Optimal rate allocation for Group Zero Forcing*

Sana Sfar, Lin Dai, and Khaled B. Letaief**, Fellow, *IEEE.*
Center for Wireless Information Technology
The Hong Kong University of Science & Technology
Clear Water Bay, HONG KONG
eesana@ust.hk, eedailin@ust.hk, eekhaled@ust.hk

Abstract— **MIMO systems have been shown to provide significant performance gains over traditional single antennas systems that fall in two categories: diversity and multiplexing rate. A tradeoff between these gains was recently put in evidence and has been quantified with the optimal detection. In this paper, we consider the evaluation of such tradeoff when group detection is applied and particularly when the Group Zero Forcing (GZF) receiver structure is considered. To do so, we will define and evaluate the outage probability per group and derive the tradeoff obtained by each of the groups. The overall system tradeoff will be then given by the minimum group tradeoff performance. Optimal rate allocation will also be proposed so as to maximize GZF tradeoff performance. Comparison for a given group partition, with equal rate allocation will show that optimal rate allocation allows us to both maximize the diversity and the multiplexing rate of GZF. Furthermore, considering a fixed number of antennas, we will find the minimum required number of groups for a given tradeoff level, as well as, the optimal group partition that maximizes the system tradeoff. Numerical results will demonstrate the optimality of this scheme. Significant diversity gains will be put in evidence demonstrating that GZF can efficiently bridge the gap between BLAST and the optimal receiver while offering lower levels of complexity.**

Keywords- *Diversity-Multiplexing tradeoff, MIMO systems, Group detection, Channel capacity, Channel multivariate statistics distribution.*

I. Introduction

Multiple Input Multiple Output systems (MIMO) have been shown to provide significant performance gains over traditional single antennas systems [1]. These gains fall into two categories: *Diversity* and *Spatial Multiplexing* [2]. Most of previously designed MIMO systems have focused on the maximization of either types of gains. Examples include Space-Time Codes (STC) [3] and orthogonal designs [4] that achieve full diversity gains. V-BLAST [5] is another example that achieves unprecedented data rates. However, these examples have been shown to present high capacity [2, 5, 6] and diversity loss [2], respectively.

Recently, it was found that both gains can be achieved simultaneously with a single system, but a fundamental tradeoff would relate how much gain of each type could be obtained [2]. Several attempts have been made to design systems that achieve a better tradeoff than the previously mentioned schemes. We cite for example the work in [7] and [8]. These systems consist in combining STC [7], or STBC [8] with V-BLAST to achieve higher data rates than STC/STBC schemes and higher diversities then BLAST type schemes. Group detection [9-10] has played an important role in the design of these schemes. In this case, signals are arranged in groups and assigned one space-time component code. Group detection is then applied at the receiver to separate the differently coded groups. Decoding is next processed inside each group independently of others. Diversity type schemes and capacity type schemes have been hence enabled to co-operate together through the use of group detection.

In this paper, we consider a MIMO system where group detection is deployed. Signals are arranged into different groups and retrieved in groups, simultaneously and in parallel. Such receiver is denoted here by GZF such as in [11]. We will evaluate here the diversity-multiplexing tradeoff of GZF. The obtained tradeoff performance will be used as a comparison benchmark for any scheme using GZF receiver. To do so, we first define and evaluate the outage probability per group. We will show such probability both lower and upper bound the group frame error rate and obtain a diversity-multiplexing tradeoff for each group. Next, we will demonstrate that the overall system tradeoff is given by the worst group tradeoff performance.

Assuming a given group partition, we will propose in this paper the optimal rate allocation algorithm that maximizes the system tradeoff performance. Comparison with the widely used rate allocation scheme, which allocates rates equally among groups, will demonstrate significant gains in both diversity and data rates. Further, we will consider a fixed number of antennas and we will find the optimal number of groups as well as the optimal group partition associated to that number which maximizes the system tradeoff assuming optimal rate allocation scheme among groups. Such a process will enable the optimization of the system configuration for a maximum performance. Results will confirm the optimality of this scheme by demonstrating unprecedented diversity gains.

