Towards a Better Diversity-Multiplexing Tradeoff in MIMO Systems♣

Lin Dai, Sana Sfar, and Khaled B. Letaief, *Fellow, IEEE*
Center for Wireless Information Technology
The Hong Kong University of Science & Technology
Clear Water Bay, HONG KONG
eedailin@ust.hk, eesana@ust.hk, eekhaled@ee.ust.hk

***Abstract*—Multiple-Input Multiple-Output (MIMO) systems can provide two kinds of gain: diversity gain and multiplexing gain. Most existing MIMO schemes, including space-time coding and layered space-time, aim at maximizing either of them. Group transmission and detection is a good way to get a tradeoff between these two gains. In this paper, the diversity-multiplexing tradeoff functions of 3 schemes based on group transmission and detection, Quasi-Orthogonal Group Space-Time (QoGST), Group Layered Space-Time (GLST) and Layered space-time block codes (LSTBC), are provided and a vivid comparison among them demonstrates that QoGST achieves the best diversity-multiplexing tradeoff. GLST also gets significant gains over LSTBC. Simulation results will validate the analysis and show that QoGST can achieve at least 3 dB gain over GLST and 12 dB over LSTBC at a FER of 10^{-3}, for instance.**

Keywords- *Group detection, MIMO systems, Space-time block coding, Layered space-time, Group layered space-time, Quasi-orthogonal group space-time, Diversity-multiplexing tradeoff.*

I. INTRODUCTION

MIMO (Multiple-Input Multiple-Output) systems have shown their ability in providing great performance improvements over the SISO (Single-Input Single-Output) system thanks to their higher spectral efficiency [1]. It has been well understood that multiple antennas can not only be used to achieve diversity gain, but also as an effective way to increase the degrees of freedom of the channel [2]. However, most existing MIMO techniques, such as space-time codes (STC) [3-4] and Layered space-time (LST) [5], aim at achieving either maximum diversity gain or maximum multiplexing gain. In order to achieve a better tradeoff between these two gains, [6] presented a combined array processing and space-time coding architecture, in which the transmit stream is partitioned into different groups and in each group STC is applied. At the receiver, group interference suppression is adopted, where each individual STC is decoded by suppressing the signals transmitted from other groups. This combination of STC and LST provides much better multiplexing gain than STC with lower decoding complexity, while at the same time achieving a much higher diversity gain than LST. [7] further developed this architecture, where Alamouti's scheme was adopted as the component encode and the transmit power was optimized to minimize the average FER, which is called Layered space-time block codes (LSTBC).

[6] and [7] presents a good example on how to get a tradeoff between multiplexing gain and diversity gain. However, there are some limitations on the designs of transmitter and receiver. First, the substreams of each group are encoded independently. No special transmit design is adopted to suppress the interference among the groups. Besides, the mapping from the substreams to the antennas is constant over the whole time interval and no interleaving gain can be achieved. Second, at the receiver, space-time decoding is performed for each group by assuming that the interference has been suppressed by virtue of a group detector. The overall performance is thus limited by the group detection step and usually is not satisfying.

[8] further proposed two novel group space-time architectures: Group Layered Space-time (GLST) and Quasi-Orthogonal Group Space-time (QoGST). GLST has the same transmitter as [7]. However, a novel detector is adopted at the receiver, where space-time block decoding is performed before group detection is applied. Specifically, for the case of m transmit and n receive antennas during T time slots, the linear nature of STBC can be exploited to obtain an equivalent $Tn \times m$ channel [9]. Group detection is then applied based on this equivalent channel. It can be seen that after decoding, the receive dimensions increase from n to Tn and thus much better performance can be achieved by group detection. QoGST not only adopts this novel detector, but also has a more efficient transmitter design. In QoGST, all the groups are encoded together via an inter-group STBC. Particularly, at each time slot t, the transmit vector of each group is regarded as one symbol and STBC is applied on all the transmit vectors. It can be seen that with this inter-group STBC, the interference among groups can be effectively suppressed because of the orthogonal nature of STBC. Therefore, QoGST should have a better interference suppressing capability. Besides, interleaving gain can be achieved due to the use of the inter-group encoder.

