

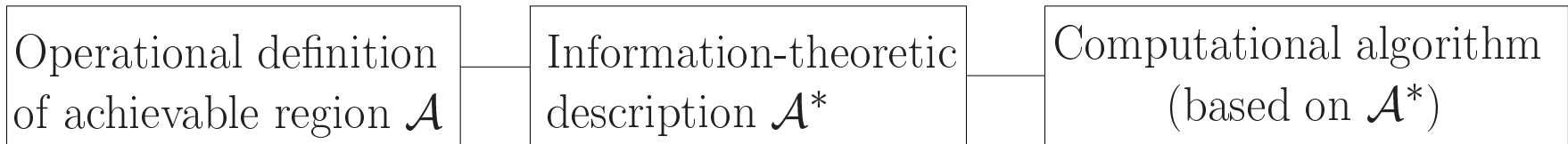
A Unified Approach to Multiterminal Source Coding

Richard Blahut and Soumya Jana

Department of Electrical and Computer engineering
University of Illinois, Urbana, IL 61801

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Mathematical Theory of Source Coding



Example: *Shannon's rate-distortion problem*

\mathcal{A} based on ϵ -achievability
(\mathcal{A} convex)

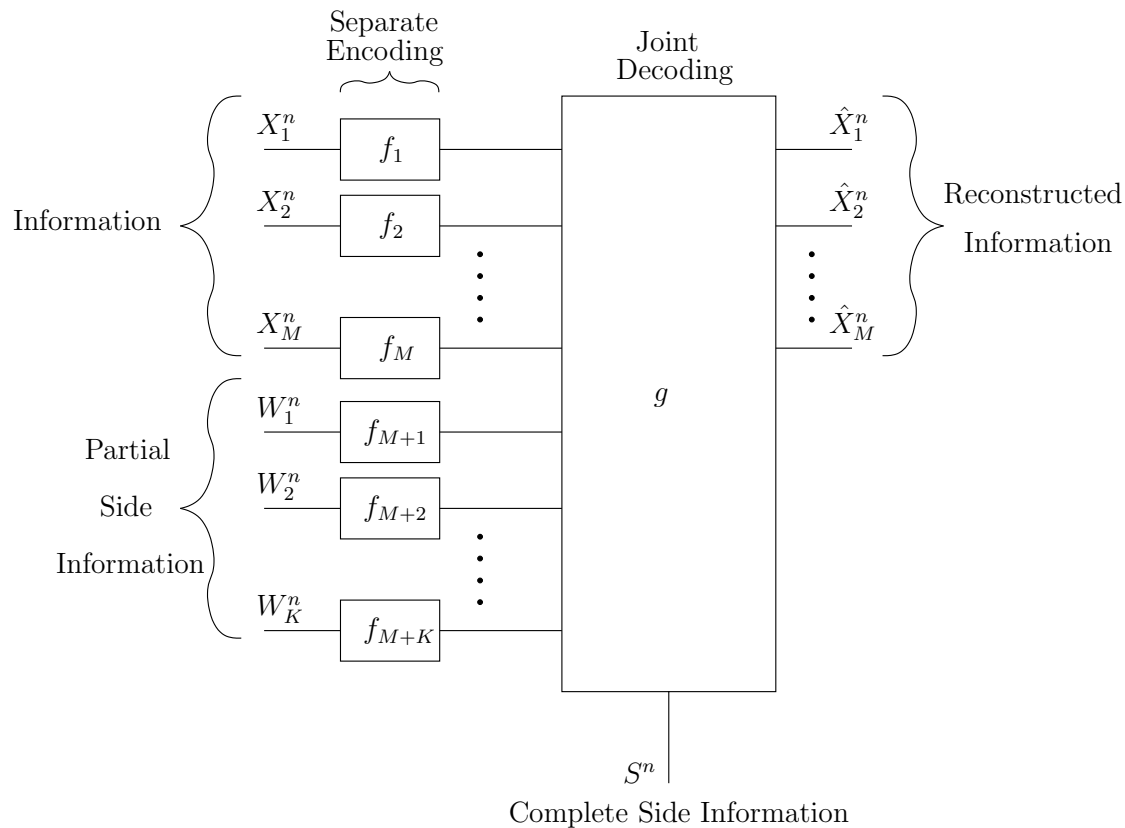
\mathcal{A}^* based on mutual information
(envelope: rate-distortion function)

Blahut algorithm based on alternating projection onto convex sets

We give a unified formula \mathcal{A}^* ($= \mathcal{A}$) for source coding problems.

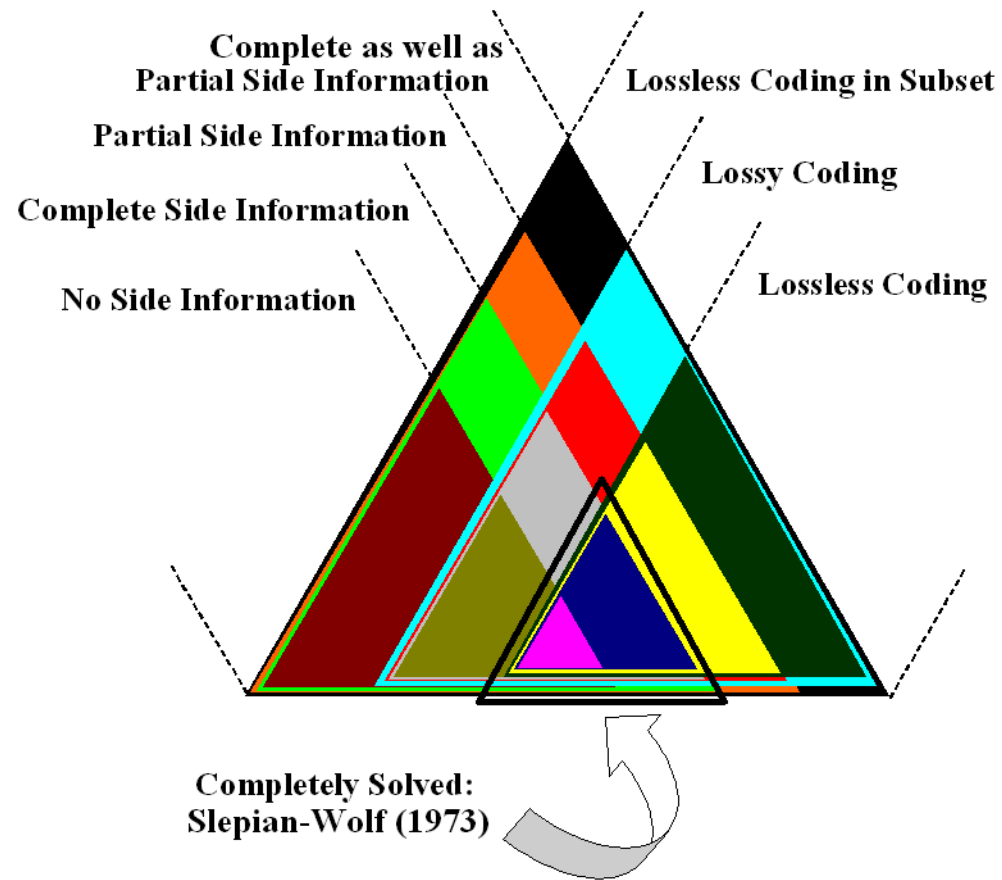
Generic Multiterminal Source Coding Problem

Consider $(X_1, \dots, X_M, W_1, \dots, W_K, S) \sim p$ and draw i.i.d. copies.

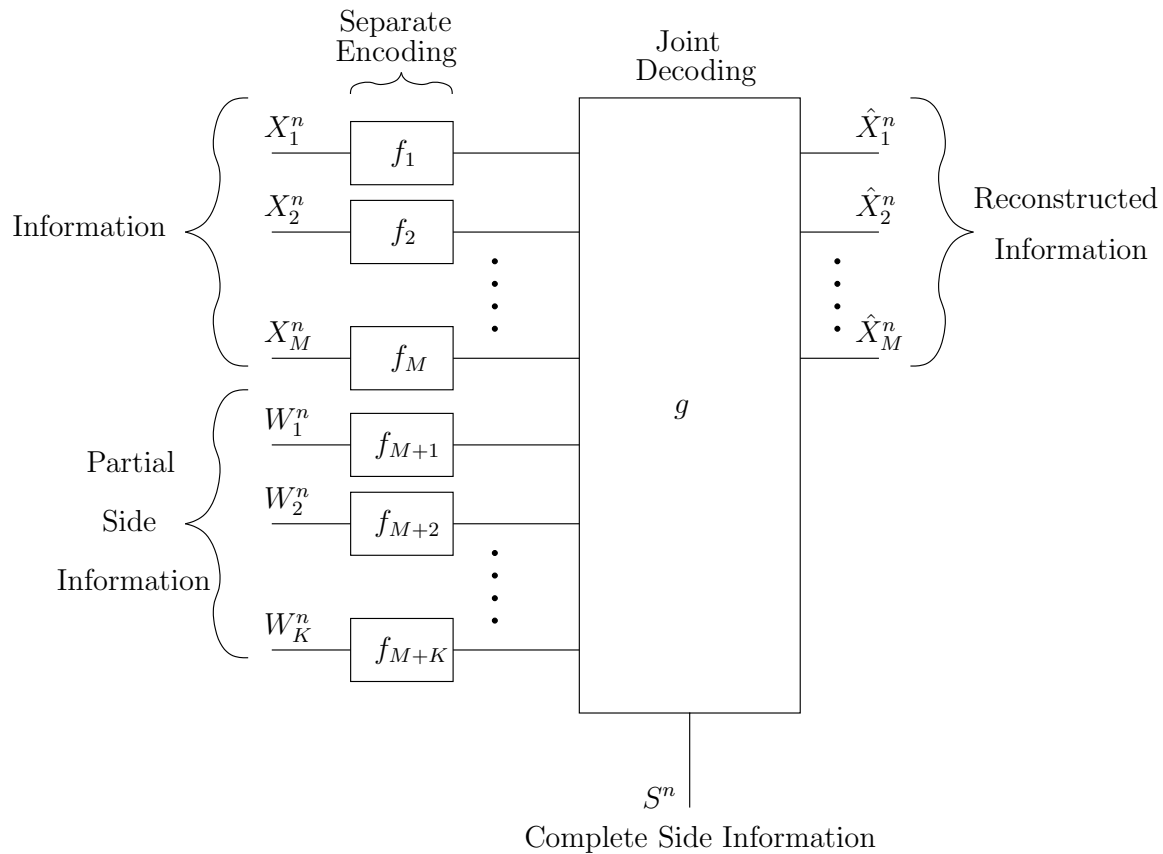


- Lossless reconstruction in $\{1, 2, \dots, J\}$
- L metrics on $\{J + 1, J + 2, \dots, M\}$
- Problem parameters: M, K, J, L , presence/absence of S .

