MPSK Modulated Constellation Design for Differential Space-Time Modulation

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Abstract—The constellation design for differential space-time modulation usually requires to construct \( L \) (constellation size) unitary matrices making the design complexity increasing rapidly with \( L \). In this correspondence, we propose a new construction technique, which exploits the rotational invariance of unitary matrices and the property of MPSK modulation. The proposed constellations are derivable from a smaller set of \( D = L/M \) unitary matrices (via rotations defined by an \( M \)-ary PSK symbol) which, in turn, are parameterized by two unitary matrices \( U_1 \) and \( U_2 \). They share some good property of group constellations thereby considerably simplifying the optimization of \( U_1 \) and \( U_2 \). They also possess other good properties which allow for simplified maximum likelihood (ML) decoding. Extensive numerical examples are presented to demonstrate that the proposed constellations in general have larger diversity products than other constellations and thus, better error performance.

Index Terms—Differential unitary space-time (DUST) modulation, diversity product.

I. INTRODUCTION

ON-COHERENT space-time modulation/coding eliminates the need for costly channel estimation, rendering it very attractive to multiple-antenna wireless systems [1]. Its use is further justified in recent theoretic studies [2], [3], which reveal that multiple-antenna systems with non-coherent reception still have large capacity. In this correspondence, we will focus on differential unitary space-time (DUST) modulation [4], [5]. The advantages of adopting differential unitary structure in space-time modulation/coding lie in its capability to approach the non-coherent system capacity [6] and minimum error probability [7] while decoding is relatively simple.

The design of DUST constellations/codes \(^1\) is usually done under the performance criterion of diversity product [4]. For a system with constellation size \( L \) and \( N_t \) transmit antennas, we need to determine \( L \) unitary matrices of size \( N_t \times N_t \) in general. To make the design tractable, it is common practice to impose certain structure upon the candidate matrices. The DUST constellations that form a group under matrix multiplication have received particular attention. In [8], all finite fixed-point-free (full diverse) group codes are thoroughly investigated and characterized. It is further shown in [9] that cyclic group codes are optimal group codes of size \( 2^p \) where \( p \) is an integer. Research efforts have also been extended to non-group codes resulting in improved cyclic codes [10], [11], Cayley codes [12], and codes from amicable designs [13]. Note that Cyclic group codes are less restrictive allowing for constellation size \( L \) to be arbitrary, at the cost of relatively inferior performance. Other group codes have fine structures and generally better performance. The tradeoff is that considerable restrictions must be imposed upon the number of transmit antennas and constellation size \( L \). The performance of non-group codes can be even better since they have more degrees of freedom in structure. The possible drawback is the increased decoding complexity.

A natural question is the likelihood to construct a non-group constellation that has good performance, relatively simple decoding, and little or no restriction on the constellation size \( L \) and the number of transmit antennas \( N_t \). We propose a class of MPSK modulated DUST (termed M-DUST) constellations which are constructed from a small set of unitary matrices via rotations defined by MPSK modulation. The proposed M-DUST constellations belong to non-group constellations since, though derivable from a subset of the infinite unitary group [14], they do not constitute a group themselves.

II. DIFFERENTIAL UNITARY SPACE-TIME MODULATION

Consider a communication system with \( N_t \) transmit and \( N_r \) receive antennas. The DUST constellation used is composed of \( L \) unitary matrices of size \( N_t \times N_t \) \( \{V_1, V_2, \ldots , V_L \} \). Let \( S_\tau \) denote an \( N_t \times N_t \) signal matrix for the \( \tau \)th block. The DUST modulation can be expressed as

\[
S_\tau = V_{z_\tau} S_{\tau-1}
\]

where the matrix \( V_{z_\tau} \in \{V_1, V_2, \ldots , V_L \} \) stands for the transmitted information in the \( \tau \)th block.

We assume that channels linking each pair of transmit and receive antennas suffer from independent and identically distributed (i.i.d.) flat Rayleigh fading. Thus, if we denote the channel matrix observed at block \( \tau \) by an \( N_t \times N_r \) matrix \( H_\tau \), its entries follow independent complex Gaussian distribution each having zero mean and unit variance. We further assume slow fading channels such that \( H_\tau = H_{\tau-1} \). The received signal matrix \( X_\tau \) of size \( N_t \times N_r \) is given by

\[
X_\tau = \sqrt{p_{\tau}} H_\tau + W_\tau, \quad \tau = 0, 1, \ldots ,
\]

where \( W_\tau \) is an \( N_t \times N_r \) additive noise matrix whose entries are independent zero-mean Gaussian variable with unit

\[
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\]
variance, and ρ signifies the signal-to-noise (SNR) ratio at each receive antenna.