[8] has shown that QoGST has a better symmetric energy than GLST. In this paper, we further obtain their diversity-multiplexing tradeoff functions and compare them to the tradeoff of LSTBC. The diversity-multiplexing tradeoff function is proposed by Zheng and Tse [2], which allows us to compare the performance between diversity-based schemes and multiplexing-based schemes. It is found that a higher spatial multiplexing gain comes at the price of sacrificing diversity and vice versa. In this paper, we follow the analysis in [2] and obtain the lower and upper bound tradeoff functions of GLST and QoGST. It is found that the lower bound tradeoff of QoGST is usually better than the upper bound tradeoff of GLST, which implies that QoGST has a better diversity-multiplexing tradeoff than GLST. Besides, the comparison among QoGST, GLST and LSTBC shows that both QoGST and GLST can achieve significant gains over LSTBC thanks to

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their efficient detectors. Simulation results will validate our analysis and show that QoGST can achieve at least 3 dB gain over GLST for m =4 and n =2 and 12 dB over LSTBC for m=6 and n=4 at a FER of 10^{-3}, for example.

This paper is organized as follows. In Section II, we review the transmitter and receiver designs of LSTBC, GLST and QoGST. Section III presents the performance analysis result, which is evaluated in terms of the diversity-multiplexing tradeoff function. Simulation results are given in Section IV. Finally, Section V summarizes and concludes this paper.

II. System Model

We begin by presenting LSTBC, and then present the details of GLST and QoGST architecture. In the following we consider a flat Rayleigh wireless link $\mathbf{H}$ with m transmit and n receive antennas. Besides, perfect channel knowledge is assumed at the receiver side only, through the use of training sequences. In this paper transmit power allocation is not considered. Therefore, all the antennas are assumed to be with the same transmit power.

A. LSTBC

A.1 Transmitter

Assume that all the m transmit antennas are partitioned into G_m groups, respectively, comprising $m_1, m_2, ..., m_{G_m}$ antennas. A block of input bits $\{b_i\}_{i=1...K}$ is divided into G_b groups, $\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_{G_b}$, and in each group, $\mathbf{b_i} = [b_{i,1}, b_{i,2}, ..., b_{i,|\mathcal{G}_i|}]'$, i=1,…,G_b, is then encoded by a component space-time block code $STBC_i$ associated with m_i transmit antennas. Here it is required that $G_m = G_b = G$. Assume that all the component codes $STBC_i$, i=1,…,G, have the same code length T, and we have $m_i = g_m$ and $|\mathcal{G}_i| = g_b$ for i=1,…,G. Then, the output $m \times T$ codeword matrix $\mathbf{X}$ over a block of T symbol intervals can be written as

$$\mathbf{X} = \left[\mathbf{S_1}', \quad \cdots, \quad \mathbf{S_G}'\right]' \tag{1}$$

where $'$ represents the transpose operator. $\mathbf{S_i} = [\mathbf{s_1^i}, \cdots, \mathbf{s_T^i}]$ is the $g_m \times T$ codeword matrix of group $\mathcal{G}_i$, i=1,…,G.

As we know, an m-antenna-T-time-slot-k-symbol STBC $\mathcal{O}_x$ can be represented as $\mathcal{O}_x = \left[\mathbf{A_1 x} + \mathbf{B_1 x}^*, \mathbf{A_2 x} + \mathbf{B_2 x}^*, ..., \mathbf{A_T x} + \mathbf{B_T x}^*\right]$, where $*$ denotes the conjugate operator, $\mathbf{x}$ is a $k \times 1$ complex variable vector and $\mathbf{A_i}, \mathbf{B_i}$ are constant coefficient matrices in $\mathcal{R}^{m \times k}$. The matrix $\mathcal{O}_x$ is called [m, T, k] STBC for short in the following. Therefore, $\mathbf{S_i}$ can be written as

$$\mathbf{S_i} = \left[\mathbf{A_{i1}} \mathbf{b_i}, ..., \mathbf{A_{iT}} \mathbf{b_i}\right] + \left[\mathbf{B_{i1}} \mathbf{b_i}^*, ..., \mathbf{B_{iT}} \mathbf{b_i}^*\right], \tag{2}$$

for i=1,…,G, where $\mathbf{A}_{ij}, \mathbf{B}_{ij}$ are constant coefficient matrices in $\mathcal{R}^{g_m \times g_b}$, j=1,…,T.