Relation among Multiterminal Problems



Special Case: Slepian-Wolf Problem



- M arbitrary
- $K = 0$
- $J = M$
- S absent/present

Slepian-Wolf Region: $M = 3$

$$H(X_1|X_2, X_3) \leq R_1$$

$$H(X_2|X_1, X_3) \leq R_2$$

$$H(X_1, X_2|X_3) \leq R_1 + R_2$$

$$H(X_3|X_1, X_2) \leq R_3$$

$$H(X_1, X_3|X_2) \leq R_1 + R_3$$

$$H(X_2, X_3|X_1) \leq R_2 + R_3$$

$$H(X_1, X_2, X_3) \leq R_1 + R_2 + R_3$$

General Setup: Salient Features of Unified Formula \mathcal{A}^*

- **Generic form** $\mathcal{A}^* = \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^*}$:
 - Formula \mathcal{A}_n^* contains information-theoretic quantities involving n -tuple $(X_1^n, \dots, X_M^n, W_1^n, \dots, W_K^n, S^n)$ and auxiliary random variables.
- **Convexity:**
 - Individual \mathcal{A}_n^* not necessarily convex;
 - \mathcal{A}^* always convex (since $\mathcal{A}^* = \mathcal{A}$).
- **In all (previously) solved special cases** (e.g., Wyner-Ziv problem),
 - \mathcal{A}_1^* convex;
 - $\mathcal{A}_n^* = \mathcal{A}_1^*$ for each n ;
 - $\mathcal{A}^* = \mathcal{A}_1^*$; coincides with known expression.

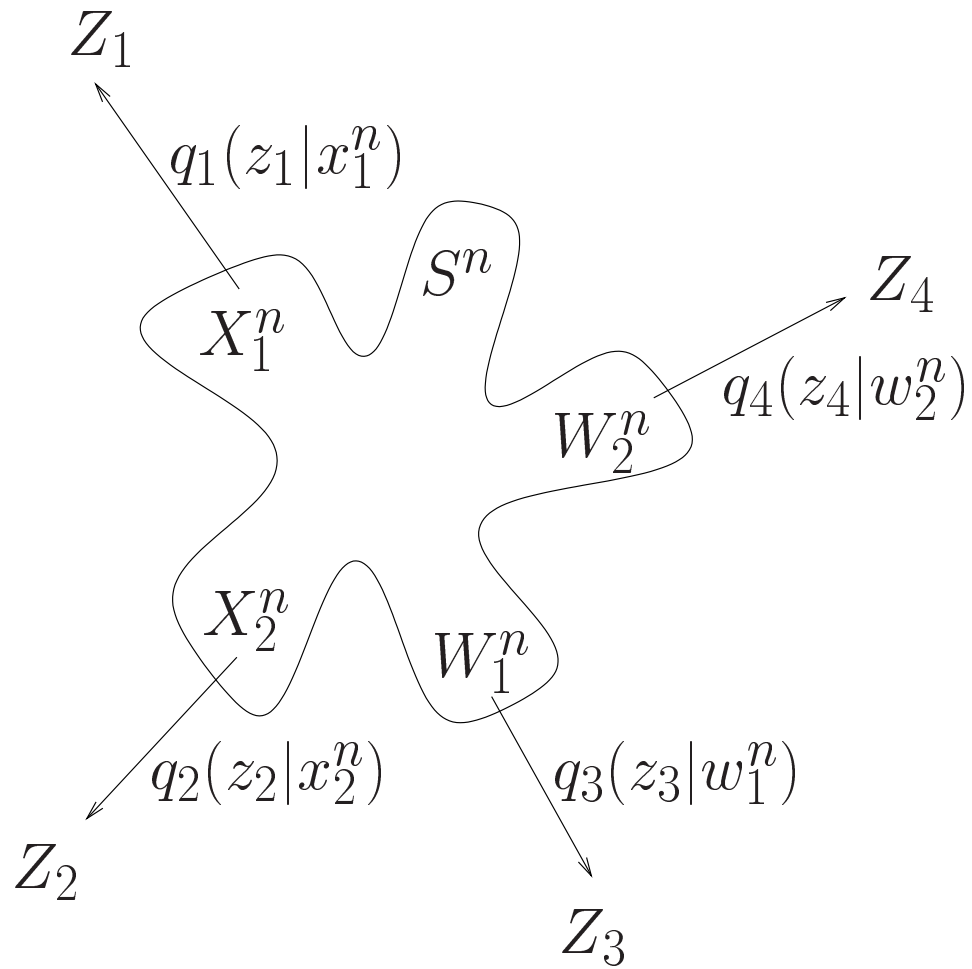
Expression for \mathcal{A}_n^* for $M = 2, K = 2, J = 0, L = 1$

Set of (R_1, R_2, R_3, R_4, D) : $\exists q_1(z_1|x_1^n), q_2(z_2|x_2^n), q_3(z_3|w_1^n), q_4(z_4|w_4^n), \psi$

$$(X_1^n, X_2^n, W_1^n, W_2^n, S^n, Z_1, Z_2, Z_3, Z_4)$$

$$\sim \prod_{k=1}^n p(x_1(k), x_2(k), w_1(k), w_2(k), s(k))$$

$$q_1(z_1|x_1^n)q_2(z_2|x_2^n)q_3(z_3|w_1^n)q_4(z_4|w_2^n)$$



$$\frac{1}{n}I(X_1^n; Z_1|Z_2, Z_3, Z_4, S^n) \leq R_1 \quad (1)$$

$$\frac{1}{n}I(X_2^n; Z_2|Z_1, Z_3, Z_4, S^n) \leq R_2 \quad (2)$$

$$\frac{1}{n}I(X_1^n, X_2^n; Z_1, Z_2|Z_3, Z_4, S^n) \leq R_1 + R_2 \quad (3)$$

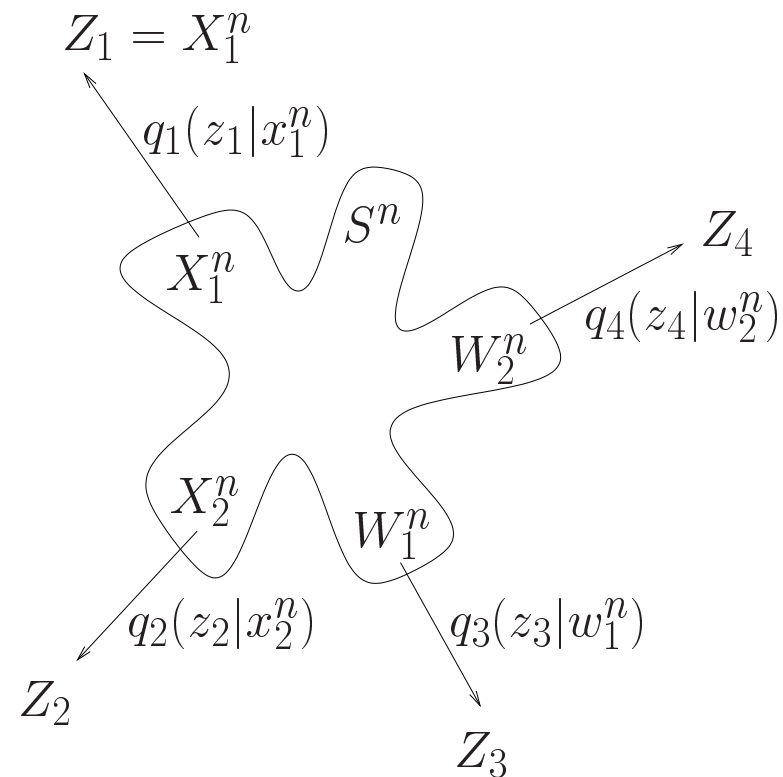
$$\frac{1}{n}I(W_1^n; Z_3|Z_1, Z_2, Z_4, S^n) \leq R_3 \quad (4)$$

$$\frac{1}{n}I(W_2^n; Z_4|Z_1, Z_2, Z_3, S^n) \leq R_4 \quad (5)$$

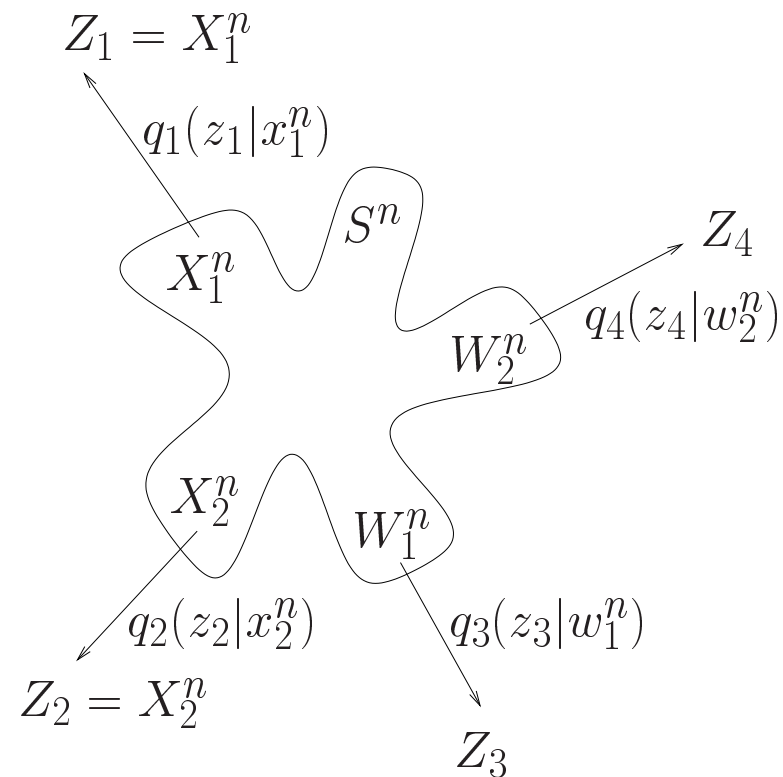
$$\frac{1}{n}I(W_1^n, W_2^n; Z_3, Z_4|Z_1, Z_2, S^n) \leq R_3 + R_4 \quad (6)$$

$$\frac{1}{n}\mathbb{E}d_n((X_1^n, X_2^n), \psi(Z_1, Z_2, Z_3, Z_4, S^n)) \leq D \quad (7)$$

