From (60), it can be combined to yield two sets of measurement representing rotational and linear accelerations at CG, respectively, as

$$y_{\dot{\omega}} = \tfrac{1}{4}(A_{o1} + A_{o2} + A_{o3} + A_{o4}) \qquad (61)$$

and

$$y_a = \begin{bmatrix} y_{ax} \\ y_{ay} \end{bmatrix} = \begin{bmatrix} \dfrac{A_{o1} - A_{o3}}{2} \\ -\dfrac{(A_{o2} - A_{o4})}{2} \end{bmatrix}. \qquad (62)$$

The state measurements in (61) and (62) do not have the unwanted ω^2 term but do have the gravity term. However, any additional measurements on $\mathbf{p}, \mathbf{V}, \theta, \omega$ can help some of the problem.

SOU-CHEN LEE
YU-CHAO HUANG
Department of System Engineering
Chung Cheng Institute of Technology
Tashi, Tao Yuan
Taiwan, 33509, R.O.C.

REFERENCES

[1] Bar-Itzhaxk, I. Y. (1993)
On two misconcepts in the theory of inertial navigation system.
AIAA93-3819-CP, 1993, 1067–1072.

[2] Schuler, A. R., et al. (1967)
Measuring rotational motion with linear accelerometer.
IEEE Transactions on Aerospace and Electronic Systems, **AES-3**, 3 (1967).

[3] Padgoankar, A. J., et al. (1957)
Measurement of angular acceleration of a rigid body using linear accelerometer.
Journal of Applied Mechanics (1957).

[4] Hu, A. S. (1977)
Rotational measurement of angular velocity without the use of gyro.
AIAA, 1977.

[5] DiNapoli, L. D. (1965)
The measurement of angular velocity without the use of gyro.
M.S. Thesis, The Moore School of Electric Engineering, University of Pennsylvania, Philadelphia, 1965.

[6] Algrain, M. C. (1991)
Accelerometer-based platform stabilization.
SPIE, **1482**, Acqusition, Tracking, and Pointing V (1991).

[7] Chen, J. H., Lee S. C., and Debra, D. B. (1994)
Gyroscope free strapdown inertial measurement unit by six linear accelerometers.
Journal of Guidance, Control and Dynamics, **17**, 2 (1994), 286–290.

[8] Franklin, G. F., Powell, J. D., and Workman, M. L. (1990)
Digital Control of Dynamic System (2nd ed.).
Reading, MA: Addison-Wesley, 1990.

[9] Aidela, V. J., and Hammel, S. E. (1983)
Utilization of modified polar coordinate for bearing-only tracking.
IEEE Transactions on Automatic Control, **AC-28**, 3 (1983), 283.

[10] Hammel, S. E., and Aidela, V. J. (1985)
Observability requirements for three-dimensional tracking via angle measurements.
IEEE Transactions on Aerospace and Electronic System, **AES-21**, 2 (1985), 200.

[11] Grossman, W. (1994)
Bearing-only tracking: A hybrid coordinate system approach.
Journal of Guidance, Control, and Dynamics, **17**, 3 (1994), 451.

[12] Stallard, D. V. (1987)
An angle-only tracking filter in modified spherical coordinates.
Presented at the AIAA Guidance, Navigation, and Control Conference, Monterey, CA, 1987.

[13] Greenwood, D. T. (1988)
Principles of Dynamics (2nd ed.).
Englewood Cliffs, NJ: Prentice-Hall, 1988.

[14] Siouris, G. M. (1993)
Aerospace Avionics Systems: A Modern Synthesis.
New York: Academic Press, 1993.

[15] Chui, C. K., and Chen, G. (1999)
Kalman Filtering with Real-Time Applications (3rd ed.).
New York: Springer-Verlag, 1999.

A Fuzzy Approach to Signal Integration

A new fuzzy concept for postdetection signal integration is presented. The fuzzy integrator is developed as a simple extension of the classic binary integrator by replacing the crisp binary threshold, which quantizes the observed data, with a fuzzy threshold. The performance of the fuzzy integrator is illustrated for detection of a simple nonfluctuating signal in Gaussian noise and is shown to exceed that of the binary integrator, approaching that of the optimal detector with Neyman–Pearson decision rule. Furthermore, the fuzzy integrator has the characteristic that the false alarm rate can be tuned using a single threshold, more easily than that of the dual-threshold binary integrator.

