Separation for Source, Channel and Network Coding

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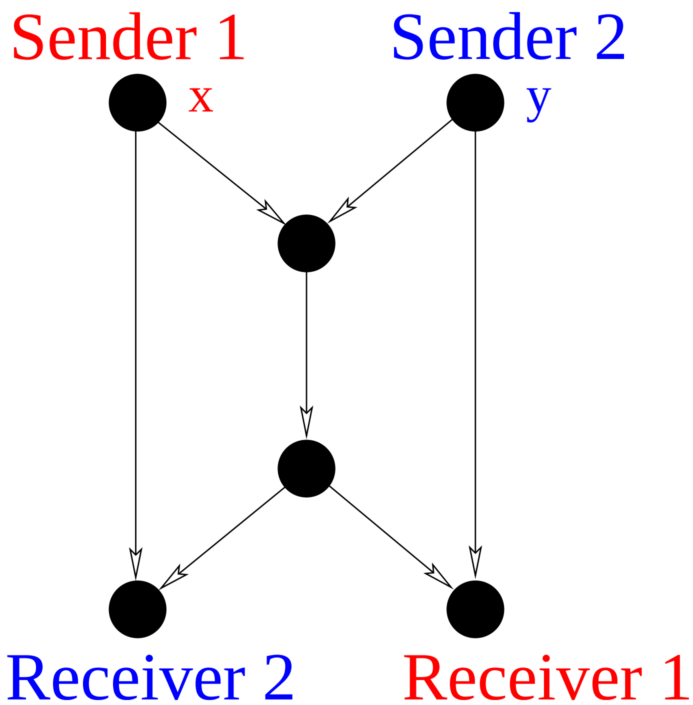
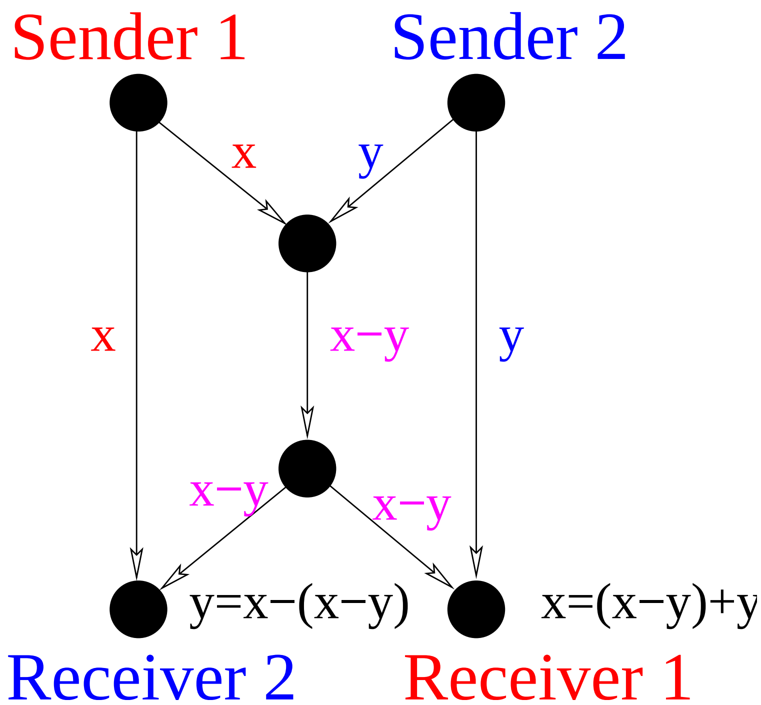
[illegible] An Example



[1] Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", IEEE-IT, vol. 46, pp. 1204-1216, 2000

[2] S.-Y. R. Li, R. W. Yeung, and N. Cai "Linear Network Coding", IEEE-IT, 2000

An Example



[1] Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, “Network Information Flow”, IEEE-IT, vol. 46, pp. 1204-1216, 2000

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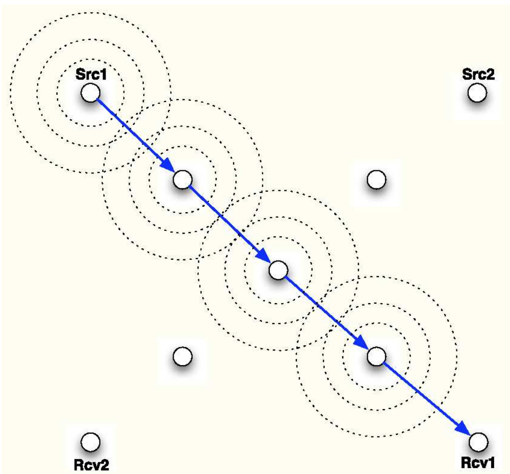
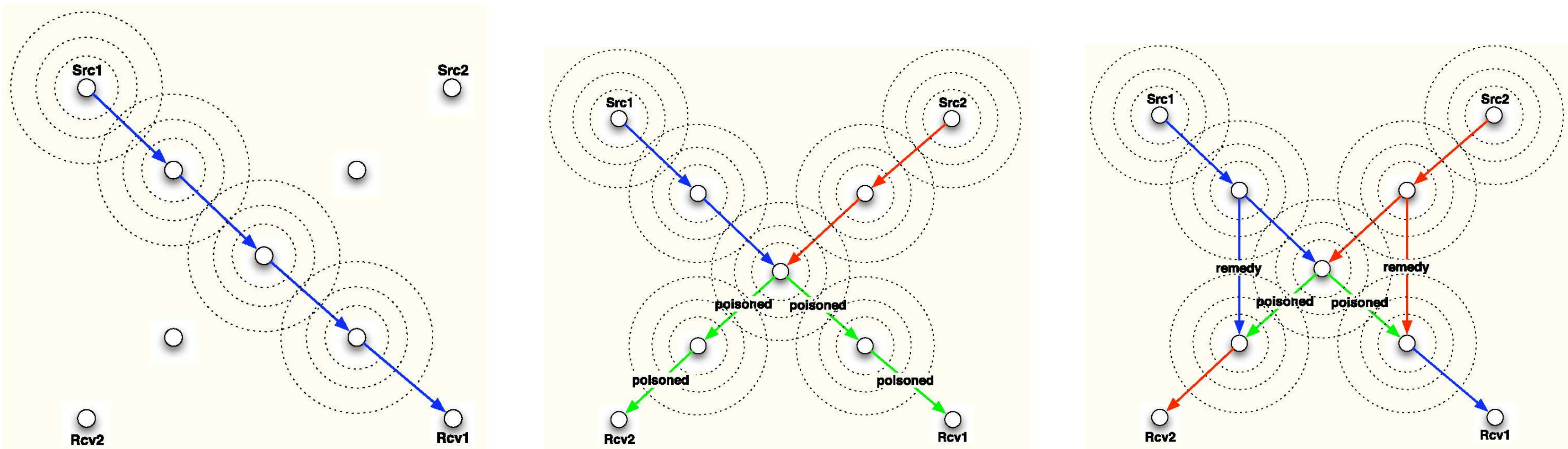