A.2 Receiver

The detector presented in [7] is to suppress signals transmitted from other groups of antennas by virtue of a group detector first, and then perform space-time decoding for the desired group. In particular, assume that a Group Zero-Forcing (GZF) receiver is adopted and group $\mathcal{G}_i$ is to be detected. Partition $\mathbf{H}$ into $\mathbf{H} = \left[\mathbf{H}_{\mathcal{G}_i}, \mathbf{H}_{\bar{\mathcal{G}}_i}\right]$, where $\mathbf{H}_{\bar{\mathcal{G}}_i}$ includes the columns of $\mathbf{H}$ corresponding to the groups except $\mathcal{G}_i$. The projection matrix $\mathbf{P}_{\mathcal{G}_i}$ is then defined as: $\mathbf{P}_{\mathcal{G}_i} = \mathbf{I_n} - \mathbf{H}_{\bar{\mathcal{G}}_i} (\mathbf{H}_{\bar{\mathcal{G}}_i}^+ \mathbf{H}_{\bar{\mathcal{G}}_i})^{-1} \mathbf{H}_{\bar{\mathcal{G}}_i}^+$, where $(\cdot)^+$ denote the complex conjugate transpose. Therefore, using the transformation $\mathbf{W_i} = \mathbf{H}_{\mathcal{G}_i}^+ \mathbf{P}_{\mathcal{G}_i}$ on the receive vector, we have

$$\mathbf{Y}_i = \sqrt{\frac{SNR}{m}} \left\{\mathbf{Q}_i^{-1} \left[\mathbf{A}_{i1} \mathbf{b}_i, ..., \mathbf{A}_{iT} \mathbf{b}_i\right] + \mathbf{Q}_i^{-1} \left[\mathbf{B}_{i1} \mathbf{b}_i^*, ..., \mathbf{B}_{iT} \mathbf{b}_i^*\right]\right\} + \mathbf{Z} \tag{3}$$

where $\mathbf{Q}_{\mathcal{G}_i}^{-1} = \mathbf{H}_{\mathcal{G}_i}^+ \mathbf{P}_{\mathcal{G}_i} \mathbf{H}_{\mathcal{G}_i}$. Then, for group $\mathcal{G}_i$, STBC decoding can be done based on $\mathbf{Y}_i$.

B. GLST

B.1 Transmitter

The transmitter design of GLST is the same as LSTBC. Therefore, we do not repeat here.

B.2 Receiver

In GLST, a new detector is adopted, where space-time decoding is performed before group detection. To do so, an equivalent channel is obtained by virtue of the linear nature of STBC. GZF is then performed. Particularly, by combining (1) and (2), the received signal vector can be written as

$$\mathbf{Y} = \sqrt{\frac{SNR}{m}} \left[[\mathbf{H}_1 \mathbf{A}_{11}, ..., \mathbf{H}_G \mathbf{A}_{G1}] \cdot \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_G \end{bmatrix}, \quad \cdots, \quad [\mathbf{H}_1 \mathbf{A}_{1T}, ..., \mathbf{H}_G \mathbf{A}_{GT}] \cdot \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_G \end{bmatrix} \right] + \tag{4}$$

$$\sqrt{\frac{SNR}{m}} \left[[\mathbf{H}_1 \mathbf{B}_{11}, ..., \mathbf{H}_G \mathbf{B}_{G1}] \cdot \begin{bmatrix} \mathbf{b}_1^* \\ \vdots \\ \mathbf{b}_G^* \end{bmatrix}, \quad \cdots, \quad [\mathbf{H}_1 \mathbf{B}_{1T}, ..., \mathbf{H}_G \mathbf{B}_{GT}] \cdot \begin{bmatrix} \mathbf{b}_1^* \\ \vdots \\ \mathbf{b}_G^* \end{bmatrix} \right] + \mathbf{Z}.$$