Expression for \mathcal{A}_n^* for $M = 2, K = 2, J = 1, L = 1$



Expression for \mathcal{A}_n^* for $M = 2, K = 2, J = 2$



$$\frac{1}{n}H(X_1^n|X_2^n, Z_3, Z_4, S^n) \leq R_1$$

$$\frac{1}{n}H(X_2^n|X_1^n, Z_3, Z_4, S^n) \leq R_2$$

$$\frac{1}{n}H(X_1^n, X_2^n|Z_3, Z_4, S^n) \leq R_1 + R_2$$

$$\frac{1}{n}I(W_1^n; Z_3|X_1^n, X_2^n, Z_4, S^n) \leq R_3$$

$$\frac{1}{n}I(W_2^n; Z_4|X_1^n, X_2^n, Z_3, S^n) \leq R_4$$

$$\frac{1}{n}I(W_1^n, W_2^n; Z_3, Z_4|X_1^n, X_2^n, S^n) \leq R_3 + R_4$$

Steps in Showing $\mathcal{A} = \mathcal{A}^*$

Proof of inner bound $\mathcal{A} \supseteq \mathcal{A}^*$:

- (*Distortion dissociation: key to unified theory*) Derivation of distortion-independent coding principle;
- Incorporation of distortion metric, when applicable.

Proof of outer bound $\mathcal{A} \subseteq \mathcal{A}^*$:

- Applying simple information-theoretic inequalities;
- Limiting set-theoretic argument.

References

1. S. Jana, “Unified Structural View of Multiterminal Source Coding,” Ph.D. dissertation, University of Illinois, Urbana, IL, 2005.
2. S. Jana, “Unified Theory of Source Coding: Part I – Two-Terminal Problems,” submitted to *IEEE Trans. on Inform. Theory*. Available from

http://arxiv.org/PS_cache/cs/pdf/0508/0508118.pdf.
3. S. Jana, “Unified Theory of Source Coding: Part II – Multiterminal Problems,” submitted to *IEEE Trans. on Inform. Theory*. Available from

http://arxiv.org/PS_cache/cs/pdf/0508/0508119.pdf.

Outline of Remaining Presentation

- Overview: One- and two-terminal problems, both solved and open
- Consistency of unified formula: Partial side information (open) problem
- **Distortion dissociation** principle
 - Shannon's rate-distortion problem (*generalization of Berger's lemma*)
 - Wyner-Ziv problem
 - Joint distortion (open) problem
- Derivation of unified solution for joint distortion problem
- On sufficiency of first-order description, etc.

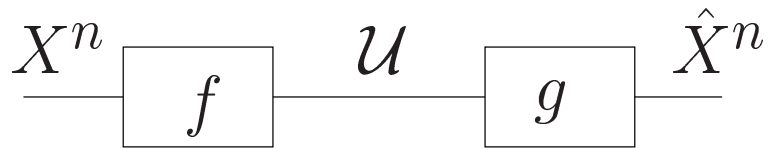
One- and Two-Terminal Problems: Completely Solved

- Shannon's lossless coding problem (1948): $M = 1, K = 0, J = 1$, no S .
- Shannon's rate-distortion problem (1959): $M = 1, K = 0, J = 0, L = 1$, no S .
- Slepian-Wolf problem (1973): $M = 2, K = 0, J = 2$, no S .
- Wyner-Ahlsvede-Körner problem (1975): $M = 1, K = 1, J = 1$, no S .
- Wyner-Ziv problem (1976): $M = 1, K = 0, J = 0, L = 1$, S : yes.
- Berger-Yeung problem (1989): $M = 2, K = 0, J = 1, L = 1$, no S .

One- and Two-Terminal Problems: Hitherto Open

- Partial side information problem: $M = 1, K = 1, J = 0, L = 1$, no S .
 - Inner bound by Berger *et al.* (1979)
- Joint distortion problem: $M = 2, K = 0, J = 0, L = 1$, no S .
 - Inner bound for separable distortion metric by Berger-Tung (1977)

Shannon's Lossless and Lossy Coding Problems



$$\frac{1}{n} \log |\mathcal{U}| \leq R + \epsilon$$

Lossless Coding (1948)

$$\Pr\{X^n \neq \hat{X}^n\} \leq \epsilon$$

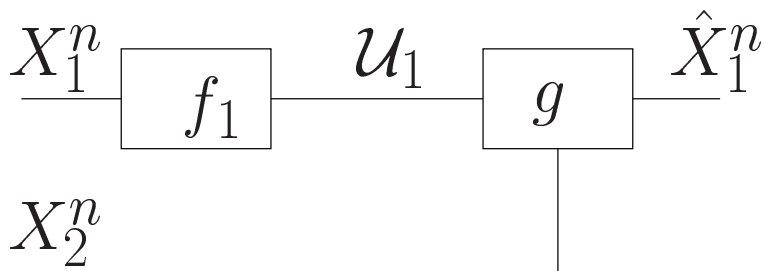
$$H(X) \leq R$$

Lossy Coding (1959)

$$\frac{1}{n} \mathbb{E} d_n(X^n, \hat{X}^n) \leq D + \epsilon$$

$$\begin{aligned} I(X; Z) &\leq R \\ \mathbb{E} d(X, Z) &\leq D \end{aligned}$$

Coding with Complete Side Information



$$\frac{1}{n} \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

Lossless Coding: Corollary of Slepian-Wolf theory (1973)

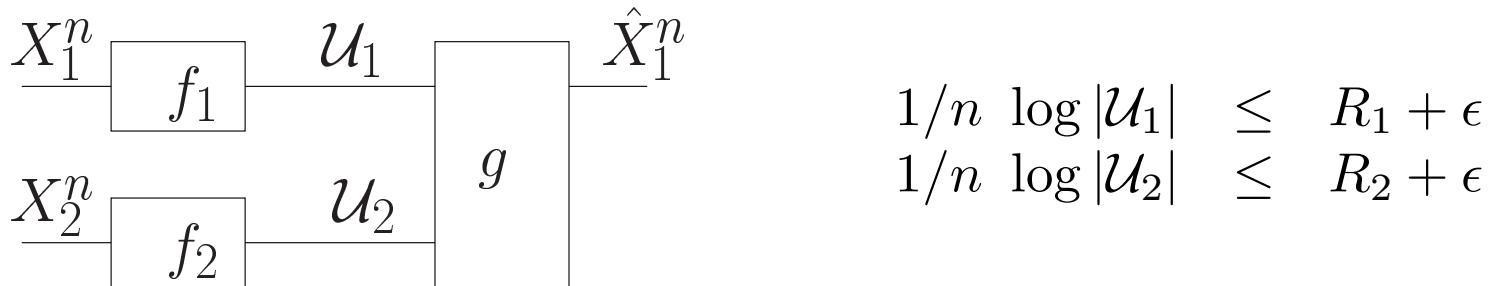
$$\Pr\{X_1^n \neq \hat{X}_1^n\} \leq \epsilon \quad H(X_1|X_2) \leq R_1$$

Lossy coding: Wyner-Ziv (1976)

$$\frac{1}{n} \mathbf{E} d_n(X_1^n, \hat{X}_1^n) \leq D + \epsilon \quad \min_{\psi} \mathbf{E} d(X_1, \psi(Z_1, X_2)) \leq D$$

$$(X_2 \rightarrow X_1 \rightarrow Z_1) \quad I(X_1; Z_1|X_2) \leq R$$

Coding with Partial Side Information



Lossless Coding: Wyner-Ahlsvede-Körner (1975)

$$\Pr\{X_1^n \neq \hat{X}_1^n\} \leq \epsilon$$

$$\begin{aligned} H(X_1|Z_2) &\leq R_1 \\ I(X_2; Z_2) &\leq R_2 \\ (X_1 \rightarrow X_2 \rightarrow Z_2) \end{aligned}$$

Coding with Partial Side Information (Continued)

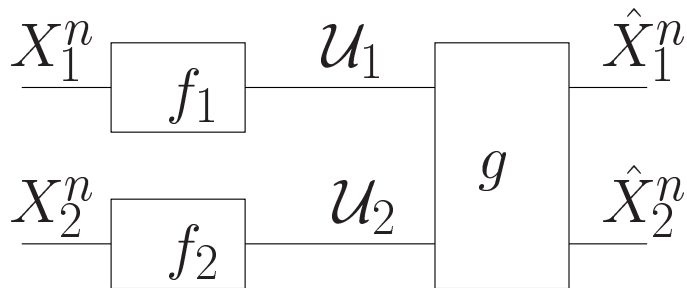
Lossless Coding: Wyner-Ahlsvede-Körner (1975)

$$\Pr\{X_1^n \neq \hat{X}_1^n\} \leq \epsilon \quad \begin{array}{l} H(X_1|Z_2) \leq R_1 \\ I(X_2; Z_2) \leq R_2 \\ (X_1 \rightarrow X_2 \rightarrow Z_2) \end{array}$$