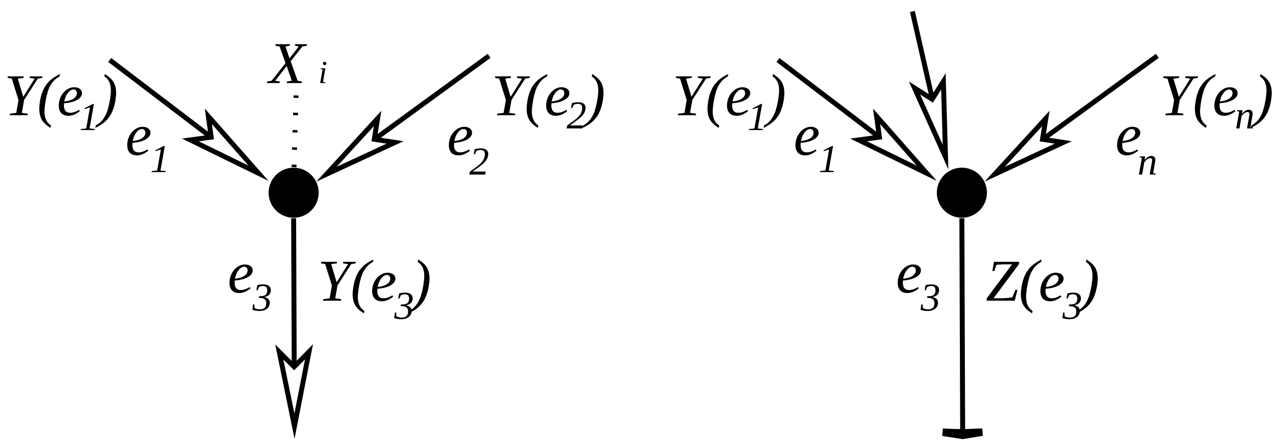
The wireless situation



The wireless situation — congested areas in the network are better utilized: packets use “car pooling”

Typical Network Coding Assumptions

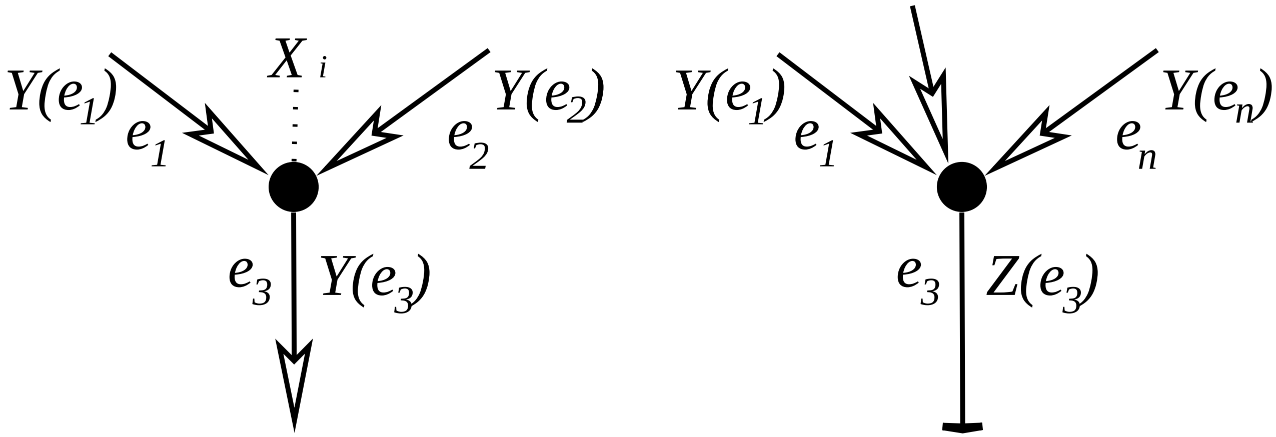
$C(e) = 1$ (links are noiseless and have the same capacity)
$H(X_i) = 1$ (sources have the same rate)
The X_i are mutually independent.



At nodes we implement deterministic functions.

Linear Network Codes

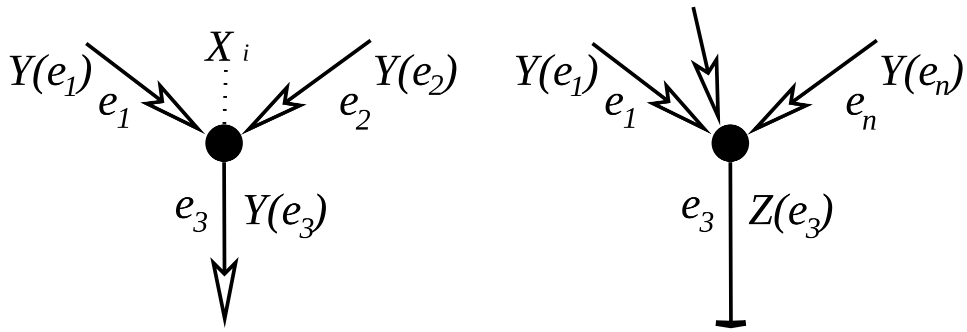
All operations at network nodes are assumed linear!



$$Y(e_3) = \sum_i \alpha_i X(v,i) + \sum_{j=1,2} \beta_j Y(e_j)$$

$$Z(v,j) = \sum_{j=1}^{n} \varepsilon_j Y(e_j).$$

Random linear network coding



$$Y(e_3) = \sum_i \alpha_i X(v,i) + \sum_{j=1,2} \beta_j Y(e_j)$$

$$Z(v,j) = \sum_{j=1}^{n} \varepsilon_j Y(e_j).$$

Operations are over a finite field $\mathbb{F}_{2^m}$ with randomly chosen coefficients. The compound effect of all the choices can be efficiently communicated in a packet header.