This paper is organized as follows. Section II describes the adopted system model, as well as, the different notations used throughout the paper. In Section III, we derive and evaluate the outage probability per group along with its corresponding tradeoff. In Section IV, the GZF tradeoff performance is provided. In Section V, the optimal rate allocation scheme is proposed to maximize the system performance. Simulations results are provided in Section VI and conclusions are given in Section VII.

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** Corresponding author: *Professor K. B. Letaief*. Email: eekhaled@ee.ust.hk. Tel:/Fax: (852) 2358 7064/1485. URL: http://www.ee.ust.hk/~eekhaled

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II. SYSTEM MODEL

We consider throughout this paper the transmission of K encoded signals, drawn from the same constellation, over ℓ time symbols through K antennas with an overall data rate R. These signals are divided into G groups and are assigned different data rates for transmission. Let the notation "$|g|$" represent the size of the g$^{\text{th}}$ group and R_g its corresponding data rate. Then,

$$K = \sum_{g=1}^{G} |g| \text{ and } R = \sum_{g=1}^{G} R_g .$$

Let the $|g|\times\ell$ matrix X_g denote the g$^{\text{th}}$ group transmitted matrix and $X = \left[X_1^T, X_2^T, \ldots, X_G^T\right]^T$. Each of these signals undergo a slowly Rayleigh fading channel, denoted by H, to reach a receiver with N antennas, such that $N \geq K$. We assume that the channel is constant within a frame of ℓ symbols and denote by H the $N \geq K$ channel realization matrix. We shall also assume perfect channel knowledge at the receiver side only, perfect symbol synchronization and equal transmission power. The discrete model of the $N\times\ell$ received complex signal vector can then be written as

$$\mathbf{Y} = \sqrt{\frac{SNR}{K}} H\mathbf{X} + \mathbf{V} \tag{1}$$

where $\mathbf{V}$ is a noise $\mathcal{N}(0,\sigma^2\mathbf{I}_{\mathbf{N}\times\mathbf{N}})$ distributed and SNR is the average received signal to noise ratio per receive antenna. When GZF is used at the receiver, groups are retrieved in parallel and simultaneously. Only one stage of detection is applied where data from each group are detected independently from each other. By doing so, a per-group sufficient statistic vector is obtained and is denoted by $\mathbf{Y}_g^{GZF}$ for the g^{th} group and is given by [9,10]

$$\mathbf{Y}_g^{GZF} = \sqrt{\frac{SNR}{K}} \mathbf{X}_g + \mathbf{V}_g \tag{2}$$

where $\mathbf{V}_g$ is $\mathcal{N}(0,\sigma^2 Q_g)$ distributed such that Q_g is the g$^{\text{th}}$ diagonal submatrix of $\left[H^\dagger H\right]^{-1}$ of size $|g|$.

Throughout this paper, for an arbitrary matrix $\mathbf{A}$, we write $\mathbf{A} \geq 0$ when it is hermitian positive and denote by $\det(\mathbf{A})$ its determinant.

III. GROUP OUTAGE PROBABILITY AND TRADEOFF

When optimal detection is applied at the receiver, [2] has derived the system diversity-multiplexing tradeoff, defined in [2] as the SNR exponent of the minimum achievable frame error rate (FER).

Consider the system model in (1) and let $P_e(SNR)$ denote its minimum FER, achieved when the best outer codes that generate the transmitted symbols are deployed. Also, let $P_{out}(R)$ denote its outage probability. Throughout this paper, the following notation will be used

$$f(SNR) \doteq b \text{ when } \lim_{SNR\to\infty} \frac{\log f(SNR)}{\log SNR} = b . \tag{3}$$

In [2], it was found that when $\ell \geq N + K - 1$, $P_{out}(R)$ both lower and upper bounds $P_e(SNR)$ to obtain

$$P_e(SNR) \doteq SNR^{-d(r)} \tag{4}$$

where $d(r)$ denotes the SNR exponent in $P_{out}(SNR)$ and $r = R/\log(SNR)$. $d(r)$ provides thus the tradeoff of the considered scheme. When optimal detection is applied, $d(r)$ is a linear piece-wise function that connects $(k, d(k))$ such that

$$d(k) = (N-k)(K-k); k = 1,\ldots,K \tag{5}$$

Similar to the approach in [2], we derive in what follows the outage probability of the g$^{\text{th}}$ group, as well as, its corresponding tradeoff.