I. INTRODUCTION

Integration is a process used in signal detection by which data from multiple observations are combined to improve detection performance. There are two

Manuscript received October 29, 1999; revised February 27 and December 4, 2001; released for publication December 4, 2001.

IEEE Log No. T-AES/38/1/02602.

Refereeing of this contribution was handled by L. M. Kaplan.

This work was supported by Competitive Earmarked Research Grants (Hong Kong) 9040627 and 9040530.

---

general categories of signal integration: predetection integration and postdetection integration. Predetection integration [1], also known as coherent integration, sums observations directly prior to processing while maintaining the phase relationship between them [2]. On the other hand, postdetection integration [1], or noncoherent integration, first processes observations, such as by passing the received signals through an envelope detector, before combining them [2]. Although, in theory, predetection integration provides superior performance, postdetection integration is sometimes preferred as it requires no knowledge of the phase of the signal [2] and sometimes provides better performance for rapidly fluctuating signals [1]. This work considers postdetection integration only. One of the simplest postdetection integration techniques is the dual-threshold binary integrator [1], also known as the M-out-of-N detector, which integrates signals by first generating a local decision for each observation and then combining the local decisions using an M-out-of-N decision rule. Although this detector is suboptimal for Gaussian noise due to the quantization effect of the threshold, it offers robust performance when the noise is non-Gaussian [1]. M-out-of-N decision rules having much the same form are also used widely for sensor fusion in distributed detection systems [3].

Soon after introducing fuzzy set theory as a method for modeling ambiguity [4], Zadeh began to look at ways in which fuzzy logic was related to and could be combined with probability theory [5]. Many authors have since investigated methods for incorporating fuzzy logic into signal detection theory. Gorzalczany and Dziech presented a series of papers discussing the transmission and detection of signals with amplitudes given by interval-valued fuzzy sets, including [6, 7]. Son and colleagues reformulated the generalized Neyman–Pearson criterion for imprecise data [8] and applied it to detection of known signals [9, 10]. Soon after, Saade and Schwarzlander developed a body of work on the testing of hypotheses involving data with both random and fuzzy components (hybrid data) [11], which they then applied to signal detection when the signal amplitude or signal shape are fuzzy [12], and when the prior probabilities and costs of a Bayesian decision rule are fuzzy [13, 14]. Leung and Minett went on to explore further the effects of parameter imprecision on the performance of detectors with Neyman–Pearson decision rules [15–17], and variants of the M-out-of-N detector using fuzzy (soft) thresholds [18, 19]. Chen and Wang devised a fuzzy theory for signal detection that incorporates the observer's belief about the presence of a signal as an attempt to model human intuition in the detection process [20]. Examining the problem of uncertainty from a different angle, Boston developed a fuzzy detection method that can "almost completely eliminate classification errors at the cost of a large number of uncertain classifications" [21]; this has recently been applied to reduce the number of errors made in QRS detection [22]. Boston has also proposed an alternative approach to modeling uncertainty in detection systems using Dempster–Shafer theory [23], finding that such a belief theoretic approach may be superior to his own fuzzy theoretic approach, although he acknowledges that the uncertainty modeled by each approach is different. Various other applications of fuzzy logic to signal detection based on feature extraction have also been proposed, including [24, 25], many of which use the fuzzy c-means algorithm [26].

In this correspondence we present a fuzzy integration concept that is a simple fuzzy extension of binary integration and follows work developed in [18, 19]. This approach is designed to overcome the reduction in performance due to quantization of the binary integrator while retaining some of its features. To achieve this, the crisp threshold used in the first stage of binary integration is replaced with a fuzzy threshold [18, 19], which smoothes the output and so retains more information than the binary integrator.

In general, detection models are constructed by making assumptions regarding the *a priori* detection environment. However, no set of *a priori* assumptions will perfectly match a real detection environment. Often the *a priori* parameters, such as the actual mean power of clutter in a typical radar detection scenario, are imprecise and so reduce the performance of the detector. The imprecision can be incorporated into more realistic detection models by treating these parameters as fuzzy numbers [12–17]. The fuzzy integration concept presented here can be combined with such detection models to model the effects of imprecise *a priori* parameters on performance, as demonstrated in [15, 17].