Network Coding vs. Transportation Model

The defining property of network coding is that nodes in a network are allowed to form outgoing symbols from incoming symbols in any way — not only time-multiplexing of data streams.

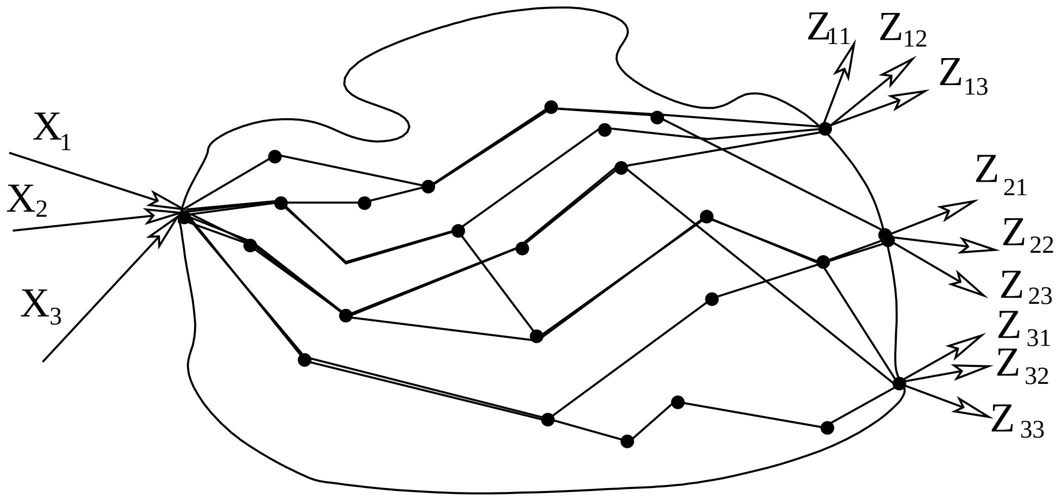
$\Rightarrow$

The *goal* of network coding is to provide a receiver R with enough evidence to solve an inference problem concerning the data that it wants to receive $\Rightarrow$ "$H(X_R|Y_R) = 0$" is the *goal*

Network coding breaks with the transportation model:

A bit is not a car!

A multicast setup



Multicast network

This becomes just a linear system with input X and output Z. The transfer matrix is a random matrix over the finite field of choice.

The main multicast theorem:

* Assume we want to transmit a source with entropy rate R.

* The maximal entropy rate that we can generate at any receiver is equal to the min-cut between the source and this receiver.

* Using network coding functions that mix data efficiently enough we can achieve the maximal entropy rate simultaneously at each receiver.

The multicast capacity is determined by the min-cut in the network and is achievable with random network coding

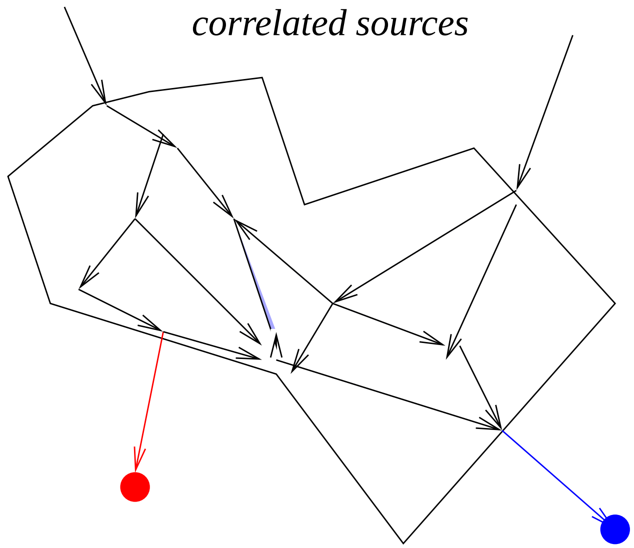
Separation

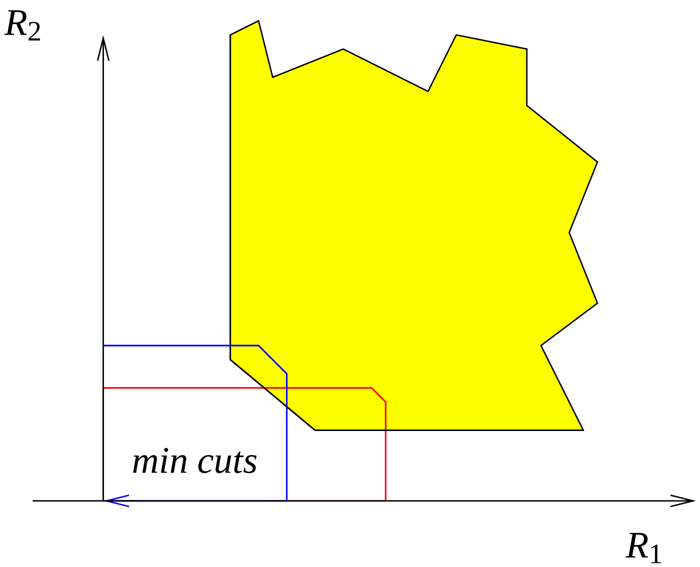
Separation between source coding, channel coding, network coding...

Source-channel coding separation: (reasonably well) understood.