After a series of linear transformation, finally we can get

$$\mathbf{r} = \sqrt{\frac{SNR}{m}} \left[\mathbf{H}_{\mathcal{G}_1}, ..., \mathbf{H}_{\mathcal{G}_G}\right] \cdot \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_G \end{bmatrix} + \mathbf{z} \tag{5}$$

where $\mathbf{r} = [\tilde{\mathbf{y}}_1', ..., \tilde{\mathbf{y}}_T']'$ and $\tilde{\mathbf{y}}_j = \begin{cases} \mathbf{y}_j, & \mathbf{B}_{ij} = \mathbf{0}_{g_m \times g_b} \\ \mathbf{y}_j^*, & \mathbf{A}_{ij} = \mathbf{0}_{g_m \times g_b} \end{cases}$, $\mathbf{y}_j$ represents the j-th column vector of $\mathbf{Y}$, j=1,…,T. For any group $\mathcal{G}_i$, i=1,…,G, the corresponding sub-channel matrix is given by

$$\mathbf{H}_{\mathcal{G}_i} = \begin{bmatrix} \mathbf{H}_i \mathbf{A}_{i1} + \mathbf{H}_i^* \mathbf{B}_{i1}^* \\ \vdots \\ \mathbf{H}_i \mathbf{A}_{iT} + \mathbf{H}_i^* \mathbf{B}_{iT}^* \end{bmatrix}. \tag{6}$$

From (5) and (6) it is clear that after obtaining this $Tn \times K$ equivalent channel of GLST, the decoding process is done. Group detection can then be applied so as to get the original transmit symbols. As explained before, here after decoding the receive dimensions have increased from n to Tn and thus much better performance can be achieved by group detection. That is why the novel detector is superior to the original one used in [6-7].

C. QoGST

C.1 Transmitter

In LSTBC and GLST, the bit streams of each group are encoded separately so that the output streams $\mathbf{S_1}$, $\mathbf{S_2}$, …, $\mathbf{S_G}$ are independent of each other. No special transmit design is adopted to suppress the interference among the groups. Besides, the mapping from different groups to the transmit antennas is always fixed over all the time slots. Therefore, no interleaving gain can be achieved. QoGST adopts a different transmit design, in which all the groups are encoded together via an "inter-group STBC" encoder. Assume that the antennas and the bit stream are divided into G_m and G_b groups, respectively. Then, the design of the inter-group STBC is given by

$$\mathbf{X}=\left[\tilde{\mathbf{A}}_1\mathbf{b},...,\tilde{\mathbf{A}}_T\mathbf{b}\right]+\left[\tilde{\mathbf{B}}_1\mathbf{b}^*,...,\tilde{\mathbf{B}}_T\mathbf{b}^*\right] \tag{7}$$

where

$$\tilde{\mathbf{A}}_j=\left[\tilde{\mathbf{A}}_{1j},...,\tilde{\mathbf{A}}_{G_b j}\right],\ \tilde{\mathbf{B}}_j=\left[\tilde{\mathbf{B}}_{1j},...,\tilde{\mathbf{B}}_{G_b j}\right], \tag{8}$$

and

$$\tilde{\mathbf{A}}_{ij}=\mathbf{A}_j^i\otimes\mathbf{I}_g,\ \tilde{\mathbf{B}}_{ij}=\mathbf{B}_j^i\otimes\mathbf{I}_g, \tag{9}$$

for i=1,…,G_b and j=1,…,T. $\mathbf{A}_j^i$ and $\mathbf{B}_j^i$ are the i^{th} column vector of $\mathbf{A}_j$ and $\mathbf{B}_j$, respectively. $\mathbf{A}_j$ and $\mathbf{B}_j$, j =1,…T, are the coefficient matrices of a $[G_m, T, G_b]$ STBC.

After inter-group STBC encoding, the mapping from the bit streams of different groups to the transmit antennas is not constant any more. Therefore, a higher diversity gain can be achieved thanks to the interleaving gain. Besides, here STBC is applied to the transmit vectors. The interference among the groups is not independent and thus can be better suppressed.

C.2 Receiver

The receiver design of QoGST is similar to GLST. From (7), the received signal vector can be written as