Lossy coding: Inner bound by Berger et al. (1979)

$$\frac{1}{n} \mathbf{E} d_n(X_1^n, \hat{X}_1^n) \leq D + \epsilon \quad \begin{array}{l} I(X_1; Z_1|Z_2) \leq R_1 \\ I(X_2; Z_2) \leq R_2 \\ \min_{\psi} \mathbf{E} d(X_1, \psi(Z_1, Z_2)) \leq D \\ (Z_2 \rightarrow X_2 \rightarrow X_1 \rightarrow Z_1) \end{array}$$

Reconstruction of Both Sources



$$1/n \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

$$1/n \log |\mathcal{U}_2| \leq R_2 + \epsilon$$

Lossless Coding: Slepian-Wolf (1973)

$$\Pr\{(X_1^n, X_2^n) \neq (\hat{X}_1^n, \hat{X}_2^n)\} \leq \epsilon$$

$$H(X_1|X_2) \leq R_1$$

$$H(X_2|X_1) \leq R_2$$

$$H(X_1, X_2) \leq R_1 + R_2$$

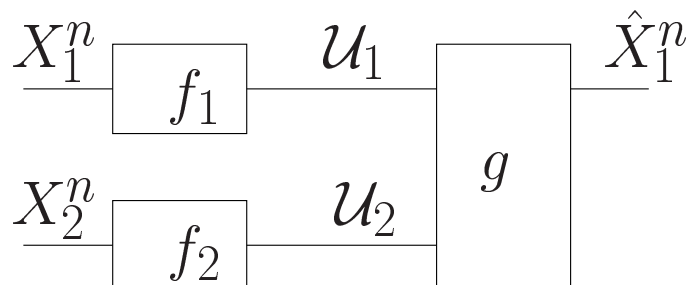
Mixed Lossless-Lossy Coding: Berger-Yeung (1989)

$$\begin{array}{rcl}
 \Pr\{X_1^n \neq \hat{X}_1^n\} & \leq & \epsilon \\
 \frac{1}{n} \mathbf{E}d_n(X_2^n, \hat{X}_2^n) & \leq & D + \epsilon \\
 \end{array}
 \quad
 \begin{array}{rcl}
 H(X_1|Z_2) & \leq & R_1 \\
 I(X_2; Z_2|X_1) & \leq & R_2 \\
 H(X_1) + I(X_2; Z_2|X_1) & \leq & R_1 + R_2 \\
 \min_{\psi} \mathbf{E}d(X_2, \psi(X_1, Z_2)) & \leq & D \\
 (X_1 \rightarrow X_2 \rightarrow Z_2) & &
 \end{array}$$

Lossy coding with separable distortion metric: Inner bound by Berger-Tung (1977)

$$\begin{array}{rcl}
 \Pr\{X_1^n \neq \hat{X}_1^n\} & \leq & \epsilon \\
 \frac{1}{n} \mathbf{E}d_{1n}(X_1^n, \hat{X}_1^n) & \leq & D_1 + \epsilon \\
 \frac{1}{n} \mathbf{E}d_{2n}(X_2^n, \hat{X}_2^n) & \leq & D_2 + \epsilon \\
 \end{array}
 \quad
 \begin{array}{rcl}
 I(X_1; Z_1|Z_2) & \leq & R_1 \\
 I(X_2; Z_2|Z_1) & \leq & R_2 \\
 I(X_1, X_2; Z_1, Z_2) & \leq & R_1 + R_2 \\
 \min_{\psi_1} \mathbf{E}d_1(X_1, \psi_1(Z_1, Z_2)) & \leq & D_1 \\
 \min_{\psi_2} \mathbf{E}d_2(X_2, \psi_2(Z_1, Z_2)) & \leq & D_2 \\
 (Z_1 \rightarrow X_1 \rightarrow X_2 \rightarrow Z_2) & &
 \end{array}$$

Complete Solution: Partial Side Information

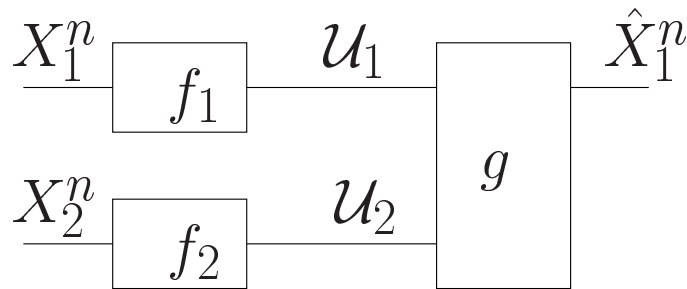


$$\begin{aligned} 1/n \log |\mathcal{U}_1| &\leq R_1 + \epsilon \\ 1/n \log |\mathcal{U}_2| &\leq R_2 + \epsilon \\ 1/n \mathbb{E} d_n(X_1^n, \hat{X}_1^n) &\leq D + \epsilon \end{aligned}$$

$$\mathcal{A}^* = \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^*} \quad \text{with } \mathcal{A}_n^*:$$

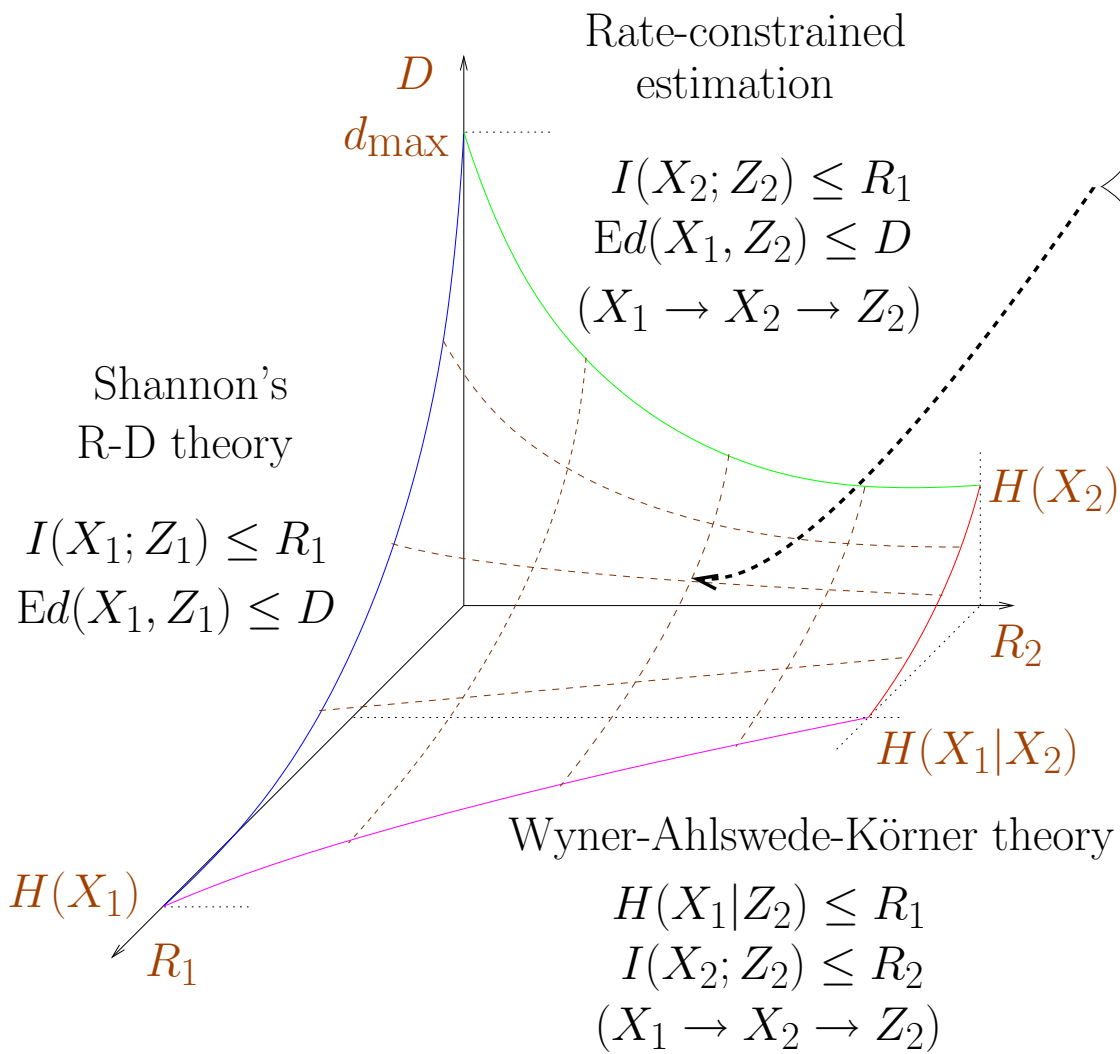
$$\begin{aligned} 1/n I(X_1^n; Z_1 | Z_2) &\leq R_1 \\ 1/n I(X_2^n; Z_2) &\leq R_2 \\ \min_{\psi} 1/n \mathbb{E} d_n(X_1^n, \psi(Z_1, Z_2)) &\leq D \\ &(Z_2 \rightarrow X_2 \rightarrow X_1 \rightarrow Z_1) \end{aligned}$$

Consistency with Known Special Cases



$$\begin{aligned} 1/n \log |\mathcal{U}_1| &\leq R_1 + \epsilon \\ 1/n \log |\mathcal{U}_2| &\leq R_2 + \epsilon \\ 1/n \mathbb{E} d_n(X_1^n, \hat{X}_1^n) &\leq D + \epsilon \end{aligned}$$

- Shannon's R-D theory: $R_2 = 0$
- Wyner-Ahlsvede-Körner theory: $D = 0$
- Wyner-Ziv theory: $R_2 = H(X_2)$
- Rate-constrained estimation: $R_1 = 0$



$$\mathcal{A}_n^*$$

$$\frac{1}{n}I(X_1^n; Z_1|Z_2) \leq R_1$$

$$\frac{1}{n}I(X_2^n; Z_2) \leq R_2$$

$$\frac{1}{n}Ed_n(X_1^n, \psi(Z_1, Z_2)) \leq D$$

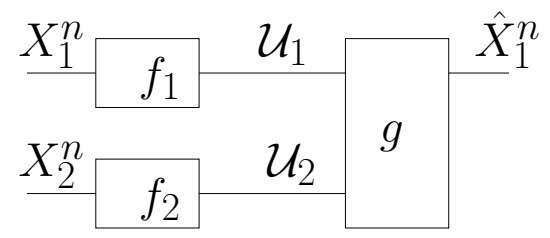
$$(Z_1 \rightarrow X_1^n \rightarrow X_2^n \rightarrow Z_2)$$