At the receiver, non-coherent maximum likelihood (ML) decoding is used to produce an estimate

\[
\hat{x}_\tau = \arg\min_{l=1,2,\ldots,L} \|X_l - V_{X_{\tau-1}}\|_F^2
\]

where \(\cdot\) denotes Frobenius norm, trace, real, and complex conjugate transpose, respectively.

### III. M-DUST CONSTELLATIONS

The design of DUST constellations is generally difficult, requiring the determination of a set of \(N_1 \times N_1\) unitary matrices \(\{V_1, V_2, \cdots, V_L\}\) even for a moderate constellation size \(L\). Here we propose a novel technique for constellation construction, which is based on a small set of unitary matrices rotated by an MPSK symbol. Specifically, assuming \(L = M \times D\) (\(M\) and \(D\) are both integer), our M-DUST constellations of size \(L\) can be written as

\[
C_{m,d} = \exp \left( j \frac{2\pi}{M} m \right) U_1^{d} U_2^{d},
\]

where \(U_1\) and \(U_2\) are \(N_1 \times N_1\) unitary matrices. Clearly, the proposed M-DUST constellations are completely characterized by the parameter \(M\) and two basis matrices \(U_1\) and \(U_2\). Let us investigate some properties of M-DUST constellations to be used in subsequent sections. The proof is straightforward and thus omitted.

**Property 1 (Diversity Product):** Let \(I_{N_1}\) denote the \(N_1 \times N_1\) identity matrix. The diversity product of M-DUST constellations, defined as

\[
\zeta \triangleq \min_{(m',d') \neq (m,d)} \frac{1}{2} \left| \det(C_{m',d'} - C_{m,d}) \right|^{1/N_1}
\]

where \(\det(\cdot)\) denotes determinant, can be simplified to

\[
\zeta = \min \{ \zeta^I_{\Delta m}, \zeta^{II}_{\Delta m, \Delta d} \}
\]

with

\[
\zeta^I_{\Delta m} = \left| \sin \left( \frac{\pi}{M} \Delta m \right) \right|, \quad \Delta m = 1, 2, \cdots, M - 1
\]

and

\[
\zeta^{II}_{\Delta m, \Delta d} = \left| \sin \left( \frac{\pi}{M} \Delta d \right) \right|, \quad \Delta m = 1, 2, \cdots, M - 1; \quad \Delta d = 1, 2, \cdots, D - 1.
\]

**Property 2 (Upper Bound on Diversity Product):** The diversity product of M-DUST constellations is upper-bounded by

\[
\zeta \leq \min_{\Delta m = 1, 2, \cdots, M - 1} \left| \sin \left( \frac{\pi}{M} \Delta m \right) \right| = \sin \left( \frac{\pi}{M} \right).
\]

Diversity product is well justified and widely used for large SNR. Union bound of block error rate, on the other hand, takes into account all error events thus characterizing the average performance, as opposed to the worst-case performance described by the diversity product. The union bound of block error rate for the M-DUST constellations is fully determined by the distance spectrum property below.

**Property 3 (Distance Spectrum and Multiplicity):** Any M-DUST constellation has only \(L - 1\) distinct pairwise diversity products, defined as

\[
\zeta(m',d'),(m,d) \triangleq \frac{1}{2} \left| \det(C_{m',d'} - C_{m,d}) \right|^{1/N_1}
\]

which, according to their expressions and multiplicities, can be categorized into two classes. Specifically, for the first class under \(m' \neq m, d' = d\), the pairwise diversity products are

\[
\zeta(m',d'),(m,d) = \left| \sin \left( \frac{\pi}{M} \Delta m \right) \right| \sin \left( \frac{\pi}{D} \Delta d \right)
\]

where \(\Delta m = |m' - m| = 1, 2, \cdots, M - 1\) and \(\Delta d = |d' - d| = 1, 2, \cdots, D - 1\). The associated multiplicities are given by \((D - \Delta d) M^2\). Taking the two classes into account, there are a total of \(M(M - 1)D + \frac{1}{2}D(D - 1)M^2 = \frac{1}{2}L(L - 1)\) pairwise diversity products as expected, but among them only \((M - 1) + M(D - 1) = L - 1\) pairwise diversity products are distinct. In other words, though not group constellations, the M-DUST constellations share a nice property of the former having \(L - 1\) distinct pairwise diversity products. This property is desirable in optimizing M-DUST constellations especially when the constellation size is relatively large.