(Lossless) Source-coding — network coding separation: very interesting leading to a study of a network version of Slepian Wolf coding (separation does not hold)

Source/Network coding

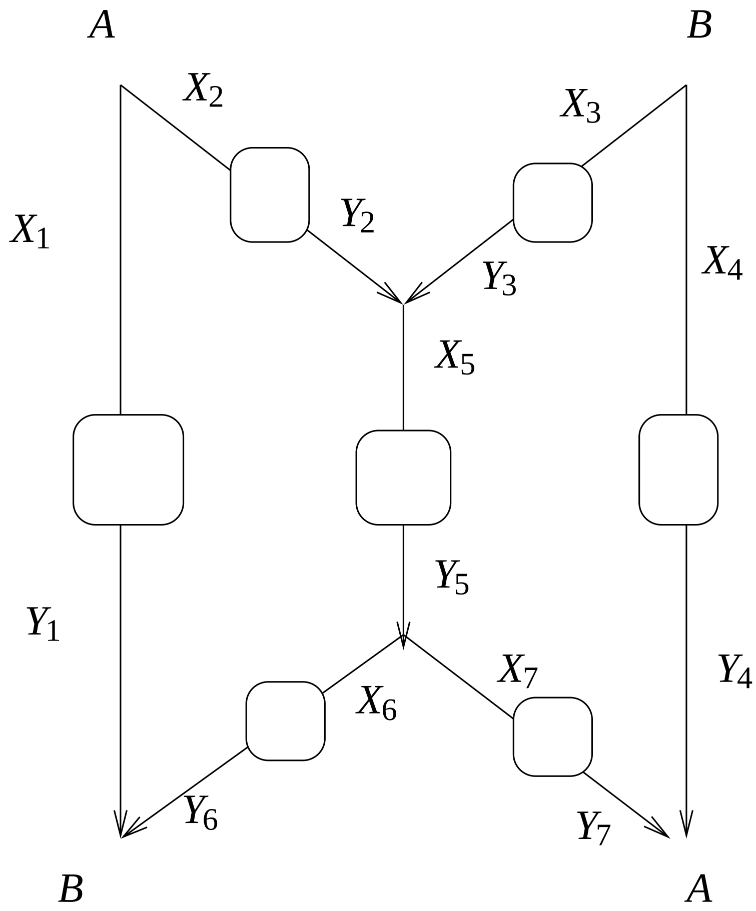




T. Ho, M. Medard, M. Effros, and R. Koetter, "Network Coding for Correlated Sources," in CISS, 2004.

A. Ramamoorthy, K. Jain, P. Chou, M Effros, "Separating Distributed Source Coding from Network Coding", IEEE-IT, 2006

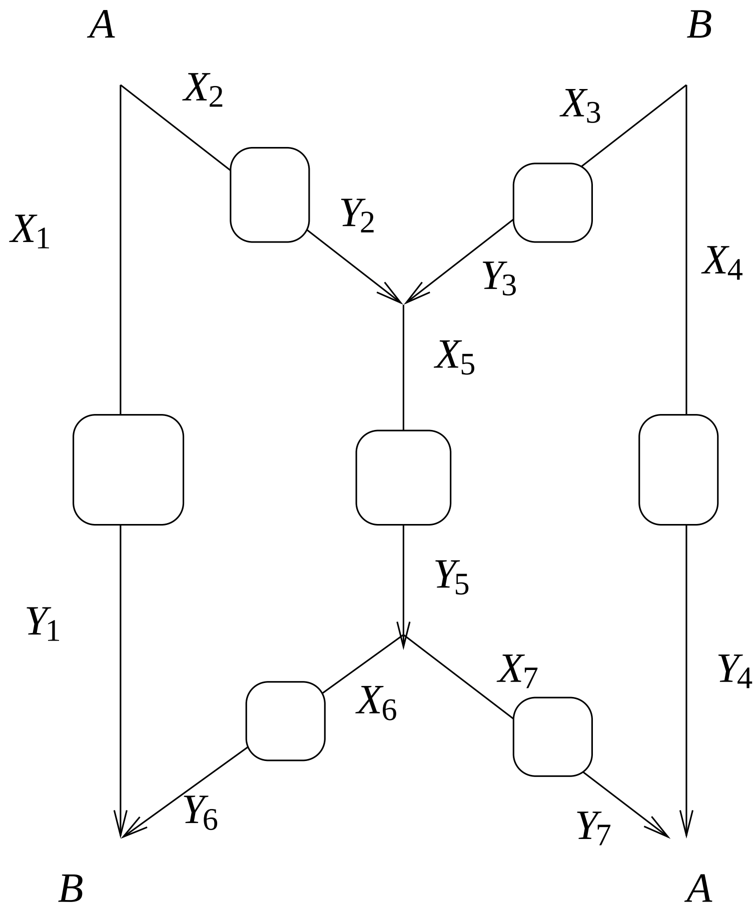
Channel/Network Coding



$R_A \leq I(Y_7; A|Y_4)$
$R_B \leq I(Y_6; B|Y_1)$
$\vdots$
$H(X_5|Y_2, Y_3) = 0$
$\vdots$
$\vdots$

$\vdots$
$B \leftrightarrow Y_6 \leftrightarrow Y_1$
$\vdots$

Channel/Network Coding



$R_A \leq I(Y_7; A|Y_4)$

$R_B \leq I(Y_6; B|Y_1)$

$\vdots$

$H(X_5|Y_2, Y_3) = 0$

$\vdots$

$\vdots$

$\vdots$

$B \leftrightarrow Y_6 \leftrightarrow Y_1$

$\vdots$

Now what?

We say that Channel/network coding separation holds for a link (v, u) in the network if we can replace a channel of capacity C with an error free link with throughput limited by C without changing the rate region.

Multicast — min-cut bounds

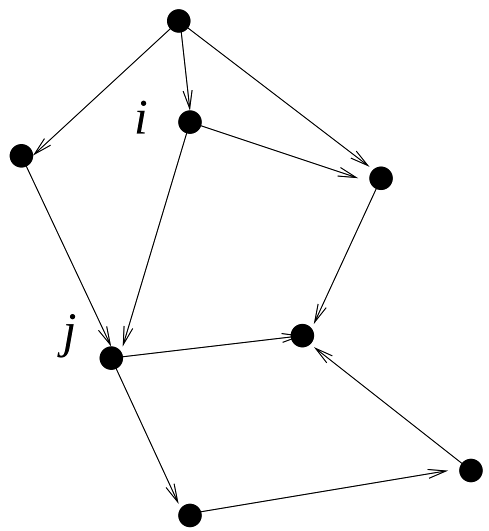
L. Song and R. W. Yeung and N. Cai, "A separation theorem for single source network coding," IEEE Transactions on Information Theory, vol. 52, no. 5, pp. 1861-1871, May 2006

Shashibhushan Borade, "Network Information Flow: Limits and Achievability," Proc. IEEE International Symposium on Information Theory, July 2002.

Separating network coding from channel coding achieves the optimal throughput in a multicast setup.