$$\mathbf{Y}=\sqrt{\frac{SNR}{m}}\left[\left[\mathbf{H}\tilde{\mathbf{A}}_{11},...,\mathbf{H}\tilde{\mathbf{A}}_{G_b1}\right]\cdot\begin{bmatrix}\mathbf{b}_1\\ \vdots\\ \mathbf{b}_{G_b}\end{bmatrix},\ \cdots,\ \left[\mathbf{H}\tilde{\mathbf{A}}_{1T},...,\mathbf{H}\tilde{\mathbf{A}}_{G_bT}\right]\cdot\begin{bmatrix}\mathbf{b}_1\\ \vdots\\ \mathbf{b}_{G_b}\end{bmatrix}\right]+$$
$$\sqrt{\frac{SNR}{m}}\left[\left[\mathbf{H}\tilde{\mathbf{B}}_{11},...,\mathbf{H}\tilde{\mathbf{B}}_{G_b1}\right]\cdot\begin{bmatrix}\mathbf{b}_1^*\\ \vdots\\ \mathbf{b}_{G_b}^*\end{bmatrix},\ \cdots,\ \left[\mathbf{H}\tilde{\mathbf{B}}_{1T},...,\mathbf{H}\tilde{\mathbf{B}}_{G_bT}\right]\cdot\begin{bmatrix}\mathbf{b}_1^*\\ \vdots\\ \mathbf{b}_{G_b}^*\end{bmatrix}\right]+\mathbf{Z}. \tag{10}$$

Similarly, we can get

$$\mathbf{r}=\sqrt{\frac{SNR}{m}}\left[\mathbf{H}_{\mathcal{G}_1},...,\mathbf{H}_{\mathcal{G}_{G_b}}\right]\cdot\begin{bmatrix}\mathbf{b}_1\\ \vdots\\ \mathbf{b}_{G_b}\end{bmatrix}+\mathbf{z} \tag{11}$$

where for i=1,…, G_b ,

$$\mathbf{H}_{\mathcal{G}_i}=\begin{bmatrix}\mathbf{H}\tilde{\mathbf{A}}_{i1}+\mathbf{H}^*\tilde{\mathbf{B}}_{i1}^*\\ \vdots\\ \mathbf{H}\tilde{\mathbf{A}}_{iT}+\mathbf{H}^*\tilde{\mathbf{B}}_{iT}^*\end{bmatrix}. \tag{12}$$

Given **r**, group detection can then be applied.

III. DIVERSITY-MULTIPLEXING TRADEOFF

In this section, we provide the main technical results of this paper. The proofs of the results stated are omitted due to limited space.

A. Tradeoff of LSTBC

The diversity-multiplexing tradeoff of LSTBC is given by

$$d_{LSTBC}(r)=g_m(n-m+g_m)(1-Tr/K) \tag{13}$$

When there is only one group, LSTBC is reduced to an orthogonal design. In this case, g_m=m. According to (13), it can be obtained that[1]: $d_{LSTBC}(r)=mn(1-Tr/K)=d_{ortho}(r)$. On the other hand, if g_m=g_b=1 and K=m=n, LSTBC is reduced to VBLAST and obviously we have: $d_{LSTBC}(r)=1-r/m=d_{VBLAST}(r)$, which is exactly the same as the one presented in [2].

B. Tradeoff of GLST

The diversity-multiplexing tradeoff of GLST is bounded by

$$d_{lower_GLST}(r)\le d_{GLST}(r)\le d_{upper_GLST}(r) \tag{14}$$

where

$$d_{upper_GLST}(r)=g_m n\left(1-Tr/K\right) \tag{15}$$

and

$$d_{lower_GLST}(r)=\left(x-Tr/G\right)\left(g_b-Tr/G\right) \tag{16}$$

with x=min$(n, Tn-K+g_b)$.

Both orthogonal design and VBLAST can be also regarded as special cases of GLST. When K=m=n=G and g_m=g_b=1, GLST is reduced to VBLAST. Notice here that n=K and T=1. Therefore, x=Tn-K+g_b=1. It can be proved that the lower bound is achieved only when x=Tn-K+g_b. Therefore, we have $d_{GLST}(r)=1-r/m=d_{VBLAST}(r)$.

On the other hand, when there is only one group, the upper bound can be achieved, i.e., $d_{GLST}(r)=mn(1-Tr/K)$. In this case, GLST is reduced to orthogonal design and obviously we have $d_{GLST}(r)=d_{ortho}(r)$.

C. Tradeoff of QoGST

The diversity-multiplexing tradeoff of QoGST is bounded by

$$d_{lower_QoGST}(r)\le d_{QoGST}(r)\le d_{upper_QoGST}(r) \tag{17}$$

where

$$d_{upper_QoGST}(r)=(G_m n-Tr/G_b)(g-Tr/G_b) \tag{18}$$

and

$$d_{lower_QoGST}(r)=\left(G_m n-m+g-Tr/G_b\right)\left(g-Tr/G_b\right). \tag{19}$$

$d_{upper_QoGST}(r)$ reflects the optimal case with no interference among the groups. In other words, $d_{QoGST}(r)$ can reach the upper bound when all the groups are orthogonal. This condition is satisfied only with the group size g=1. In this case, QoGST is reduced to an orthogonal design and obviously we have $d_{QoGST}(r)=mn(1-Tr/K)=d_{ortho}(r)$ (r can only take values from $\{0, K/T\}$).