**Property 4 (Simplified ML Decoding):** The number of matrix multiplications and trace operations required for ML decoding of M-DUST signals is \(D = L/M\), which is smaller than \(L\). The quantity \(D/L\) can be used as a metric of decoding complexity reduction for our M-DUST constellations.

Finally, a full-diversity special case is presented below, which can be regarded as a direct extension of MPSK modulation to space-time modulation.

**Property 5 (Special Case: Full Diversity):** The diversity product of M-DUST constellations with \(M = L\) becomes

\[
\zeta = \min_{\Delta l = 1, 2, \cdots, L - 1} \left| \sin \left( \frac{\pi}{L} \frac{\Delta l}{L} \right) \right| = \sin \left( \frac{\pi}{L} \right) > 0.
\]

The parameter \(M\) is crucial to the design of M-DUST constellations. A large value of \(M\) will lower the upper bound on the diversity product as indicated in (9). On the other hand, increase in \(M\) implies a complexity reduction in decoding an M-DUST constellation. As a consequence, the choice of \(M\) is a trade-off between the performance and ML decoding complexity. The upper bound on diversity product given by (9) can serve as guidance in designing an M-DUST constellation to meet a target diversity product.

### IV. DESIGN OF M-DUST CONSTELLATIONS

To search for good M-DUST constellations, we need to determine the parameter \(M\) and to optimize the basis unitary
matrices $U_1$ and $U_2$. Once $M$ is selected, the optimization problem can be expressed as

$$\left(U_1, U_2\right)_{opt} = \arg \max_{U_1, U_2} \zeta.$$  \hspace{1cm} (14)

It is worth noting that our performance criterion is the diversity product, which, in addition to characterizing the potential coding gain, reflects the achievable diversity order since a non-zero coding gain (i.e., non-zero diversity product) always means full diversity. Consequently, the optimization procedure (14) always (in the numerical sense) leads to M-DUST constellations with full diversity and maximized coding gain.

So far, the constellation size $L$ is assumed to have a factored form of $L = M \times D$. This condition is not really a restriction since very often, $L$ is an integer power of 2. If in certain cases where $L$ is a prime, we can replace $L$ by a close non-prime number such as $L + 1$.

A. Selection of $M$

Since $M$ controls both the performance (characterized by the upper bound on diversity product) and decoding complexity (measured by the number of complex matrix multiplications and trace operations), it is necessary to make a judicious choice by taking into consideration both the performance and decoding complexity. A rule of thumb is that $M$ is selected as large as possible while keeping the upper diversity-product bound of candidate M-DUST constellations to meet the performance requirement.

B. Optimization of $U_1$ and $U_2$

It remains to optimize the unitary matrices $U_1$ and $U_2$. We perform optimization in two steps.

1) Givens Parameterization: Unitary matrices are well studied in mathematics. There are several different representations for them. In this correspondence, we use Givens parameterization as Givens matrices are easy to generate and the resulting unitary matrices have excellent numerical stability. From [15, p. 297] we know that any $N_t \times N_t$ unitary matrix can be expressed as a product of a diagonal unitary matrix and $N_t(N_t - 1)/2$ complex Givens matrices. In other words, any $N_t \times N_t$ unitary matrix is completely parameterized by $N_t^2$ rotation angles.