Definition 1: *Let $\Omega_{g_{out,H}}$ denote the g^{th} group outage event, associated with an input $\mathbf{X}_g$, an output $\mathbf{Y}_g$, a channel realization $\mathbf{H}$=H and a data rate R_g. It is defined as the event where the g^{th} group does not meet its required data rate R_g and is written as*

$$\Omega_{g_{out,H}} = \left\{H : I_{g,H} < R_g\right\}$$

where $I_{g,H}$ denotes the g^{th} group mutual information given by $I(\mathbf{X}_g;\mathbf{Y}_g \mid \mathbf{H} = H)$. Its corresponding probability will be denoted by $P_{g,out}(R_g)$.

According to the system model in (2), we have

$$I(\mathbf{X}_g;\mathbf{Y}_g \mid \mathbf{H} = H) = \log\det\left(\mathbf{I}_{|g|} + SNR\times Q_g^{-1}\right)$$
$$= \log\prod_{i=1}^{|g|}\left(1 + SNR\times\mu_i\right)$$

where $\left\{\mu_1, \mu_2, \ldots, \mu_{|g|}\right\}$ are the ordered eigenvalues of Q_g^{-1}. We note here that since $Q_g \geq 0$ and is of full rank $|g|$, then so is Q_g^{-1}. It can be shown that Q_g^{-1} is Wishart distributed [12] and that the joint probability density function (pdf) of its eigenvalues satisfies

$$p(\mu_1,\mu_2,\cdots,\mu_{|g|}) = C_{|g|,N-|\bar{g}|}^{-1} e^{-\sum_{i=1}^{|g|}\mu_i} \prod_{i=1}^{|g|} \mu_i^{|N-|\bar{g}|-|g||} \prod_{j<i} (\mu_j - \mu_i)^2 \tag{6}$$

where $|\bar{g}|$ denotes the number of interferers to the g$^{\text{th}}$ group, and is given by $|\bar{g}| = K - |g|$. Due to space limitations, we do not provide details about Q_g^{-1} distribution in this paper.

Let $R_g = r_g \log(SNR)$ and $d_g^{GZF}(r_g)$ denote the SNR exponents in $P_{g,out}(R_g)$. A similar approach as the one in [2] permits to obtain

$$P_{e,g}(SNR) \doteq P_{g,out}(R_g) \tag{7}$$

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where $P_{e,g}(SNR)$ denotes the g^{th} group FER. $d_g^{GZF}(r_g)$ is hence its tradeoff function. Let $\ell_g = N-K+2|g|-1$. According to [2], when $\ell \geq \ell_g$, $d_g^{GZF}(r_g)$ is a linear piece-wise function that connects $(k, d_g^{GZF}(k))$ such that

$$d_{g,out}^{GZF}(k) = (N-|\overline{g}|-k)(|g|-k),\ k \in \{1,...,|g|\}. \quad (8)$$

IV. GZF TRADEOFF EVALUATION

Let $\Omega_{out,H}$ denote the outage event of the overall system when GZF is used, written as

$$\Omega_{out,H} = \{H : I_H < R\}$$

where $I_H = I(\mathbf{X};\mathbf{Y} \mid \mathbf{H} = H)$ is the mutual information of the system in (1) conditioned on $\boldsymbol{H}$=*H*. Also, let $P_{out}^{GZF}(R)$ denote the overall system outage probability.

When group detection is applied, an overall outage event indicates that any of the considered groups is in outage. Hence,

$$\Omega_{out,H} = \left\{ H : \bigcup_{g=1}^{G} \{ I_{g,H} < R_g \} \right\}$$

According to (2), the *G* group outage events are independent of each other resulting in

$$\Omega_{out,H} = \Omega_{1_{out,H}} \cup \Omega_{2_{out,H}} \cup \ldots \cup \Omega_{G_{out,H}}.$$

Hence, the overall system outage probability satisfies

$$P_{out}^{GZF}(R) = 1 - \prod_{g=1}^{G}\left[1 - P_{g,out}(R_g)\right]. \quad (9).$$