When there is only one group, i.e., G_m=G_b=1, QoGST turns into an m-transmit n-receive system with a maximum likelihood detector over Rayleigh quasi-static channels. In this case, the upper bound and lower bound converge to

[1] In [2] the diversity-multiplexing tradeoff of the orthogonal design with 2 transmit antennas and n receive antennas has been presented as $d(r)=2n(1-r)$. Through a similar analysis, the tradeoff of the orthogonal design with an arbitrary m and n can be derived as $d(r)=mn(1-Tr/K)$. We omit the proof due to space limitation.

$d_{upper_QoGST}(r)= d_{lower_QoGST}(r)=(n-r)(m-r)$. Therefore, the diversity-multiplexing tradeoff function of QoGST can be exactly given by $d_{QoGST}(r)=(n-r)(m-r)$, which is exactly the same as the optimal tradeoff function for an (m, n) over Rayleigh channels as shown in [2].

D. Tradeoff Comparison

Fig. 1 presents the diversity-multiplexing tradeoff curves of GLST and QoGST for $m = K = 4$, $n = 2$ and 2 groups, i.e., $g=g_m=g_b=2$ and $G=G_m=G_b=2$. Here the diversity gain of LSTBC is always zero since its detector cannot work when $n \le m - g_m$. For comparison, we also show the optimal tradeoff curve for an (m, n) over Rayleigh channels. Both GLST and QoGST needs $T=2$ time slots. Since $x=Tn-K+ g_b =2$, the lower bound tradeoff function of GLST can be achieved. Therefore, from (16), (18) and (19), the tradeoff function of GLST and the lower and upper bound tradeoff functions of QoGST can be obtained. As Fig. 1 shows, the tradeoff curve of GLST is exactly the same as that of the lower bound of QoGST. This implies that QoGST always has a better diversity-multiplexing tradeoff than GLST. Besides, the upper bound tradeoff of QoGST overlaps with the optimal one. Actually, it is not always the same as the optimal tradeoff. As Fig. 2 shows, when n increases to 3, the maximum multiplexing gain r_{max} of the optimal one will increase to 3, while r_{max} of the upper bound of QoGST is only 2. In this case, the lower bound tradeoff of QoGST is even better than the upper bound tradeoff of GLST, thus, indicating a more significant gain. The diversity-multiplexing tradeoff curve of LSTBC is also plotted in Fig. 2. Obviously it is much lower than the lower bound of GLST.

Fig. 3 and Fig. 4 show the case of $m = K = 6$ and $n = 4$. Since 6 symbols (antennas) can be divided into 2 groups with 3 symbols (antennas) in each group, or 3 groups with 2 symbols (antennas) in each group, we consider both of these possible partitions. In Fig. 3, the bit stream and transmit antennas are assumed to be separated into 2 groups, i.e., $g=g_m=g_b=3$ and $G=G_m=G_b=2$. Recall that in QoGST, STBC is adopted among the groups. Therefore, for a 2-symbol-2-antenna transmission, only $T_{QoGST}=2$ time slots are needed. However, for GLST and LSTBC, STBC is adopted inside the group. We take the [3,4,3] STBC code given in [10] (pp. 2485, Eqn. (99)), and so $T_{GLST}=4$. As Fig. 3 shows, QoGST can always get a much better diversity-multiplexing tradeoff than GLST as its lower bound is always better than the upper bound tradeoff of GLST. GLST and LSTBC can only get a maximum multiplexing gain of 1.5, which is lower than that of QoGST. This is because they need more time slots to transmit all K symbols. LSTBC again gets the worst diversity-multiplexing tradeoff among these 3 schemes. Its maximum diversity gain is much lower than that of GLST.