2) Hybrid Optimization: The objective function to be optimized is a highly nonlinear multimodal function characterized by $2N_t^2$ rotation angles. The conventional gradient-based methods usually fail to yield satisfactory results because they are most likely to get stuck at local optimum. Here we adopt a hybrid optimization approach, aiming to incorporate the advantages of both global and local search. The local search is gradient based whereas the global search relies on a stochastic algorithm such as the real-coded (i.e., with real variables) genetic algorithm (GA) [16]. We explore the global search capability of GAs [17] and the efficiency of gradient based algorithms in local optimization, so that a near optimal solution is reached. \footnotemark[2]

\footnotetext[2]{In fact, as shown subsequently, we do obtain global optima in some cases.}

readers are referred to [17]–[21] and the references therein for more details about GAs and their applications in communications. In this work, a real-coded GA with the non-uniform mutation [16, p. 293] and the BLX-$\alpha$ crossover [16, p. 288] is used. The hybrid search procedure can be divided into two phases:

**Phase 1: Coarse Search**

In this phase, only the real-coded GA is applied so that most local optima can be surpassed.

**Phase 2: Fine Search**

In the fine-search phase, a gradient-based search implemented by the MATLAB function `fmincon` is employed to improve the convergence speed of the GA.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the proposed M-DUST constellations, obtained by numerical optimization of the diversity product, are compared with DUST constellations available in the literature. We assume that ML decoding is used and the number of receive antennas is 1 ($N_r = 1$). In certain cases, we replace the constellation size $L$ (even if $L$ is not prime) by a close integer to make the factorization $L = M \times D$ more flexible. In some numerical examples, we can obtain global optima (equal to upper bound on diversity product), which will be marked by superscript * in the sequel.

A. Comparison with Cyclic, Quaternion, Orthogonal, and Parametric Constellations

Comparison of diversity product among M-DUST, cyclic, quaternion, orthogonal, and parametric constellations is shown in Table I, where the reference diversity products $\zeta_{ref}$ are taken from Table I of [8, p. 2339] for the cyclic, quaternion, and orthogonal constellations, and Table I of [11, p. 2298] for the parametric constellations. It is observed that in general, M-DUST constellations have larger diversity products than other constellations.

B. Comparison with Fixed-Point-Free Group Constellations

Comparison between M-DUST and fixed-point-free constellations is listed in Table II, where the reference diversity products $\zeta_{ref}$ are taken from Table III of [8, p. 2357]. We see that M-DUST constellations outperform the $G_{m,r}$ group constellations. The latter include the cyclic group constellations as a special case and are the least restrictive group constellations. On the other hand, when compared with other group constellations including $E_{m,r}, F_{m,r}, I_{m,r},$ and $K_{m,r}$, M-DUST constellations have larger or comparable diversity products. But in the special case of $N_t = 4$ and $L = 240$, the group constellation $K_{1,1,-1}$ is clearly better than its M-DUST counterpart. This means that group constellations excluding $G_{m,r}$ are generally have good performance. Their construction, however, needs to impose restrictions on key parameters such as the constellation size $L$ and the number of transmit antennas $N_t$ (see Table II of [8, p. 2345]). For M-DUST constellations, there are no such limitations. It is worth noting that for the case of $N_t = 2$ and $L = 120$, the M-DUST constellation provided has the same diversity product as the famous group constellation $J_{1,1}$ while enjoying a simplified ML decoding complexity ($\frac{2}{r} = \frac{1}{2}$).
TABLE I
COMPARISON OF DIVERSITY PRODUCT: CYCLIC, QUATERNION, ORTHOGONAL DESIGN, AND M-DUST CONSTELLATIONS