At high SNR values, we have

$$P_{out}^{GZF}(R) \approx \max_{g=1,...,G} P_{g,out}^{GZF}(R_g). \quad (10)$$

According to (7), $d_g^{GZF}(r_g)$ is given by (8) when $\ell \geq \ell_g$. Let $\ell_{\max}^{GZF} = \max_{g=1,...,G} \ell_g$. It follows that when $\ell \geq \ell_{\max}^{GZF}$,

$$P_e^{GZF}(R) \doteq P_{out}^{GZF}(R) \doteq SNR^{-d^{GZF}(r)} \quad (11)$$

where $P_e^{GZF}(R)$ is the system FER. The overall tradeoff function is hence given by

$$d^{GZF}(r) = \left\{ \min_{g=1\cdots G}\{d_g^{GZF}(r_g)\}; \sum_{g=1}^{G} r_g = r \right\}. \quad (12)$$

where $d_g^{GZF}(r_g)$ are given in (8).

V. OPTIMAL RATE ALLOCATION

In this section, we provide the optimal rate allocation algorithm that maximizes the overall system tradeoff. Due to space limitations, the derivation of the algorithm will not be explained and only the final results will be given. Consider a group partition *{p}*, and let $d^{\{p\}}(r)$ denote its corresponding tradeoff when the data rate is optimally allocated among the groups. We have

$$d^{\{p\}}(r) = \max_{r_1,...,r_G}\left\{ \min_{g=1\cdots G}\left\{ d_g^{GZF}(r_g) \right\} \right\}$$

subject to:

$$\left\{ \sum_{g=1}^{G} r_g = r; r_g \in [0 \quad |g|], \forall g = 1,...,G \right\}.$$

In what follows, we assume $\ell \geq \ell_{\max}^{GZF}$ and denote by $g_1^{\{p\}}$ the group with the largest size, by $g_2^{\{p\}}$ with the second largest size and so on. Hence, we have $|g_1^{\{p\}}| \geq |g_2^{\{p\}}| \geq ... \geq |g_G^{\{p\}}|$.

With this scheme, the allocation is performed in *G* slots such that only "*j*" groups are allocated rates different than zero in the j^{th} slot. Let $\{r_0^{\{p\}}, r_1^{\{p\}}, ..., r_G^{\{p\}}\}$ denote the borders of these slots, and let $d^{\{p\}}(j)$ be their corresponding diversity levels. $d^{\{p\}}(r)$ will then connect the points with coordinates $\left(r_j^{\{p\}}, d^{\{p\}}(j)\right)$. These coordinates, as well as, $d^{\{p\}}(r)$ are computed according to the following algorithm

- Let

  $$d^{\{p\}}(j) = (N - K_{(j)}^{\{p\}})\,|g_{j+1}^{\{p\}}|,\ j = 0,...,G-1 \quad (13)$$

  and

  $$r_j^{\{p\}} = Nj/2 + (1-j/2)K_{(j)}^{\{p\}} - j/2\sqrt{\Delta_j^{\{p\}}} \quad (14)$$

  for $j = 1,...,G-1$, where

  $$K_{(j)}^{\{p\}} = \sum_{k=1}^{j} |g_k^{\{p\}}|,\ A_j^{\{p\}} = N - K_{(j)}^{\{p\}} \quad (15)$$

  $$\text{and } \Delta_j^{\{p\}} = \left(A_j^{\{p\}}\right)^2 + 4A_j^{\{p\}}\,|g_{j+1}^{\{p\}}|. \quad (16)$$

  Also let $r_0^{\{p\}} = 0$, $r_G^{\{p\}} = K$ and $d^{\{p\}}(G) = 0$.

  When $r = r_j^{\{p\}}$, GZF tradeoff with optimal allocation is given by $d^{\{p\}}(j)$.

- When $r \in [r_{j-1}^{\{p\}}, r_j^{\{p\}}]$: only the set of groups $\{g_1^{\{p\}}, g_2^{\{p\}}, ..., g_j^{\{p\}}\}$ are allocated a non zero rate, $r^*_{g_i^{\{p\}}}$, such that

  $$r^*_{g_i^{\{p\}}} = \frac{1}{j}\left(r + j\,|g_i^{\{p\}}| - K_{(j)}^{\{p\}}\right). \quad (17)$$

  The optimal tradeoff is given by

  $$d^{\{p\}}(r) = \left(N - \frac{r}{j}\right)(K_{(j)}^{\{p\}} - \frac{r}{j}). \quad (18)$$

A close observation of $d^{\{p\}}(r)$ in this case, indicates that the obtained tradeoff is equivalent to the optimal tradeoff [2] achieved with a system of *N* receiving antennas and $K_{(j)}^{\{p\}}$ antennas transmitting with a rate r/j.