In Fig. 4, the latter partition, i.e., $g_m=g_b=g=2$ and $G=G_m=G_b=3$ is shown with the solid lines. Here LSTBC cannot work due to $n = m - g_m$. Besides, GLST needs 2 time slots, while QoGST needs 4 time slots. From Fig. 4, it can be seen that in this case despite a much higher diversity gain, QoGST has a smaller maximum multiplexing gain due to its lower transmission rate. Nevertheless, if we compare this

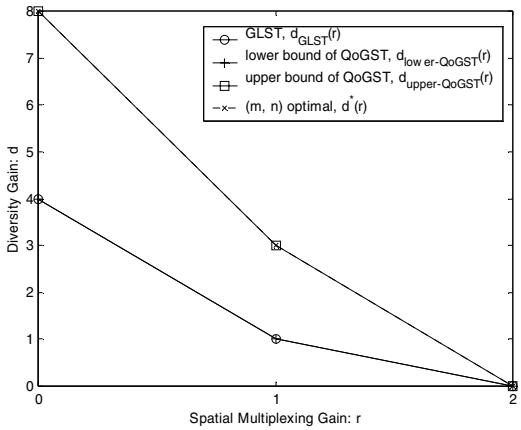

Fig. 1: Diversity-multiplexing tradeoff curves of GLST, the upper and lower bounds of QoGST and the optimal one when $m = K = 4$, $n = 2$. $G= G_m=G_b=2$, and $g=g_m=g_b=2$

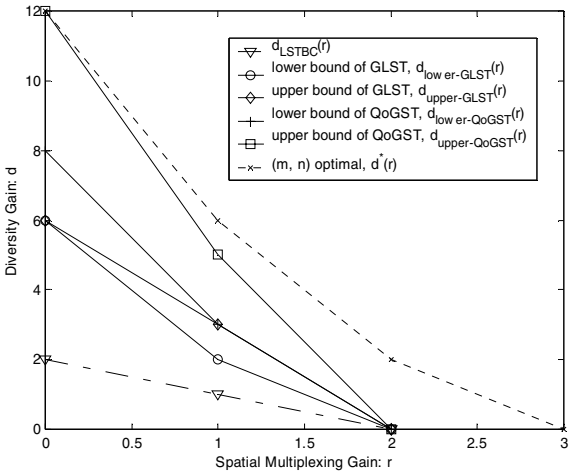

Fig. 2: Diversity-multiplexing tradeoff curves of LSTBC, the upper and lower bounds of GLST and QoGST, and the optimal one when $m = K = 4$, $n = 3$. $G= G_m=G_b=2$, and $g=g_m=g_b=2$.

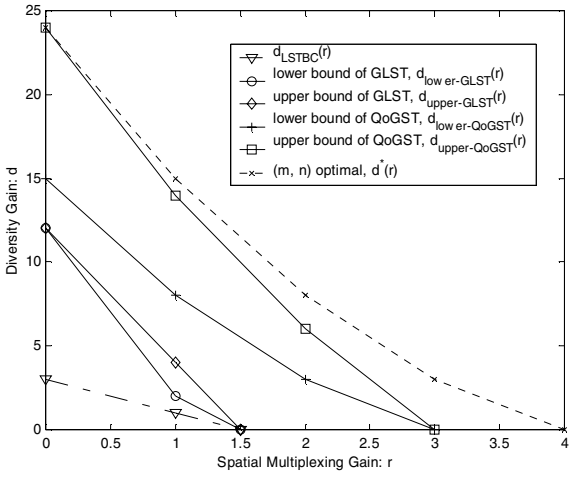

Fig. 3: Diversity-multiplexing tradeoff curves of LSTBC , the upper and lower bounds of GLST and QoGST, and the optimal one when $m=K=6$, $n=4$. $G=G_m=G_b=2$, and $g_m=g_b=g=3$.

tradeoff of QoGST to the tradeoff of GLST with $g_m = g_b = 3$ and $G = 2$ (which is plotted with dash lines), it is found that QoGST can always achieve a better diversity gain while keeping the same *multiplexing gain* as GLST. As Fig. 4 shows, in this case the lower bound tradeoff curve of QoGST is always higher than the upper bound tradeoff curve of GLST. The same conclusion holds true for the comparison of QoGST with $g = 3$, $G_m = G_b = 2$ and GLST with $g_m = g_b = 2$, $G = 3$. Therefore, we can conclude that QoGST always has a better diversity-multiplexing tradeoff than GLST.