Wyner-Ziv theory

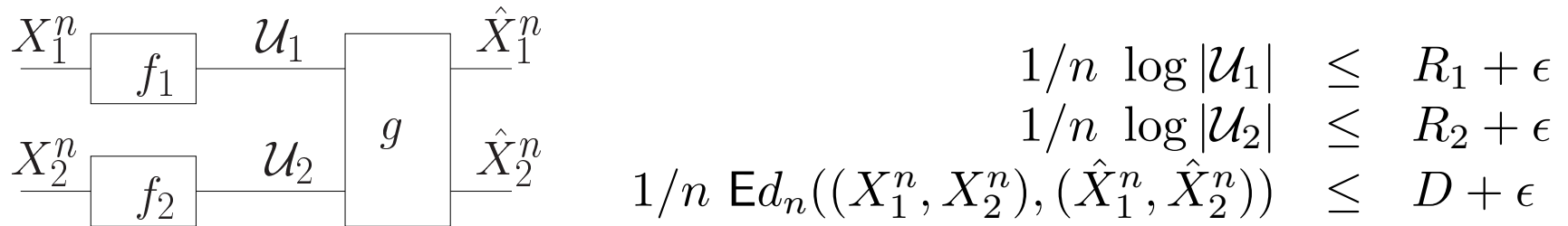
$$I(X_1; Z_1|X_2) \leq R_1$$

$$Ed(X_1, \psi(Z_1, X_2)) \leq D$$

$$(Z_1 \rightarrow X_1 \rightarrow X_2)$$



Complete Solution: Joint Distortion Problem



$$\mathcal{A}^* = \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^*} \quad \text{with } \mathcal{A}_n^*:$$

$$1/n I(X_1^n; Z_1 | Z_2) \leq R_1$$

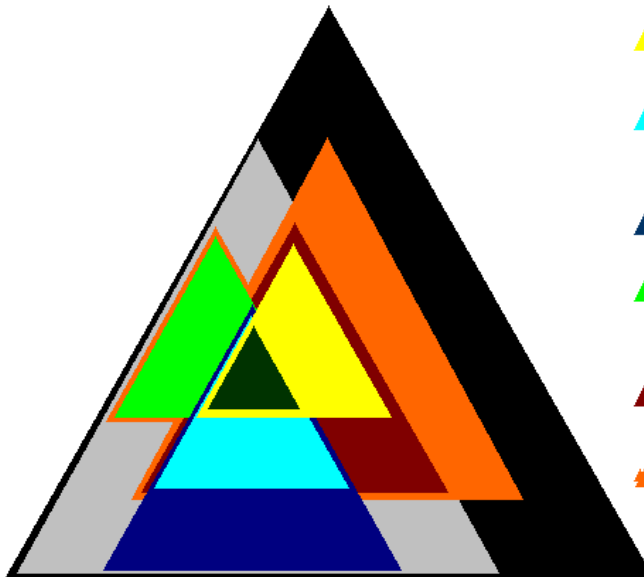
$$1/n I(X_2^n; Z_2 | Z_1) \leq R_2$$

$$1/n I(X_1^n, X_2^n; Z_1, Z_2) \leq R_1 + R_2$$

$$\min_{\psi} 1/n \mathbf{E} d_n((X_1^n, X_2^n), \psi(Z_1, Z_2)) \leq D$$

$$(Z_2 \rightarrow X_2 \rightarrow X_1 \rightarrow Z_1)$$

Relation among One- and Two-Terminal Problems



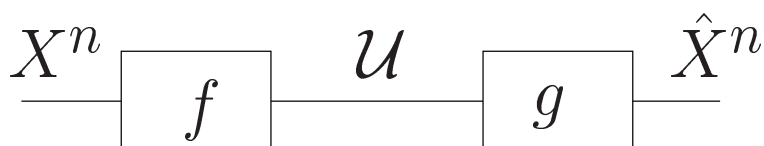
- ▲ **Shannon's Lossless Coding (1948)**
- ▲ **Shannon's Lossy Coding (1959)**
- ▲ **Lossless Coding with Complete Side Information: Slepian-Wolf (1973)**
- ▲ **General Slepian-Wolf Problem (1973)**
- ▲ **Lossless Coding with Partial Side Information: Wyner/Ahlsvede-Korner (1975)**
- ▲ **Wyner-Ziv Problem (1976)**
- ▲ **Partial Side Information Problem: Inner Bound on achievable region by Berger *et. al* (1979); Complete solution hitherto unknown**
- ▲ **Berger-Yeung Problem (1989)**
- ▲ **Joint Distortion Problem: Hitherto unsolved [Inner bound for separable distortion metric by Berger-Tung (1977)]**

Distortion Dissociation Principle: Key to Unification

Distortion-independent problem:

- Depends on structure
- Based on strong typicality

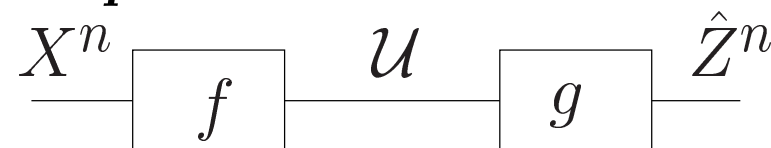
Shannon problem



$$1/n \log |\mathcal{U}| \leq R + \epsilon$$

$$1/n \mathbf{E}d_n(X^n, \hat{X}^n) \leq D + \epsilon$$

Step 1:



$$1/n \log |\mathcal{U}| \leq R + \epsilon$$

$$\Pr\{(X^n, \hat{Z}^n) \notin \mathcal{T}_\epsilon^{(n)}(X, Z)\} \leq \epsilon$$

Step 2:

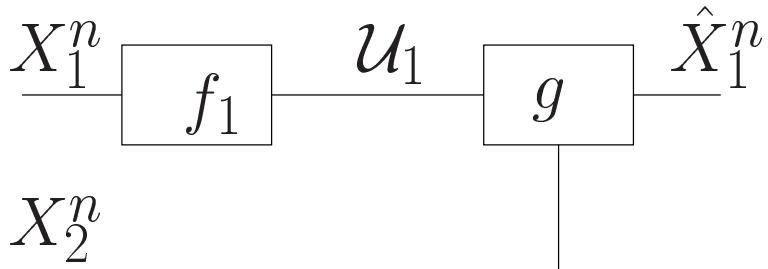
Choose $q(z|x)$: $\mathbf{E}d(X, Z) \leq D$.

Then

$$1/n \mathbf{E}d_n(X^n, \hat{Z}^n) \leq D + \epsilon d_{\max}.$$

Set $\hat{X}^n = \hat{Z}^n$.

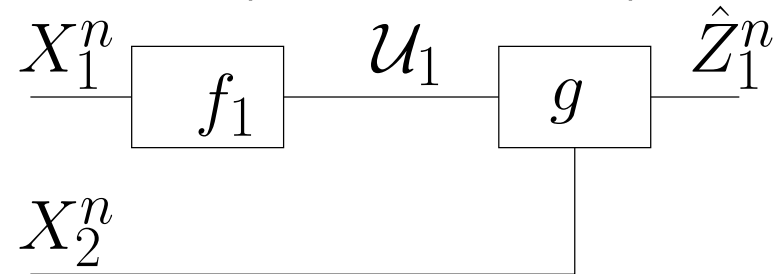
Wyner-Ziv problem



$$1/n \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

$$1/n \mathbf{E}d_n(X_1^n, \hat{X}_1^n) \leq D + \epsilon$$

Step 1: ($X_2 \rightarrow X_1 \rightarrow Z_1$)



$$1/n \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

$$\Pr\{(X_1^n, X_2^n, \hat{Z}_1^n) \notin \mathcal{T}_\epsilon^{(n)}(X_1, X_2, Z_1)\} \leq \epsilon$$

Step 2:

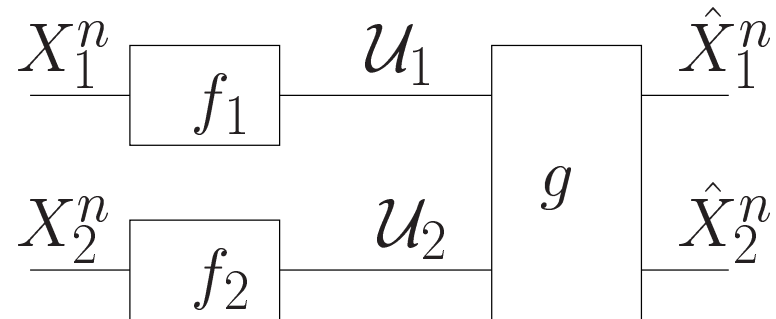
Choose $q_1(z_1|x_1)$:

$\mathbf{E}d(X_1, \psi(Z_1, X_2)) \leq D$. Then

$1/n \mathbf{E}d_n(X_1^n, \psi_n(\hat{Z}_1^n, X_2^n)) \leq D + \epsilon d_{\max}$.

Set $\hat{X}_1^n = \psi_n(\hat{Z}_1^n, X_2^n)$.