How about general non-multicast demands?

A general network of DMCs



Each channel (i, j) is a DMC

A DMC:

$(\mathcal{X}_i, P_{ij}(y|x), \mathcal{Y}_j)$

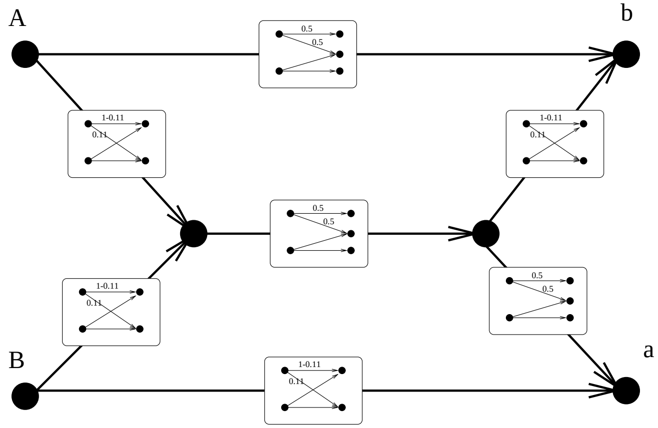
Input alphabet: $\mathcal{X} = (x_1, x_2, \ldots)$

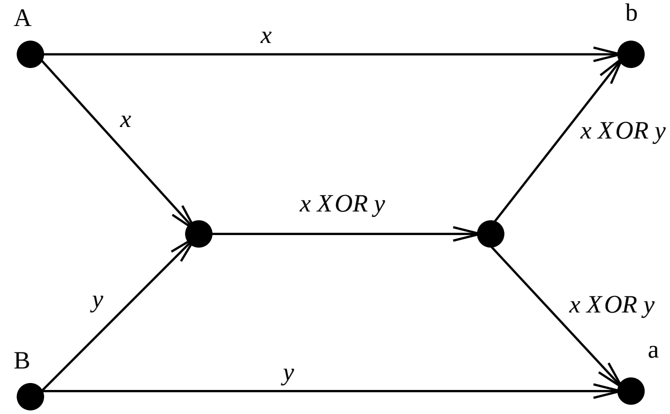
Output alphabet: $\mathcal{Y} = (y1, y2, , \ldots)$

Transition probabilities: $P_{ij}(y|x)$

What is the rate region of the network for specific (in particular, non-multicast) demands?

“Noisy” Channels



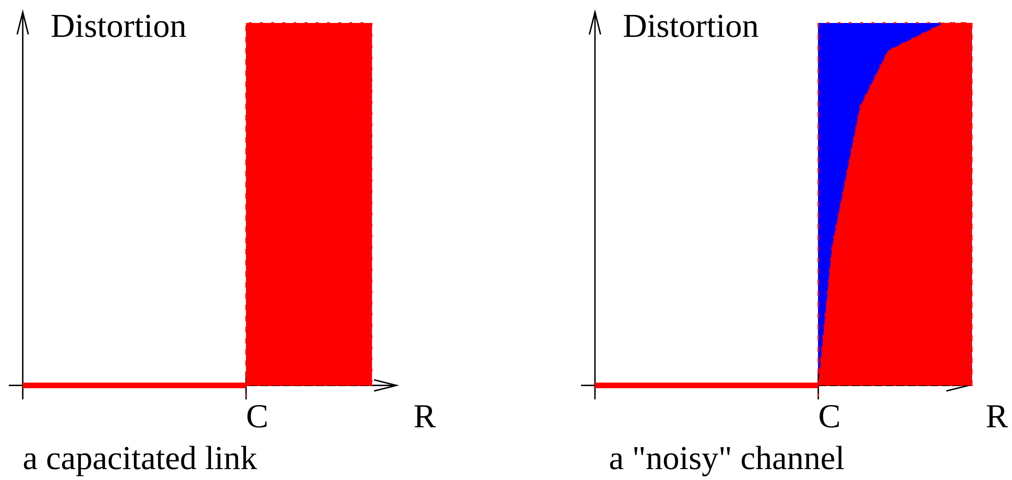


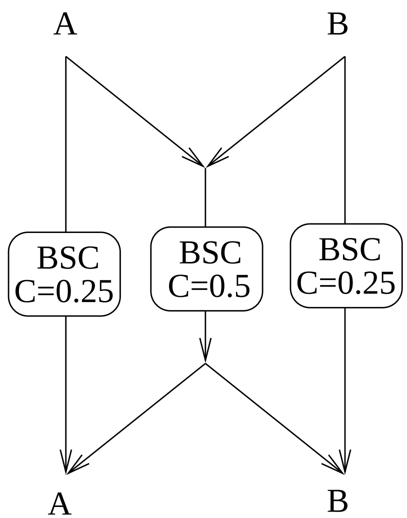
(all channels have capacity 1 bit/(channel use))

Are these two networks “essentially” the same?

Intuitively, since the locally introduced noise is uncorrelated to any other random variable it cannot be of any help....

The characteristic of a noisy channel vs. a capacitated link.

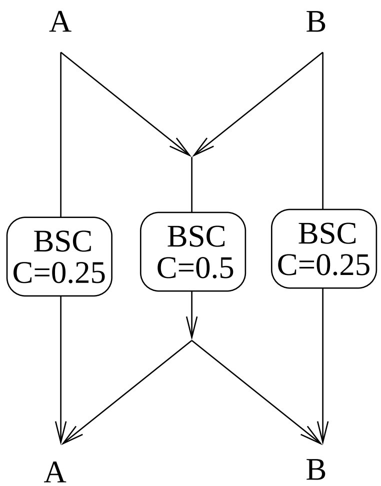


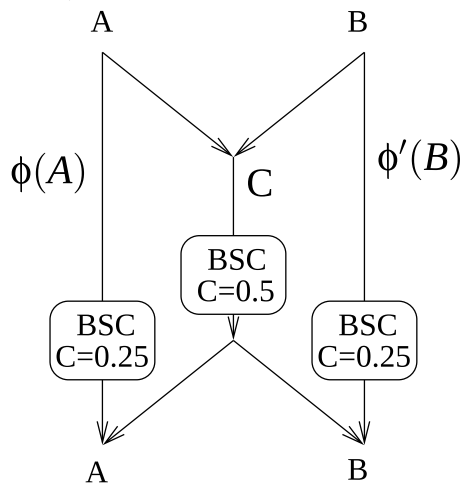


Can the blue area help?