Here, we consider the classical problem of detecting a constant amplitude signal in Gaussian interference. Of course, when the signal amplitude and noise variance are known exactly, the Neyman–Pearson criterion, discussed throughout the literature, e.g. [2, 27], is well known to be optimal. Equally, when the noise variance is unknown or, more generally, when "very coarse information about observations is available" [3], nonparametric detectors, such as the Wilcoxon rank detector [3, 28], can be used effectively. Our aim here, rather, is to describe a signal integration concept using elements of fuzzy logic that provides good detection performance. We illustrate the characteristics of the fuzzy integrator for a nonfluctuating signal in Gaussian interference; this allows the essential features of the fuzzy integrator to be described conveniently. Nevertheless, the fuzzy integrator can be developed for other interference models in much the same way, such as for Weibull clutter [18]. Since the fuzzy integrator is based on the binary integrator, we first review the binary integrator.

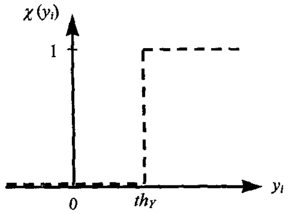


Fig. 1. Characteristic function of the binary integrator.

II. SUMMARY OF THE BINARY INTEGRATOR

For a constant amplitude signal in Gaussian interference, the task at hand is to select one of two hypotheses,

$$H_0: \quad Y \sim N(0,\sigma^2) : \text{“No Signal”}$$

$$H_1: \quad Y \sim N(m,\sigma^2) : \text{“Signal”}$$

based on a set of n independent observations Y_i, $1 \leq i \leq n$, of a random variable Y, where $Y \sim N(m,\sigma^2)$ denotes that Y has a Gaussian distribution with mean m and variance σ^2. The binary integrator applies a crisp threshold to quantize each observation, the output of which is given by the characteristic function, shown in Fig. 1,

$$\chi(y_i) := \begin{cases} 0 : y_i < th_y \\ 1 : y_i \geq th_y \end{cases} \tag{1}$$

where th_y is the crisp threshold, and y_i is the observed value of Y_i. We refer to this threshold as the *local decision threshold* and values of the characteristic function as *local decisions*. The local decisions are then summed and compared to a second threshold T producing the decision rule

$$\sum_{i=1}^{n} \chi(y_i) \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{2}$$

For integration of n observations, T is chosen to be an integer in the range $[1,n]$ and indicates the minimum number of local decisions that must have the value 1 (i.e., that imply H_1 locally) for hypothesis H_1 to be selected.

The probability that a single observation exceeds the local decision threshold is

$$\Pr(Y_i \geq th_y \mid Y_i \sim \mathrm{N}(0,\sigma^2)) = 1 - \Phi(th_y/\sigma) \tag{3}$$

where Φ is the cumulative distribution function of the standard Gaussian distribution [29] given by

$$\Phi(z) := \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2}\right) d\xi. \tag{4}$$

Hence, the false alarm rate can be evaluated as the probability that at least T out of n observations exceed this threshold:

$$P^{\mathrm{FA}} = \sum_{j=T}^{n} \binom{n}{j} [1 - \Phi(th_y/\sigma)]^j [\Phi(th_y/\sigma)]^{n-j}. \tag{5}$$

When a signal with amplitude m is present, the probability that a single observation exceeds the local decision threshold is

$$\Pr(Y_i \geq th_y \mid Y_i \sim \mathrm{N}(m,\sigma^2)) = 1 - \Phi(th_y - m/\sigma). \tag{6}$$

The detection rate of the binary integrator can be evaluated as the probability that at least T observations exceed the threshold:

$$P^{\mathrm{D}} = \sum_{j=T}^{n} \binom{n}{j} [1 - \Phi(th_y - m/\sigma)]^j [\Phi(th_y - m/\sigma)]^{n-j}. \tag{7}$$

III. FUZZY INTEGRATOR

As discussed earlier, the quantization effect of the binary integrator gives rise to significantly suboptimal detection performance. In order to retain more information, the local decision threshold is smoothed in the fuzzy integrator by replacing the characteristic function (1) of the binary integrator with a fuzzy membership function [30] with value denoted by $\mu(y)$. This fuzzy threshold [18, 19] allows a smooth transition of each local decision from No Signal to Signal.

We define the decision rule of the fuzzy integrator as

$$\sum_{i=1}^{n} \mu(y_i) \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{8}$$

This is similar to the binary decision rule (2) except that the threshold T can be any real number in the range $(0,n)$, not necessarily an integer. Similarly, each local decision may be any real number in the range $[0,1]$.