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VI. SIMULATION RESULTS

We assume throughout this section that $K=N=8$ and $G \in \{2,3,..,K\}$. We shall denote by $\{a_1, a_2, \dots, a_G\}$ an ordered group partition where a_i refers to the size of the i^{th} detected group. Finally, we assume that the coding block length, $\ell \geq \ell_{\max}^{GZF}$ and that all transmitted symbols have the same constellations.

First, we investigate the diversity-multiplexing tradeoff of GZF for a given group partition. Particularly, we consider the *{3,2,2,1}* and the *{2,2,2,2}* partitions when $G=4$. We evaluate and compare in this case GZF tradeoff with both the optimal rate allocation and the equal rate allocation. With the first scheme, rates are optimally allocated according to Section V. As for the equal rate allocation scheme, groups are equally allocated rates for transmission. Results are provided in Figure 1. Clearly, the optimal allocation scheme outperforms the equal rate one with both partitions. When $\{p\} = \{3,2,2,1\}$, a close observation of Figure 1 indicates that the maximum multiplexing rate has improved by 4 sym/sec with the optimal scheme. A gain of the order of 25 in the system diversity is also observed. When $\{p\} = \{2,2,2,2\}$, we notice that GZF performance with the optimal allocation scheme presents diversity degradation, while both diversity and multiplexing rate gains are achieved with the equal rate scheme. We also observe that the equal rate scheme performs similarly to the optimal one when r>2 sym/sec.

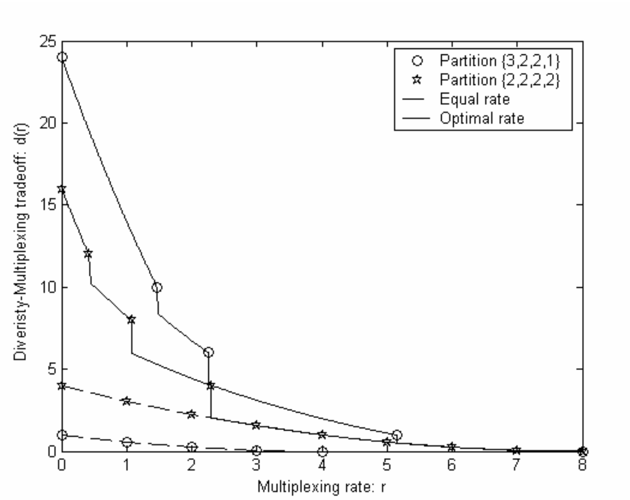

Fig. 1: O*ptimal versus equal rate allocation tradeoff performance with a random partition. N=K=8.*

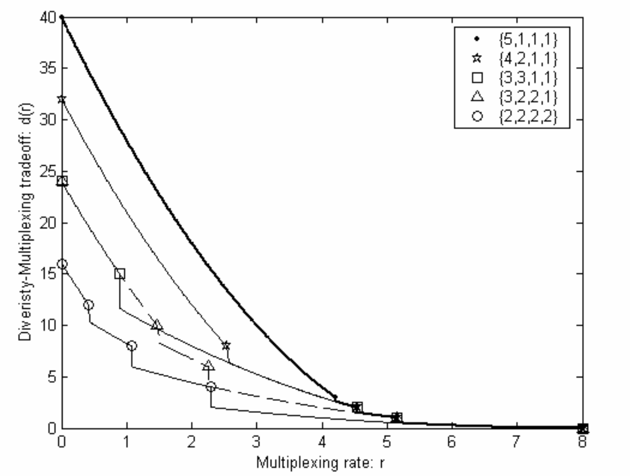

Fig. 2: Tradeoff o*ptimization with optimal rate allocation. N=K=8 and G=4.*

Next, we find the optimal group partition that maximizes the tradeoff when $G=4$ and $K=N=8$. To do so, we evaluate the tradeoff for each partition and choose the best tradeoff for each r. Figure 2 illustrates such process when rates are allocated optimally among the four groups. A close observation of this figure indicates that the {5,1,1,1} partition outperform all others, and that the {2,2,2,2} partition performs the worst. Such results hint that GZF tradeoff is maximized when the first retrieved group has the largest possible size, i.e. $K-G+1$, and is minimized when all groups have the same size. This can be easily confirmed using the results in (17)-(18). In what follows, we denote the obtained tradeoff with such process by the optimized tradeoff.