Using the center link uncoded, achieves capacity based on a decoder with side information at the receivers.

Uncoded transmissions may be useful....





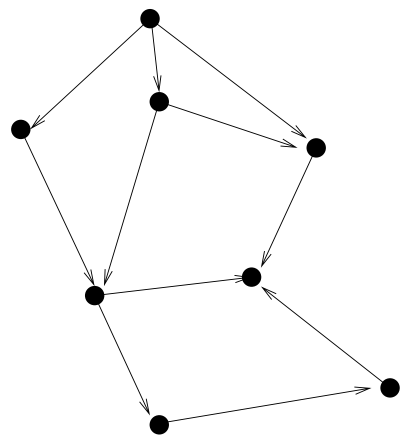
$$A, B \in \mathcal{X}^{n/2}$$

$$\phi(A), \phi'(B), C \in \mathcal{X}^{n}$$

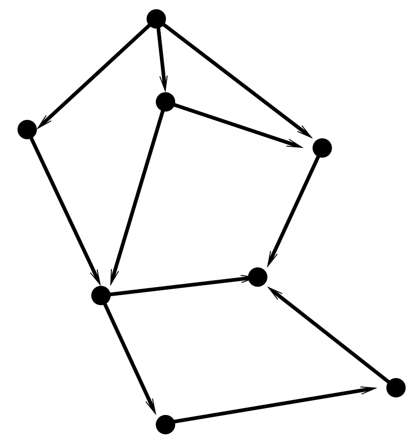
$$C = (A : B)$$

achieves the min-cut capacity rate-region!

S. Shamai and S. Verdu, “Capacity of channels with uncoded side information,” Europ. Trans. Telecommun., vol. 6, no. 5, pp. 587-600, Sept.-Oct. 1995.



Network A of DMCs (i, j) with capacity C_{ij}



Network B of error-free bit-pipes with throughput C_{ij}

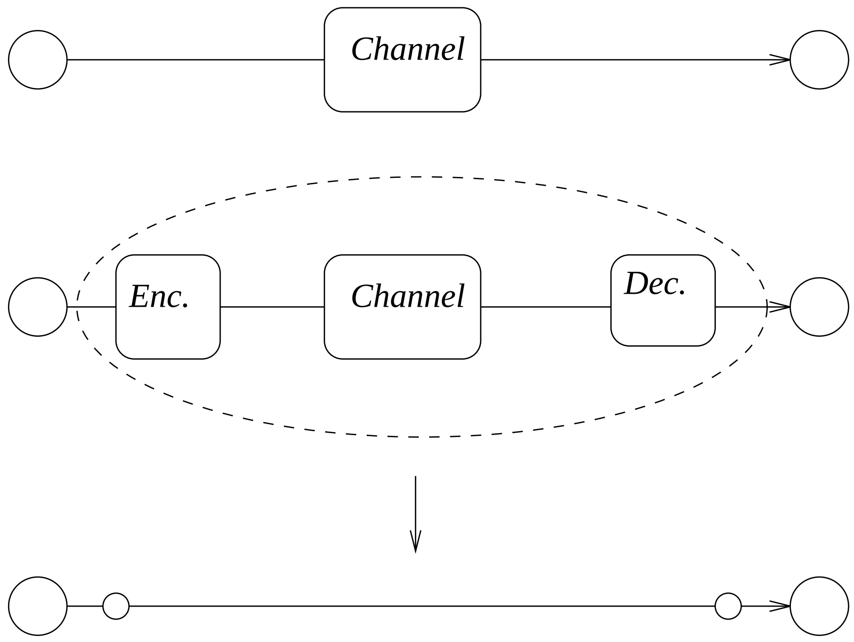
For any set of demands the rate regions of networks A and B coincide!

This makes the problem of rate regions of networks of DMCs into a network coding problem! In order to make a statement about "capacity" of such networks we (just) have to solve the network coding problem!

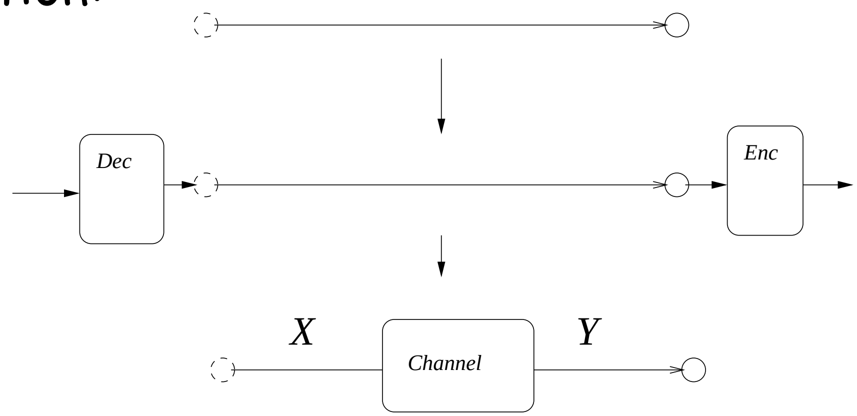
Theorem Let a network of independent DMCs be given with arbitrary demands on source and sink pairs. The achievable rate region is not changed if any DMC in the network is replaced by another DMC with the same capacity.

Proof sketch...

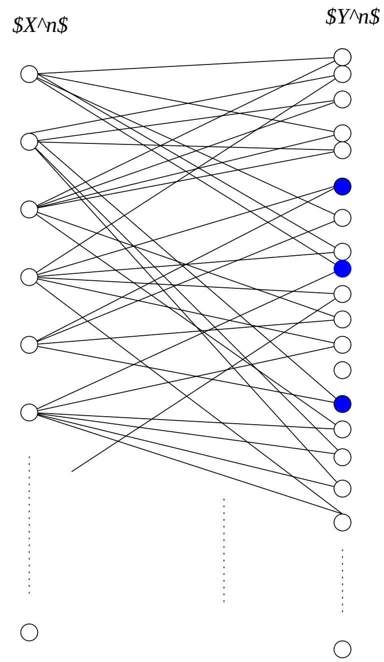
One direction is easy: Any DMC of capacity C can emulate an error free link with capacity C:



The other direction:



We have to find an Dec./Enc. pair that implements the typical behaviour of the relevant joint types.