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>$L$</th>
<th>$\zeta_{ref}$</th>
<th>$\zeta_{M-DUST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.3827 (quaternion)</td>
<td>0.3827 (cyclic)</td>
<td>0.5946 (M = 2)</td>
</tr>
<tr>
<td></td>
<td>0.3827 (cyclic)</td>
<td>0.5946 (orthogonal)</td>
<td>0.5946 (parametric)</td>
</tr>
<tr>
<td>32</td>
<td>0.1951 (quaternion)</td>
<td>0.2494 (cyclic)</td>
<td>0.4043 (M = 4)</td>
</tr>
<tr>
<td></td>
<td>0.2494 (cyclic)</td>
<td>0.5946 (orthogonal)</td>
<td>0.5946 (parametric)</td>
</tr>
<tr>
<td>64</td>
<td>0.0980 (quaternion)</td>
<td>0.1985 (cyclic)</td>
<td>0.3614 (M = 8)</td>
</tr>
<tr>
<td></td>
<td>0.1985 (cyclic)</td>
<td>0.5946 (orthogonal)</td>
<td>0.5946 (parametric)</td>
</tr>
<tr>
<td>128</td>
<td>0.0191 (quaternion)</td>
<td>0.1498 (cyclic)</td>
<td>0.2666 (M = 8)</td>
</tr>
<tr>
<td></td>
<td>0.1498 (cyclic)</td>
<td>0.2104 (M = 8)</td>
<td>0.2104 (M = 8)</td>
</tr>
<tr>
<td>256</td>
<td>0.0221 (quaternion)</td>
<td>0.0980 (cyclic)</td>
<td>0.2104 (M = 8)</td>
</tr>
<tr>
<td></td>
<td>0.0980 (cyclic)</td>
<td>0.1379 (orthogonal)</td>
<td>0.1379 (orthogonal)</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>0.2104 (cyclic)</td>
<td>0.3827 (M = 8)</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0.5453 (cyclic)</td>
<td>0.5946 (M = 4)</td>
</tr>
<tr>
<td></td>
<td>0.5453 (cyclic)</td>
<td>0.5946 (orthogonal)</td>
<td>0.5946 (parametric)</td>
</tr>
<tr>
<td>5</td>
<td>256</td>
<td>0.2208 (cyclic)</td>
<td>0.2201 (M = 8)</td>
</tr>
<tr>
<td></td>
<td>0.2201 (M = 8)</td>
<td>0.2201 (M = 8)</td>
<td>0.2201 (M = 8)</td>
</tr>
</tbody>
</table>

TABLE II
COMPARISON OF DIVERSITY PRODUCT: FIXED-POINT-FREE GROUP AND M-DUST CONSTELLATIONS

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>$L$</th>
<th>$\zeta_{ref}$</th>
<th>$\zeta_{M-DUST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>24</td>
<td>0.5000 ($E_{4,1}$)</td>
<td>0.5287 (M = 5, L = 25)</td>
</tr>
<tr>
<td></td>
<td>48</td>
<td>0.8868 ($E_{4,1}, -1$)</td>
<td>0.4156 (M = 5, L = 50)</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>0.3090 ($J_{11}$)</td>
<td>0.3090 (M = 5)</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>0.2293 ($T_{15.1,11}$)</td>
<td>0.2293 (M = 10)</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>0.3853 ($Q_{24.4}$)</td>
<td>0.4091 (M = 3)</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>0.1985 ($Q_{24.4}$)</td>
<td>0.1985 (M = 3)</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>0.2201 ($T_{15.1,11}$)</td>
<td>0.2201 (M = 6)</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>0.2208 ($Q_{20.5,16}$)</td>
<td>0.2208 (M = 6)</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>0.1787 ($Q_{20.5,16}$)</td>
<td>0.2143 (M = 5)</td>
</tr>
</tbody>
</table>

C. Comparison with Non-group Constellations

Given that the availability of group constellations is limited, non-group constellations are also discussed in [8]. Due to the close connection between the non-group and group constellations, the number of available non-group constellations is also limited. Comparison between the M-DUST and non-group constellations is listed in Table III, where the reference diversity products $\zeta_{ref}$ are extracted from Table IV of [8, p. 2358]. We see that except for the specific case of $N_t = 3$ and $L = 57$, the M-DUST constellations in general perform better than, or at least comparably with, the non-group constellations. In particular, for the cases of $N_t = 2$ and $L = 81, 289, 1089$, the M-DUST constellations outperform their counterparts and yet, are simpler in decoding ($\frac{Q}{L} = \frac{1}{\sqrt{L}}$).