Clearly, the performance of the fuzzy integrator depends on the choice of membership function. Logically, the membership function should obey the following rules:

$$\mu(y) \in [0,1] \qquad \forall \quad y \tag{9a}$$

$$\mu(y_i) \geq \mu(y_j) \Leftrightarrow y_i \geq y_j \tag{9b}$$

$$\begin{aligned} &\lim\nolimits_{y \to -\infty} \mu(y) = 0 \\ &\lim\nolimits_{y \to +\infty} \mu(y) = 1. \end{aligned} \tag{9c}$$

Rule (9a) guarantees that local decisions are confined to the interval $[0,1]$ and so are consistent with the binary local decisions. Rule (9b) ensures that stronger signals are assigned greater signal membership. Rule (9c) states the obvious limiting requirements for large magnitude signals. These rules suggest a graph of μ much like that shown in Fig. 2. The cumulative

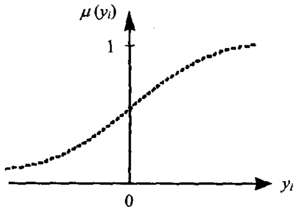


Fig. 2. Membership function of fuzzy integrator.

distribution function of Y under H_0 is one possible choice for μ, which for zero-mean Gaussian noise is

$$\mu(y) := \Phi(y/\sigma). \tag{10}$$

However, a compelling reason for this choice of membership function is that the local decisions are then distributed uniformly on $[0,1]$ under the no signal hypothesis. This is true because the random variable formed by applying the cumulative distribution function to any continuous random variable is itself uniformly distributed [31] (and approximately uniform when the noise variance is not specified precisely). Therefore, the distribution of each local decision is independent of the distribution of Y, allowing the false alarm rate of the fuzzy integrator to be analyzed simply as follows.

The false alarm rate of the fuzzy integrator for a constant amplitude signal in Gaussian interference is

$$P^{\mathrm{FA}} = \Pr\left(\sum_{i=1}^{n} \mu(y_i) \geq T \mid Y_i \sim \mathrm{N}(0,\sigma^2)\right). \tag{11}$$

Under hypothesis H_0, the distribution of the local decision sum is given by the convolution of n uniform distributions [31] and so has the cumulative distribution function

$$F(z) = \sum_{i=0}^{z} (-1)^i \binom{n}{i} \frac{(z-i)^n}{n!}. \tag{12}$$

The false alarm rate can therefore be written as a function of threshold T as

$$P^{\mathrm{FA}} = 1 - F(T). \tag{13}$$

In practice, an integrator is tuned by selecting threshold values that achieve a desired false alarm rate. The false alarm rate of the binary integrator in (5) depends on T, th_x, and the interference distribution. However, as demonstrated in (13), the false alarm rate of the fuzzy integrator depends on T only; knowledge of the interference distribution is included implicitly in the definition of the membership function (10). This allows the fuzzy integrator to be tuned far more easily than the binary integrator.

The detection rate of the fuzzy integrator is

$$P^{\mathrm{D}} = \Pr\left(\sum_{i=1}^{n} \mu(y_i) \geq T \mid Y_i \sim \mathrm{N}(m,\sigma^2)\right). \tag{14}$$

The local decisions, $\mu(y_i)$, are not uniform under H_1; no analytic expression for the detection rate can be easily determined. We therefore use Monte-Carlo simulation to estimate the detection rate.

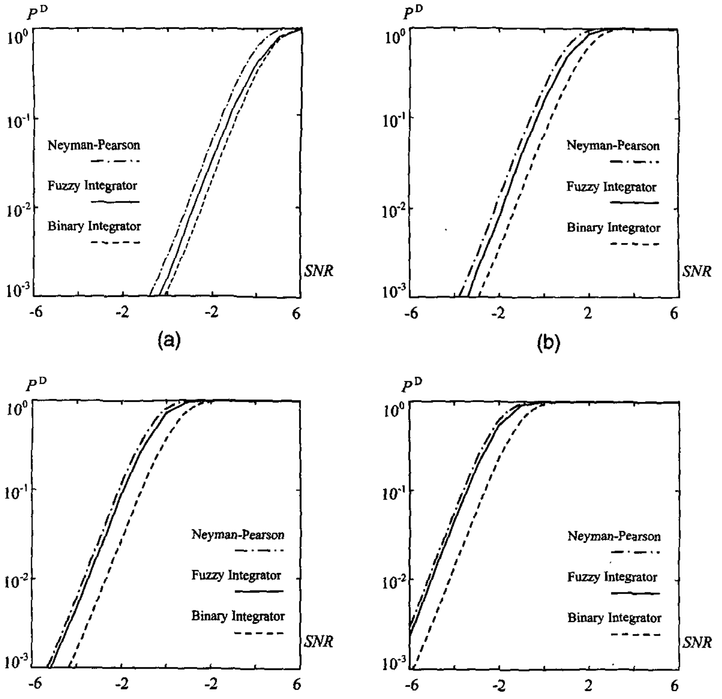
IV. COMPARISON OF PERFORMANCE OF THE FUZZY INTEGRATOR AND BINARY INTEGRATOR FOR GAUSSIAN INTERFERENCE