Let a bipartite, biregular graph be given with vertex classes V_1, V_2 of degree d_1, d_2. There exists of subset U of k vertices of V_2 such that every vertex in V_1 is adjacent to U and $k \leq \frac{|V_2|}{d_1}(1 + \log(d_2))$.

(a weak form of the Johnson-Stein-Lovasz Theorem)

$\Rightarrow$

Here $|U| \leq 2^{n(I(X,Y)+o(1))} \Rightarrow$ we can emulate the "type" by transmitting not more than $I(X,Y) + o(1)$ bits.

In other words the "randmoness" due to the channel is provided by the random representations of the types...

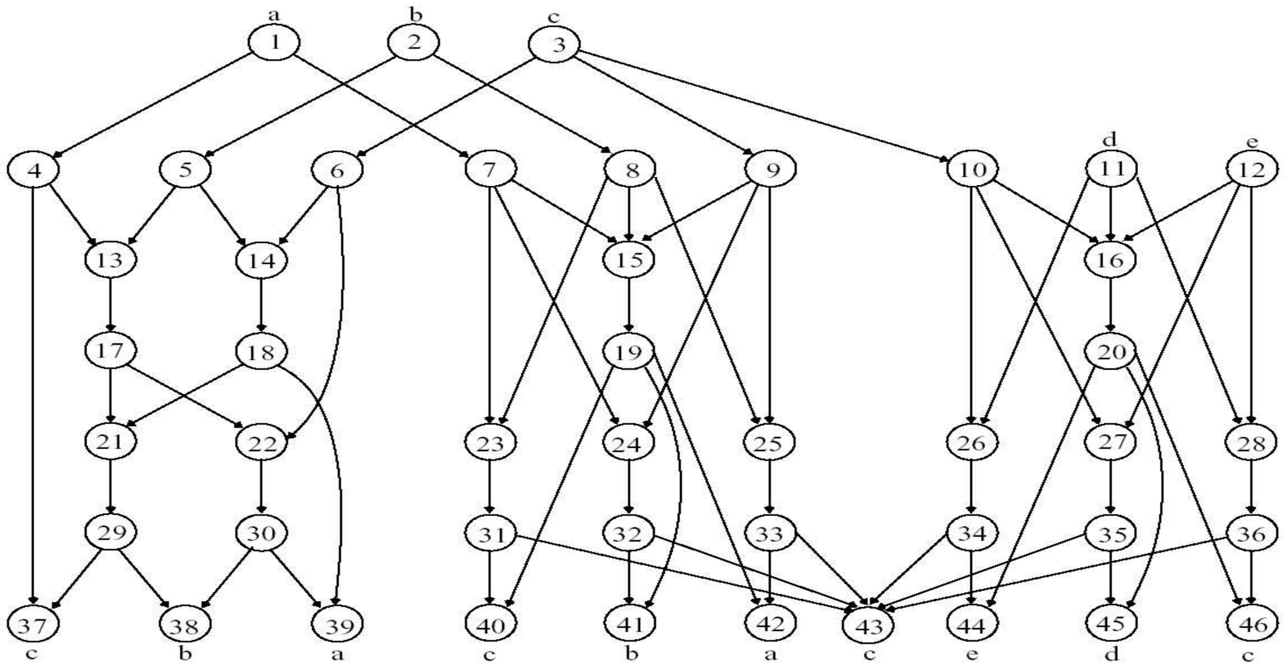
——————-

Other issues: - we have to consider all possible ways to use a channel

- error exponents are far from equal (and rather poor)

Network coding as combinatorial problem

R. Dougherty, C. Freiling, and K. Zeger, "Insufficiency of Linear Coding in Network Information Flow", preprint, 2004



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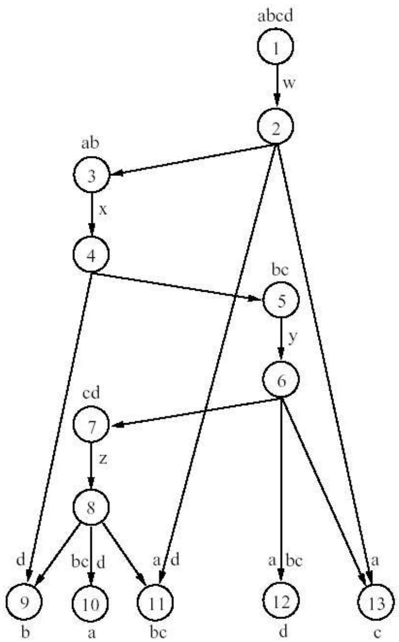
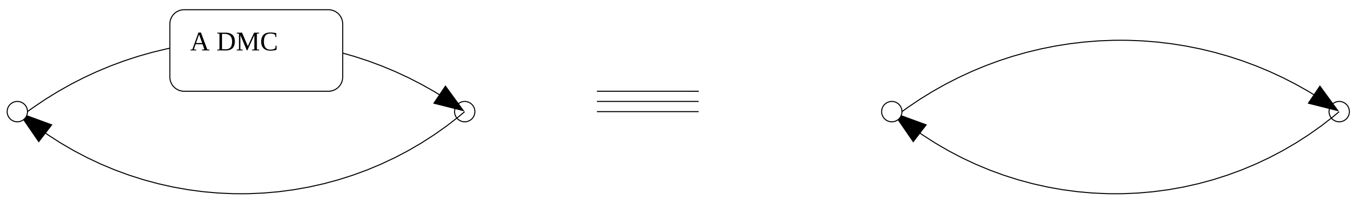


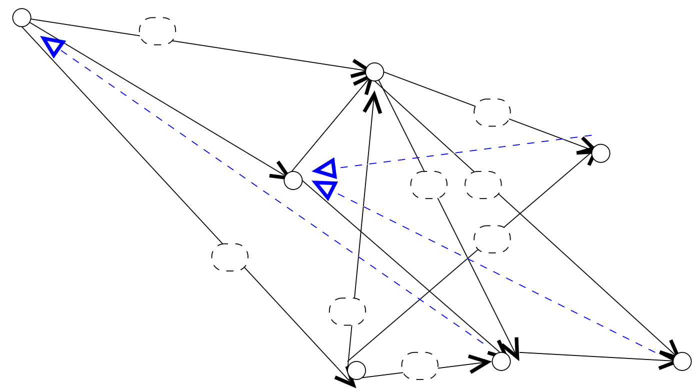
The capacity region of this network is not characterized by classical Shannon type inequalities (i.e. inequalities of type $I(q;r|p) > 0$ for collections of random variables q, r, p are not sufficient to characterize rate regions of networks)

Some (classical) results

* Feedback doesn't help in single source networks of DMCs (even if nodes in the network have memory)[Song et al.]



