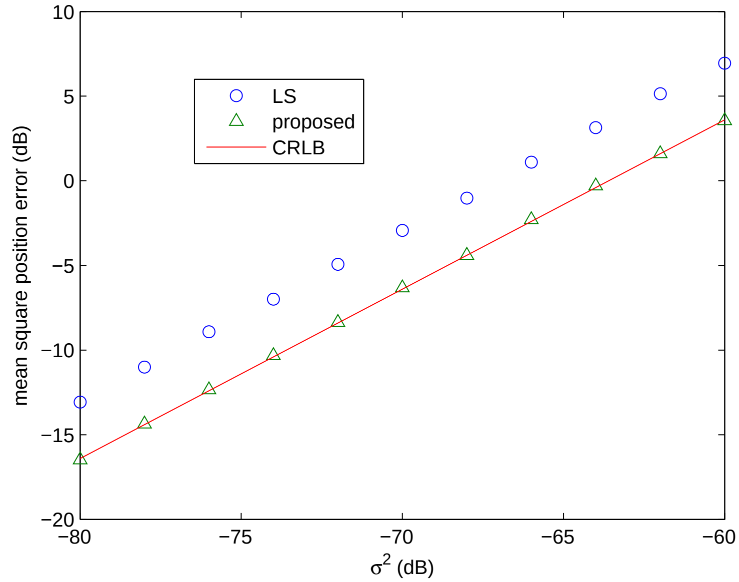

Figure 1: Mean square position errors

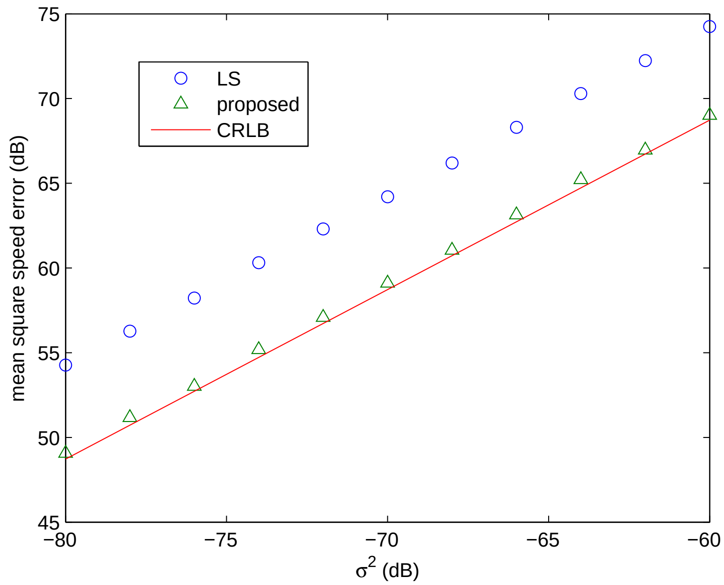

Figure 2: Mean square speed errors