TABLE III
COMPARISON OF DIVERSITY PRODUCT: NON-GROUP AND M-DUST CONSTELLATIONS

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>$L$</th>
<th>$\zeta_{ref}$</th>
<th>$\zeta_{M-DUST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>81</td>
<td>0.1967 ($L_A = 9$)</td>
<td>0.3130 (M = 9)</td>
</tr>
<tr>
<td></td>
<td>289</td>
<td>0.1026 ($L_A = 17$)</td>
<td>0.1838 (M = 17)</td>
</tr>
<tr>
<td></td>
<td>1089</td>
<td>0.0794 ($L_A = 33$)</td>
<td>0.0900 (M = 33)</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>0.4845 ($Q_{9.3}$)</td>
<td>0.4461 (M = 3)</td>
</tr>
<tr>
<td></td>
<td>122</td>
<td>0.1863 ($L_A = 23$)</td>
<td>0.2234 (M = 10, L = 530)</td>
</tr>
<tr>
<td></td>
<td>289</td>
<td>0.0105 ($L_A = 17$)</td>
<td>0.0000 (M = 3)</td>
</tr>
<tr>
<td></td>
<td>333</td>
<td>0.6500 ($Q_{9.3}$)</td>
<td>0.6500 (M = 3)</td>
</tr>
<tr>
<td>5</td>
<td>1369</td>
<td>0.2307 ($L_A = 37$)</td>
<td>0.2310 (M = 5, L = 1370)</td>
</tr>
</tbody>
</table>

D. Comparison with Amicable Constellations

In [13], unitary constellations are derived from Amicable orthogonal designs that have been used for the construction of orthogonal space block codes. We consider the case of $N_t = 4$. Amicable constellations are defined by eq. (17) of [13]. Simulated error performance comparison between the M-DUST and Amicable constellations is shown in Fig. 1, where for $L = 64$ the Amicable constellation with 4PSK has $\zeta_{ref} = 0.4083$ while the M-DUST constellation with $M = 4$ has $\zeta_{M-DUST} = 0.4677$, and for $L = 512$ the Amicable constellation with 8PSK has $\zeta_{ref} = 0.2208$ while the M-DUST constellation with $M = 8$ has $\zeta_{M-DUST} = 0.2738$. It can be seen that the M-DUST constellations outperform their counterparts, especially when the constellation size is large. Note that amicable constellations have linear decoding complexity, however, the number of available amicable constellations is very limited due to the stringent requirements of Amicable orthogonal designs.

E. Comparison with Cayley Constellations

Cayley transform is used in [12] to construct unitary constellations from skew-Hermitian matrices. The Cayley constellations obtained have a relatively low ML decoding (sphere decoding) complexity. Unfortunately, unlike the group and M-DUST constellations, the number of different pairwise diversity products for a Cayley constellation of size $L$ can be as large as $L(L - 1)/2$. To overcome the difficulty in performance evaluation, a statistical (rather than exhaustive but deterministic) criterion is used for the optimization of Cayley constellations [12]. This approach may incur a performance penalty. For a fair comparison, we choose the Cayley constellation provided in [12, p. 1495] with parameters $Q = 4, r = 8$, and $N_t = 2$. The comparison of simulated error performance between the proposed M-DUST ($\zeta_{M-DUST} = 4.9068 \cdot 10^{-2}, M = 64$) and Cayley ($\zeta_{ref} = 7.4157 \cdot 10^{-4}$) constellations is shown in Fig. 2. It is clear that the M-DUST constellation outperforms the Cayley constellation significantly while allowing for simplified ML decoding ($\frac{Q}{L} = \frac{1}{\sqrt{L}}$).
VI. CONCLUSION

We have proposed M-DUST constellations for differential space-time modulation, which are derivable from MPSK modulation and two basis unitary matrices \( U_1 \) and \( U_2 \). An arbitrary M-DUST constellation of size \( L \) shares the same property as group constellations that only \( L - 1 \) pairwise diversity products are distinct thereby making the constellation design and performance evaluation very simple. The most important feature of the M-DUST constellations is that the parameter \( M \) has conflicting influence on the error performance and ML decoding complexity. This feature, along with the simple upper-bound on diversity product of M-DUST constellations, can serve as a design guide for the selection of \( M \). Numerical examples show that in most cases the optimized M-DUST constellations outperform other alternatives while allowing for simplified ML decoding. Moreover, the M-DUST constellations are general and flexible in the sense that with a possible slight modification, their constellation size \( L \) and number of transmit antennas can take on any integer.

For ease of comparison with existing constellations, we use diversity product as the main design criterion for the M-DUST constellations. However, other criteria such as minimum Euclidean distance or union bound can be readily applied. The proposed M-DUST constellations can be easily extended to the non-coherent case as well for which tall, instead of square, unitary matrices need to be considered [22]. Finally, given the block structure of M-DUST constellations (intuitively, useful for set-partitioning), it is interesting to investigate their performance and optimization when trellis-coded differential unitary space-time modulation [23] is employed.

REFERENCES