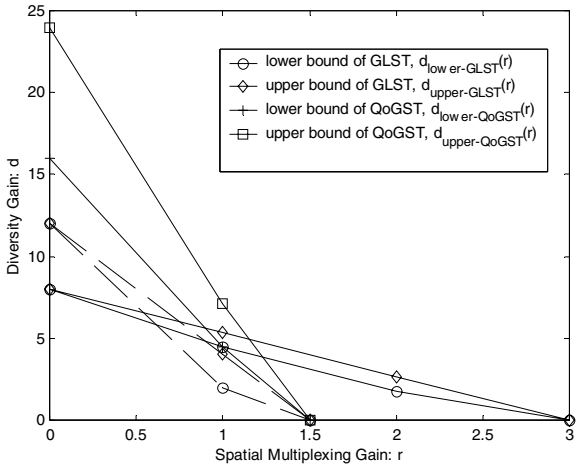

Fig. 4: Diversity-multiplexing tradeoff curves of the upper and lower bounds of GLST and QoGST when $m = K = 6$, $n = 4$. Solid line: $G = G_m = G_b = 3$, $g_m = g_b = g = 2$; Dash line: $g_m = g_b = 3$, $G = 2$.

IV. FER RESULTS AND DISCUSSIONS

We have shown that among 3 schemes QoGST has the best diversity-multiplexing tradeoff and LSTBC is the worst. In this section, we further compare their FER performance. QPSK is assumed to be adopted. As Fig. 5 shows, when $m = 4$ and $n = 2$, QoGST can achieve a gain of 3 dB over GLST at a FER of 10^{-3}. Besides, in high-SNR conditions, the FER curve of QoGST has a larger slope than that of GLST, which implies that QoGST has a better diversity gain. Notice that these two schemes have the same spectral efficiency. As a result, we can conclude that QoGST achieves a better diversity-multiplexing tradeoff, which is consistent with our analysis in Section III. Here we did not show the FER curve of LSTBC since it cannot work in this case.

When $m = 6$ and $n = 4$, the transmit antennas can be divided into 2 or 3 groups. To keep the same transmission rate, the group size of QoGST as well as GLST and LSTBC is assumed to be 3 and 2 (or 2 and 3), respectively. As Fig. 5 shows, for the case of $g_{QoGST} = G_{GLST} = 3$, QoGST can achieve at least 4 dB gain at a FER of 10^{-3}, and the FER curve of QoGST is much steeper than that of GLST, which implies a much better diversity gain. Again LSTBC cannot work in this case. When $g_{QoGST} = G_{GLST} = G_{LSTBC} = 2$, QoGST will not achieve such a significant gain over GLST as before. Nevertheless, a larger slope and 1 dB gain can be seen. LSTBC gets the worst performance. At a FER of 10^{-3}, 12 dB and 11 dB gains can be achieved by QoGST and GLST, respectively. These observations clearly validate our analysis on the diversity-multiplexing tradeoff.

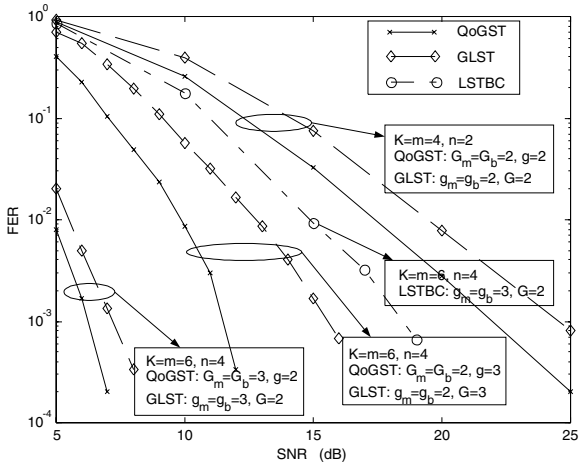

Fig. 5: FER vs. SNR curves of QoGST, GLST and LSTBC. Solid line: QoGST; Dash Line: GLST; Dash-dot Line: LSTBC.

V. CONCLUSIONS

In this paper, the diversity-multiplexing tradeoffs of 3 schemes based on group transmission and detection, QoGST, GLST and LSTBC, were presented. The comparison showed that QoGST can always achieve the best diversity-multiplexing tradeoff thanks to its inter-group STBC encoder and effective detector. GLST is not as good as QoGST, while substantial gains over LSTBC can still be observed.

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