Joint distortion problem

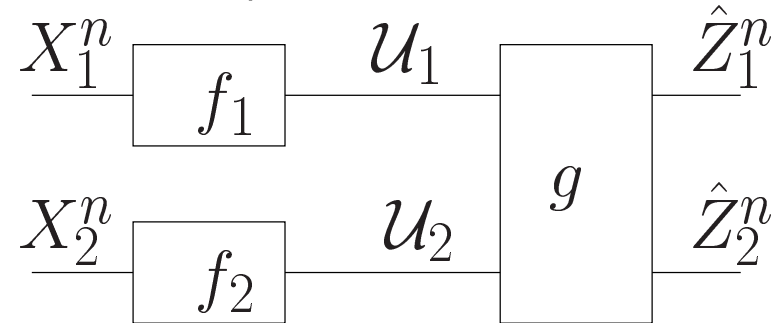


$$1/n \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

$$1/n \log |\mathcal{U}_2| \leq R_2 + \epsilon$$

$$1/n \mathbf{E}d_n((X_1^n, X_2^n), (\hat{X}_1^n, \hat{X}_2^n)) \leq D + \epsilon$$

Step 1: ($Z_2 \rightarrow X_2 \rightarrow X_1 \rightarrow Z_1$)



$$1/n \log |\mathcal{U}_1| \leq R_1 + \epsilon$$

$$1/n \log |\mathcal{U}_2| \leq R_2 + \epsilon$$

$$\Pr\{(X_1^n, X_2^n, \hat{Z}_1^n, \hat{Z}_2^n) \notin \mathcal{T}_\epsilon^{(n)}(X_1, X_2, Z_1, Z_2)\} \leq \epsilon$$

Step 2:

Choose $q_1(z_1|x_1)$ and $q_2(z_2|x_2)$:

$\mathbf{E}d((X_1, X_2), \psi(Z_1, Z_2)) \leq D$. Then

$$1/n \mathbf{E}d_n((X_1^n, X_2^n), \psi_n(\hat{Z}_1^n, \hat{Z}_2^n)) \leq D + \epsilon d_{\max}.$$

Set $(\hat{X}_1^n, \hat{X}_2^n) = \psi_n(\hat{Z}_1^n, \hat{Z}_2^n)$.

Distortion-Independent Problems: Inner Bound on Achievable Rate-Region

- Shannon problem: $I(X; Z) \leq R$
- Wyner-Ziv problem: $I(X_1; Z_1|X_2) \leq R_1$
- Joint distortion problem:

$$I(X_1; Z_1|Z_2) \leq R_1$$

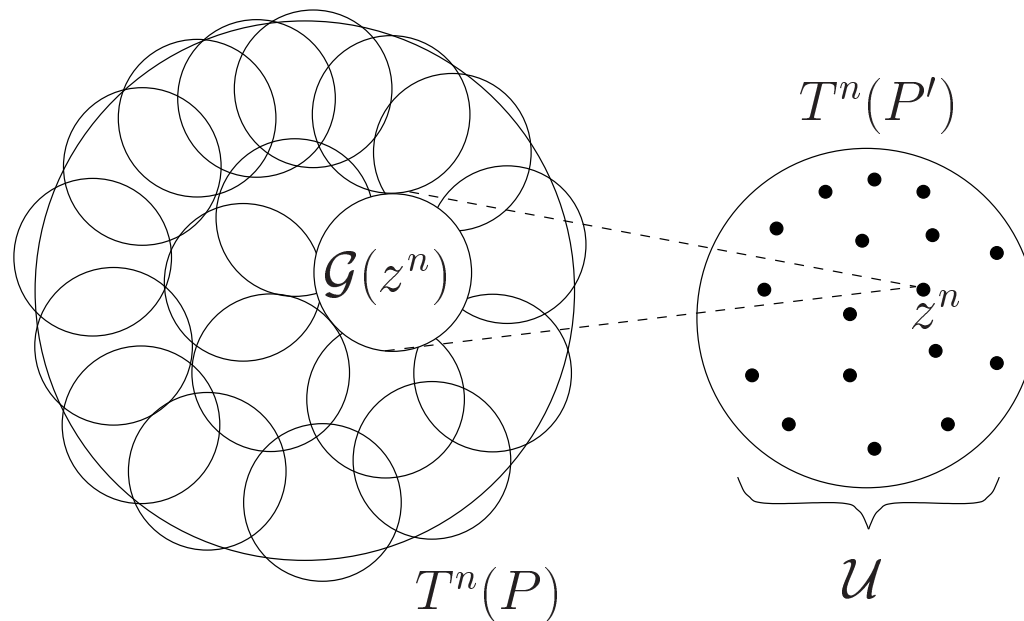
$$I(X_2; Z_2|Z_1) \leq R_2$$

$$I(X_1, X_2; Z_1, Z_2) \leq R_1 + R_2$$

Distortion-Independent Shannon Problem

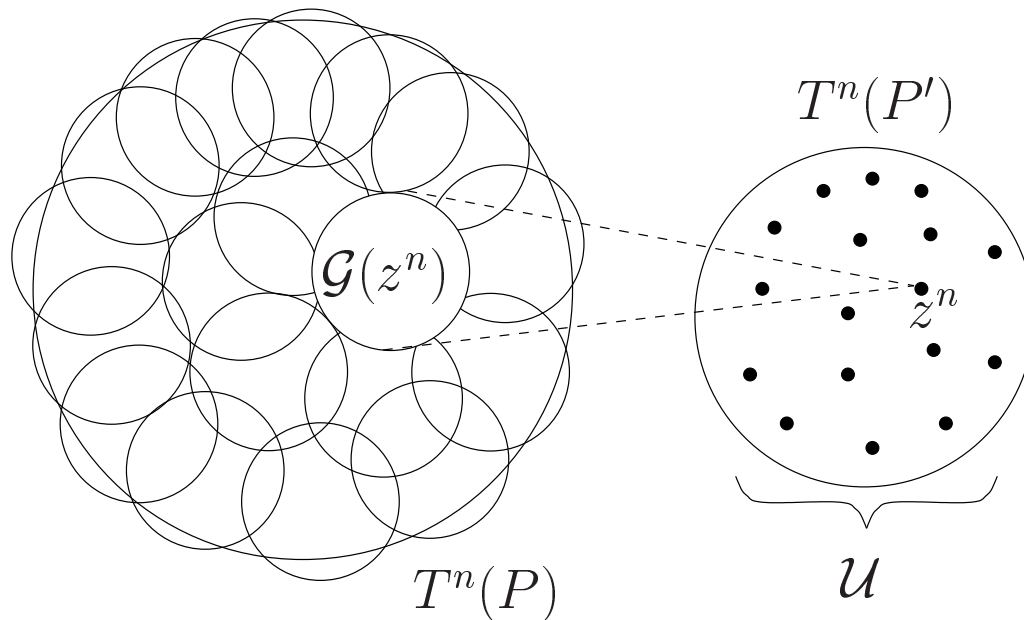
- Inner bound: $I(X; Z) \leq R$
- Derivation:
 - Using strong typicality: Concise, little insight
 - Using method of types: Combinatorial insight (type covering), generalization of Berger's lemma

Our Type Covering Result



- $\bar{P}(x, z) = P(x)Q(z|x)$
and P' is Z -marginal.
- $\mathcal{G}(z^n) = \{x^n\}$: (x^n, z^n) of
joint type \bar{P} .
- $|\mathcal{G}(z^n)| \approx 2^{nH(X|Z)}$.
- $\frac{1}{n} \log |\mathcal{U}| \leq I(X; Z) + \epsilon$.
- $\mathcal{G}(z^n)$'s cover $T^n(P)$.

Distortion-Dependent Choice



- For any $Q(z|x)$ such that $\mathbb{E} d(X, Z) \leq D$,

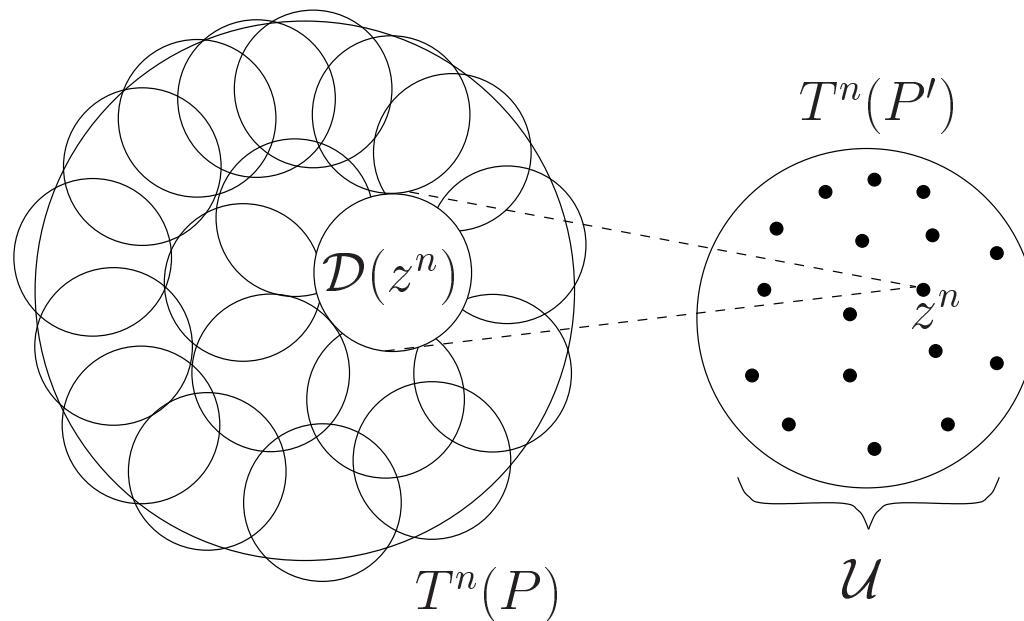
$$\frac{1}{n} d_n(x^n, z^n) \leq D,$$

$$\forall x^n \in \mathcal{G}(z^n), z^n \in \mathcal{U}.$$

- Required bit-rate

$$\inf_Q \frac{1}{n} \log |\mathcal{U}| \leq \inf_Q I(X; Z) + \epsilon.$$

Berger's Lemma as Corollary



- $\mathcal{D}(z^n)$: D -ball around z^n .
- (Recall) $\mathcal{G}(z^n) \subseteq \mathcal{D}(z^n)$ for any D -admissible $Q(z|x)$.
- $\mathcal{D}(z^n)$'s cover $T^n(P)$.
- $\frac{1}{n} \log |\mathcal{U}| \leq \inf I(X; Z) + \epsilon$,
inf over admissible $Q(z|x)$'s.