To investigate whether the local decision rule of the fuzzy integrator has indeed retained more information than that of the binary integrator, we now compare the performance of each detector for a constant amplitude signal in Gaussian interference. Fig. 3 summarizes the performance gain of the fuzzy integrator over the binary integrator, plotting detection rate against signal-to-noise ratio at false alarm rate 10^{-6}. The performance of the optimal Neyman–Pearson detector for Gaussian noise [27] is also shown as a reference against which to compare the performance of the fuzzy and binary integrators. For integration of 4 observations, the gain of the fuzzy integrator over the binary integrator is not more than 0.4 dB, falling short of the performance of the ideal Neyman–Pearson scheme. For 16 observations, the gain of the fuzzy integrator increases to 0.6 dB, approaching the performance of the Neyman–Pearson scheme. For both 32 and 64 observations, the fuzzy integrator is far superior to the binary integrator with performance similar to the Neyman-Pearson detector.

V. CONCLUSION

This paper presents a new postdetection signal integration concept, fuzzy integration, using elements of fuzzy logic and which is an extension of binary integration. Although it presumes that the distribution of interference is known *a priori*, it provides room for modeling imprecision in the *a priori* parameters to be included in the model. Once the membership function is defined appropriately, the false alarm rate depends only on a single threshold, allowing performance to be tuned more easily than for the binary integrator. It has been shown that for nonfluctuating signals in Gaussian distributed interference, the fuzzy integrator outperforms the binary integrator and its performance approaches that of the optimal Neyman–Pearson detector.

S. W. LEUNG
J. W. MINETT
Y. M. SIU
M. K. LEE
Department of Electronic Engineering
City University of Hong Kong
Tat Chee Avenue, Hong Kong
People's Republic of China
E-mail: (eewl@cityu.edu.hk)



(a) (b) (c) (d)

Fig. 3. Plot of detection rate against signal-to-noise ratio for fuzzy integrator, binary integrator, Neyman–Pearson detector. (a) 4 observations, false alarm rate: 10^{-6}. (b) 8 observations, false alarm rate: 10^{-6}. (c) 16 observations, false alarm rate: 10^{-6}. (d) 32 observations, false alarm rate: 10^{-6}.

REFERENCES

[1] Skolnik, M. (Ed.) (1990)
Radar Handbook (2nd ed.).
New York: McGraw-Hill, 1990, ch. 2, 8.

[2] Minkoff, J. (1991)
Signals, Noise, and Active Sensors: Radar, Sonar, Laser Radar.
New York: Wiley, 1991.

[3] Varshney, P. K. (1997)
Distributed Detection and Data Fusion.
New York: Springer-Verlag, 1997.

[4] Zadeh, L. A. (1965)
Fuzzy sets.
Information and Control, **8**, 3 (1965), 338–353.

[5] Zadeh, L. A. (1968)
Probability measures of fuzzy events.
Journal of Mathematical Analysis and Applications, **23** (1968), 421–427.

[6] Gorzalczany, M. B. (1987)
A method of inference in approximate reasoning based on interval-valued fuzzy sets.
Fuzzy Sets and Systems, **21** (1987), 1–17.

[7] Dziech, A., and Gorzalczany, M. B. (1987)
Decision making in signal transmission problems with interval-valued fuzzy sets.
Fuzzy Sets and Systems, **23** (1987), 191–203.

[8] Son, J. C., Song, I., and Kim, H. Y. (1992)
A fuzzy decision problem based on the generalized Neyman–Pearson criterion.
Fuzzy Sets and Systems, **47** (1992), 65–75.

[9] Son, J. C., Kim, S. Y., Song, I., and Kim, H. M. (1992)
A fuzzy set theoretic approach to weak known-signal detection.
Signal Processing, **28** (1992), 183–194.

[10] Son, J. C., Song, I., Kim, S. Y., and Park, S. I. (1993)
An application of the generalized Neyman–Pearson fuzzy test to stochastic-signal detection.
IEEE Transactions on Systems, Man, and Cybernetics, **23**, 5 (1993), 1474–1481.