* Feedback does not help for any network of DMCs and arbitrary demands



* Any link that does not increase rate in the case of error free connections may be discarded)

* Finding the rate region of networks of DMCs is essentially a combinatorial problem and not a problem of finding the distribution of random variables that maximizes a classically information theoretic quantity.

* The achievable rates for networks are connected to the characterization of matroids. Non-linear matroids give many results for networks of DMCs

* Deciding if a rate tuple restricted to <u>scalar, linear</u> network coding lies within the rate region of a network of DMCs is essentially solved.

Open/research problems — as far as I know..

* Channels with memory ?

* Channels with continuous alphabets ?

* Separation for source coding/ non-multicast network coding?

* Matroids and network coding ??

Let E be a finite set:

1. The empty set is independent.
2. Every subset of an independent set is independent
3. If A and B are two independent sets and A has more elements than B, then there exists an element in A which is not in B and when added to B still gives an independent set

* Is there ay way to include the Broadcast/Relay/MAC channel in a separtion framework?

Some observations and statements

- Network coding is a way to trade excess capacity in parts of the network for bottleneck capacity somewhere else.

- Network coding and channel coding separate in DMCs which makes network coding (to some extent) the missing link between information theory and networking

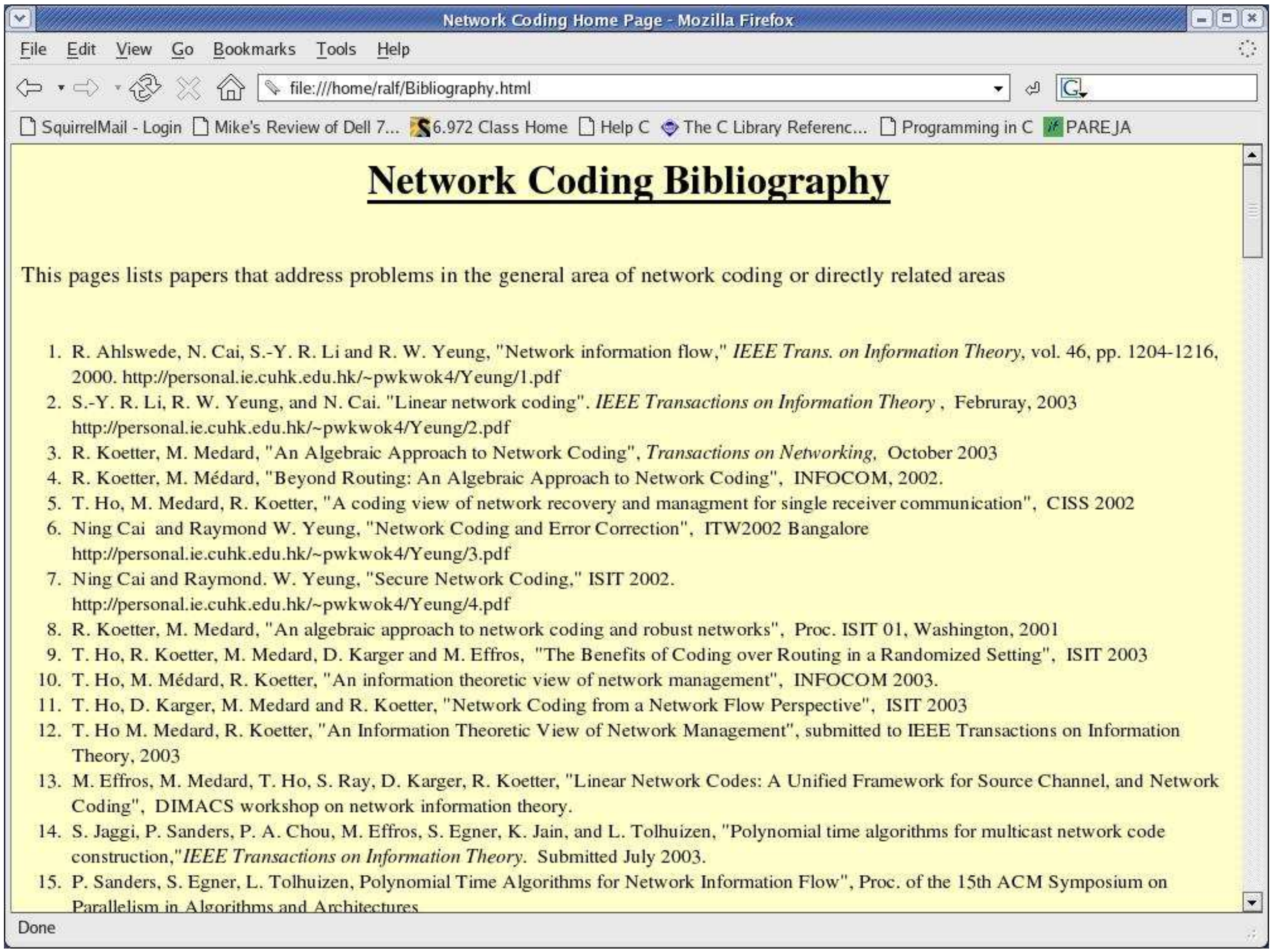
- The non-multicast case (multiple unicast case) is a combinatorial problem and far more difficult with even decideability of vector linear problems unknown.

- In general, linear solutions are not sufficient to achieve capacity

- A major benefit of network coding is the opportunity to cast network problems as flow problems which are efficiently solvable

- Network coding is characterized as transmitting evidence rather than information directly, We get all the benefits of diversity (not only the benefits of diversity routing).

Thanks! www.networkcoding.info



Network Coding Bibliography

This pages lists papers that address problems in the general area of network coding or directly related areas

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