Joint-Distortion Problem: Rate-Region in Distortion-Independent Version

- Inner bound:

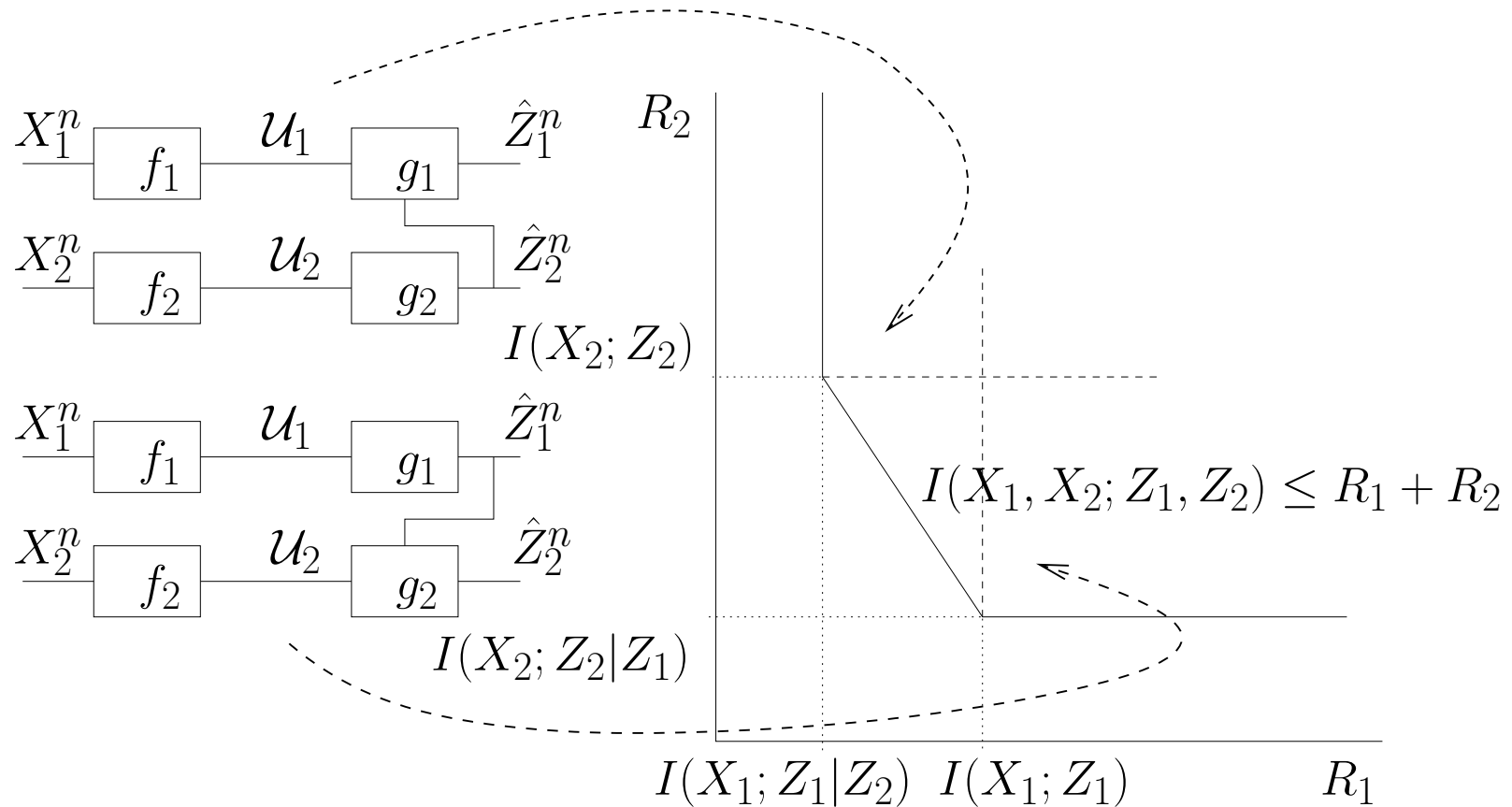
$$I(X_1; Z_1|Z_2) \leq R_1$$

$$I(X_2; Z_2|Z_1) \leq R_2$$

$$I(X_1, X_2; Z_1, Z_2) \leq R_1 + R_2$$

- Derivation using distortion-independent
 - Shannon problem: $I(X; Z) \leq R$
 - Wyner-Ziv problem: $I(X_1; Z_1|X_2) \leq R_1$

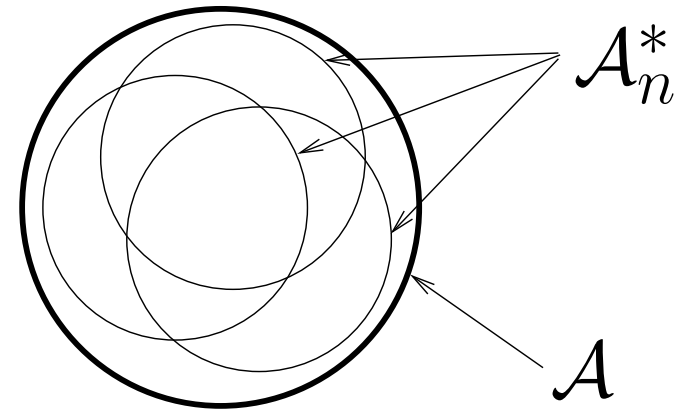
Joint-Distortion Problem: Distortion-Independent Region



Proof of Inner Bound: $\mathcal{A} \supseteq \mathcal{A}^*$

- Incorporation of distortion: $\mathcal{A} \supseteq \mathcal{A}_1^*$
- n -tuple $\Rightarrow \mathcal{A} \supseteq \mathcal{A}_n^*$
- Taking union and closure:

$$\mathcal{A} \supseteq \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^*} = \mathcal{A}^*.$$



Proof of Outer Bound: $\mathcal{A} \subseteq \mathcal{A}^*$

$$(R_1, R_2, D) \in \mathcal{A}$$

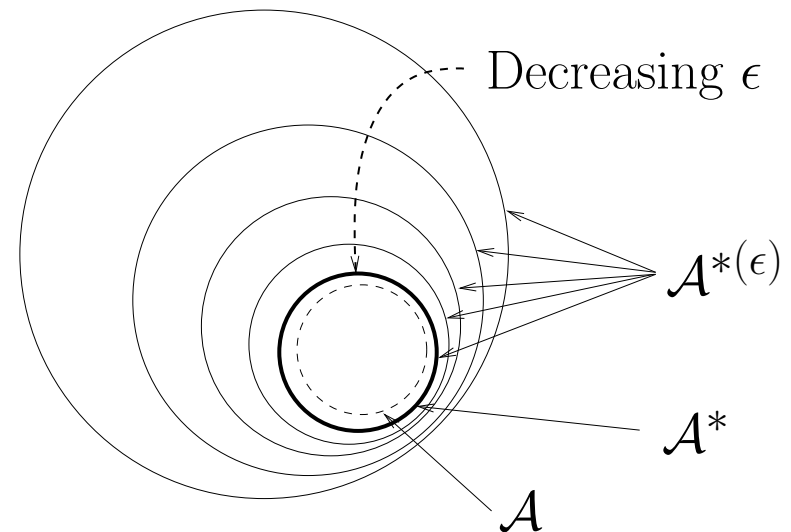
\Rightarrow For any $\epsilon > 0$ and some $n(\epsilon)$,

$$(R_1 + \epsilon, R_2 + \epsilon, D + \epsilon) \in \mathcal{A}_n^*$$

(i.e., $(R_1, R_2, D) \in \mathcal{A}_n^{*(\epsilon)}$)

$$\Rightarrow (R_1, R_2, D) \in \mathcal{A}^{*(\epsilon)} = \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^{*(\epsilon)}}$$

$$\Rightarrow (R_1, R_2, D) \in \mathcal{A}^* = \bigcap_{\epsilon > 0} \mathcal{A}^{*(\epsilon)}$$

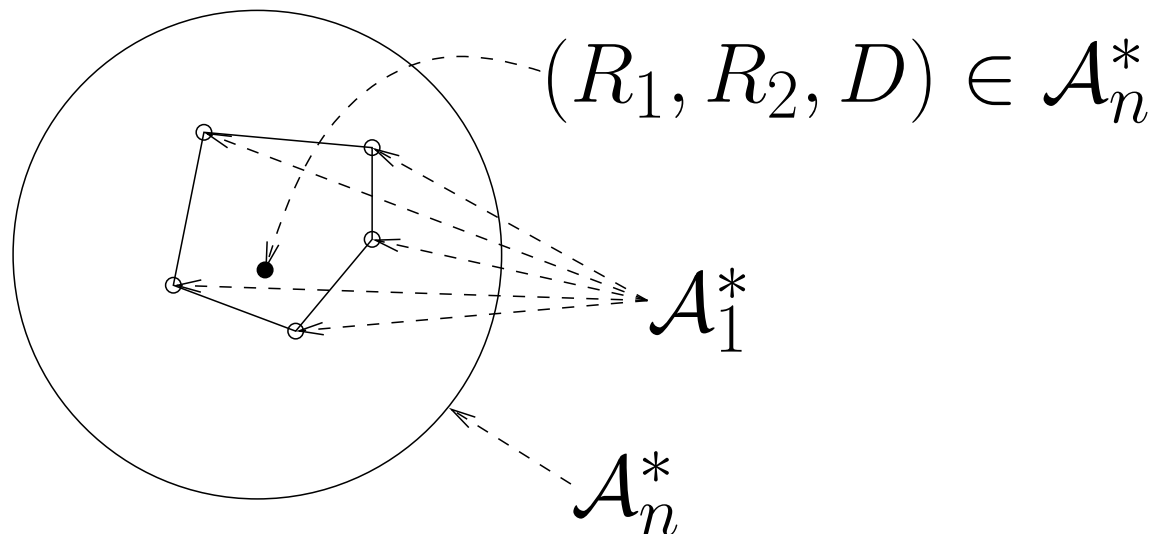


Representation by Convex Hulls of \mathcal{A}_n^*

- We know: $\mathcal{A} = \mathcal{A}^* = \overline{\bigcup_{n=1}^{\infty} \mathcal{A}_n^*}$ is convex.
- Define: $\mathcal{B}_n^* = \text{convex hull of } \mathcal{A}_n^*$; $\mathcal{B}^* = \overline{\bigcup_{n=1}^{\infty} \mathcal{B}_n^*}$.
- Then: $\mathcal{A} = \mathcal{B}^*$.