[11] Saade, J. J., and Schwarzlander, H. (1990)
Fuzzy hypothesis testing with hybrid data.
Fuzzy Sets and Systems, **35** (1990), 197–212.

[12] Saade, J. J., and Schwarzlander, H. (1994)
Application of fuzzy hypothesis testing to signal detection under uncertainty.
Fuzzy Sets and Systems, **62** (1994), 9–19.

[13] Saade, J. J. (1994)
Extension of fuzzy hypothesis testing with hybrid data.
Fuzzy Sets and Systems, **63** (1994), 57–71.

[14] Saade, J. J. (1994)
Towards intelligent radar systems.
Fuzzy Sets and Systems, **63** (1994), 141–157.

[15] Minett, J. W., and Leung, S. W. (1999)
Detecting fixed amplitude signals in a fuzzy Gaussian environment.
In *Proceedings of the 1999 IEEE International Conference on Systems, Man and Cybernetics*, Tokyo, Japan, Oct. 12–15, 1999.

[16] Leung, S. W., Minett, J. W., and Lee, M. K. (2000)
An application of fuzzy hypothesis testing to signal integration.
In *Proceedings of the 2000 IEEE International Conference on Fuzzy Systems*, San Antonio, TX, May 7–10, 2000.

[17] Leung, S. W., Minett, J. W., Siu, Y. M., and Lee, M. K. (2001)
A method for modeling imprecise detection parameters as fuzzy numbers for a Rician target in Rayleigh background.

In *Proceedings of the 2001 IEEE Radar Conference*, Atlanta, GA, May 1–3, 2001.

[18] Leung, S. W., and Minett, J. W. (1996)
CFAR data fusion using Fuzzy Integration.
In *Proceedings of the Fifth IEEE International Conference on Fuzzy Systems*, New Orleans, LA, Sept. 8–11, 1996, 1291–1295.

[19] Leung, S. W., and Minett, J. W. (2000)
The use of fuzzy spaces in signal detection.
Fuzzy Sets and Systems, **114** (2000), 175–184.

[20] Chen, T., and Wang, M.-J. J. (1995)
Applying fuzzy set approach to signal detection theory.
Fuzzy Sets and Systems, **72** (1995), 39–49.

[21] Boston, J. R. (1997)
Effects of membership function parameters on the performance of a fuzzy signal detector.
IEEE Transactions on Fuzzy Systems, **5**, 2 (1997), 249–255.

[22] Boston, J. R., and Akyol, G. (1999)
Using an uncertainty measure in a fuzzy QRS detector.
In *Proceedings of The First Joint BMES/EMBS Conference*, Atlanta, GA, Oct. 13–16, 1999, 916.

[23] Boston, J. R. (2000)
A signal detection theory based on Dempster-Shafer theory and a comparison to fuzzy detection.
IEEE Transactions on Systems, Man, and Cybernetics—Part C, **30**, 1 (2000), 45–51.

[24] Minett, J. W., Leung, S. W., Siu, Y. M., Ng, K. T., and Chau, W. N. (2001)
Estimating the length of a radar shadow in shadow-feature-enhanced detection using a fuzzy system.
In *Proceedings of the Tenth IEEE International Conference on Fuzzy Systems*, Melbourne, Australia, Dec. 2–5, 2001.

[25] Söderholm, K., Ihalainen, H., and Ritala, R. (1999)
Fuzzy gate based feature detection.
In *Proceedings of Information, Decision and Control '99*, Adelaide, Australia, Feb. 9–10, 1999, 545–549.

[26] Bezdek, J. C. (1981)
Pattern Recognition with Fuzzy Objective Function Algorithms.
New York: Plenum Press, 1981.

[27] Van Trees, H. L. (1968)
Detection, Estimation, and Modulation Theory, Part I.
New York: Wiley, 1968.

[28] Helstrom, C. W. (1995)
Elements of Signal Detection and Estimation.
Englewood Cliffs, NJ: Prentice-Hall, 1995.

[29] DeGroot, M. H. (1986)
Probability and Statistics (2nd ed.).
Reading, MA: Addison-Wesley, 1986.

[30] Kaufmann, A., and Gupta, M. M. (1985)
Introduction to Fuzzy Arithmetic: Theory and Applications.
New York: Van Nostrand Reinhold, 1985.

[31] Evans, M., Hastings, N., and Peacock, B. (1993)
Statistical Distributions (2nd ed.).
New York: Wiley, 1993.