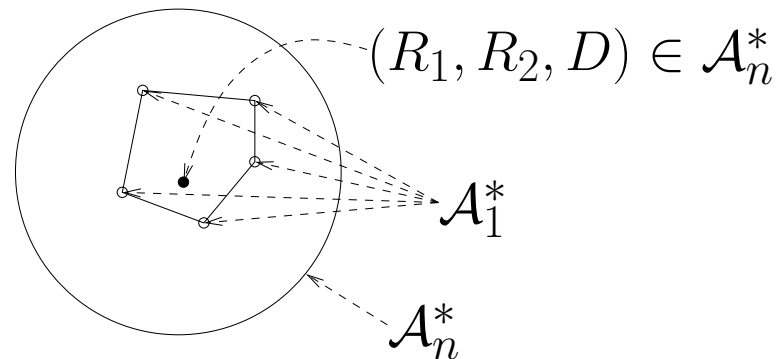
Necessary and Sufficient Condition: $\mathcal{A} = \mathcal{B}_1^*$

For each n and any $(R_1, R_2, D) \in \mathcal{A}_n^*$:



Necessary and Sufficient Condition: $\mathcal{A} = \mathcal{A}_1^*$

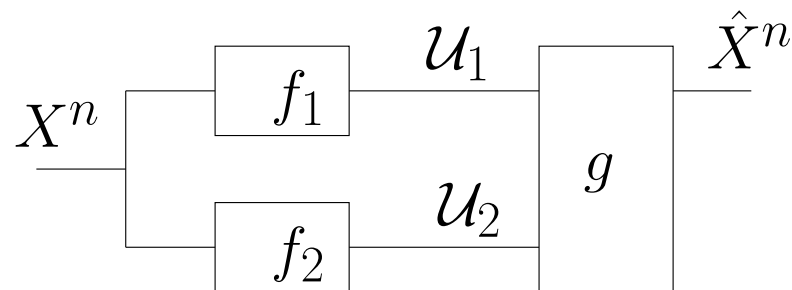
1. Condition for $\mathcal{A} = \mathcal{B}_1^*$:



2. Convexity of \mathcal{A}_1^* .

Illustration of Nonconvexity of \mathcal{A}_1^*

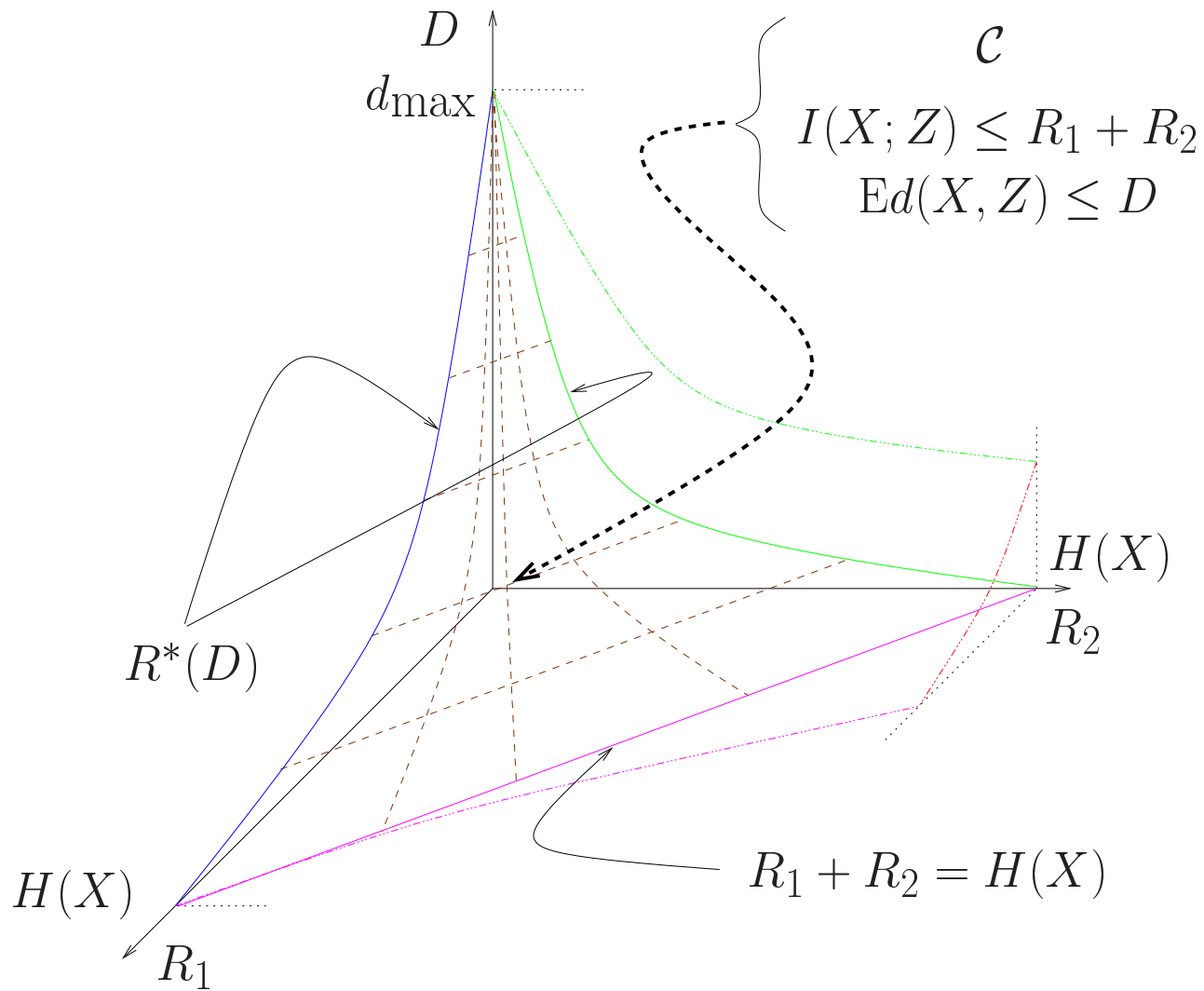
Partial side information problem: $X_1 = X_2 = X \sim (1/2, 1/2)$



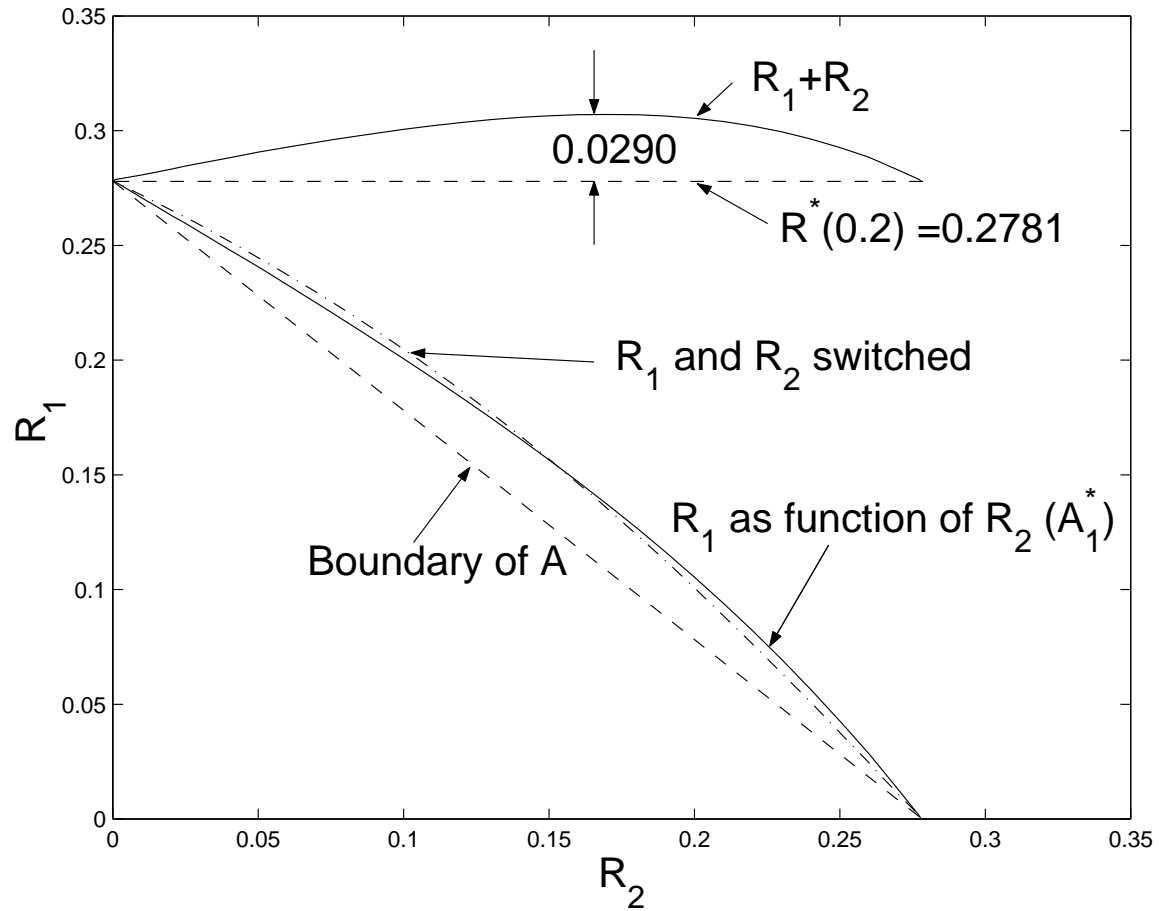
Rate-distortion region: $\mathcal{C} (= \mathcal{A})$

$$I(X; Z) \leq R_1 + R_2$$

$$Ed(X, Z) \leq D$$



$$A_1^* \neq C = A = B_1^*$$



Open Questions

1. Condition for first-order description
2. Topology of \mathcal{A}_n^*
3. Computational algorithm