In Figure 3, we compare the optimized tradeoff obtained when rates are optimally or equally allocated among the four groups. Results demonstrate that the optimal allocation scheme achieves an unprecedented diversity gains at low values of r. For example, when $r=0$, the maximum diversity achieved with the equal rate scheme is around *4* versus *32* with the optimal scheme. Such gains are also observed to reduce exponentially with r and to vanish when $r>4$. Indeed, the optimized tradeoff performance obtained with equal rate allocation scheme is observed when $r>4$ in Figure 3 to approach the optimized performance when rates are optimally allocated among groups. This implies that the equal rate allocation scheme can be used instead of the optimal allocation one when groups are optimally partitioned to maximize the system tradeoff. By doing so, similar levels of performance along with significant complexity reduction are obtained.

Finally, we investigate the effect of G on the optimized tradeoff of GZF with the optimal rate allocation scheme. Figure 4 provides the obtained results for $G=2,\dots,8$. We note here that when G=8, GZF is equivalent to V-BLAST and our results confirm those in [2]. The optimal tradeoff given in (5) when group detection is not used at the receiver but rather the optimal detection [2] is also shown in this figure for comparison. Results indicate in this case that the optimized tradeoff is enhanced when G is decreased. Indeed, when G=2, the GZF approaches the optimal tradeoff significantly, while G=8 presents the worst performance. We note here that when G=2, we found that the best tradeoff is obtained with the {7,1} partition. This further confirms our results listed in Figure 2. We can conclude here that group detection is efficiently bridg-

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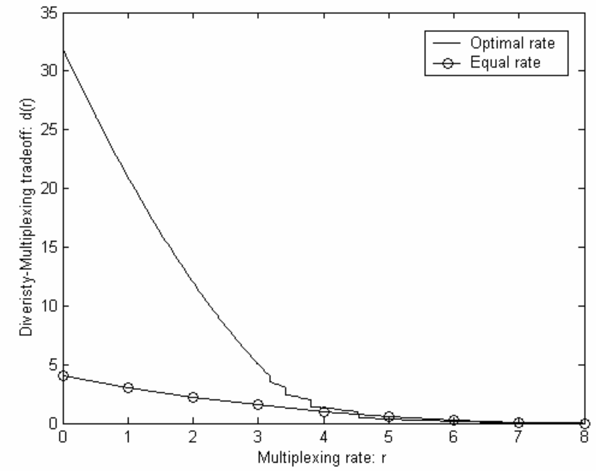

Fig. 3: *Optimized tradeoff performance with equal rate and optimal rate allocation schemes. N=K=8 and G=4.*

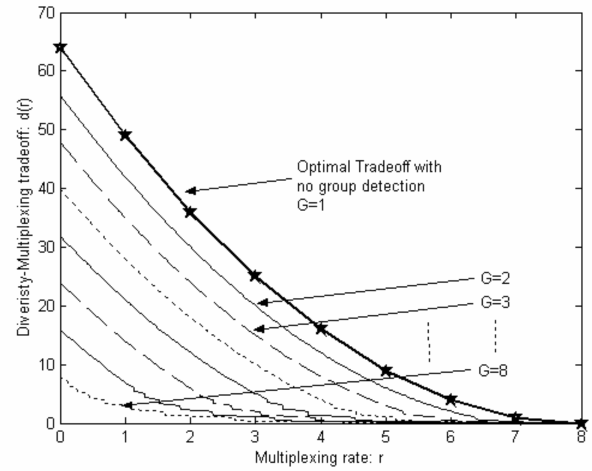

Fig. 4: *Effect of the number of groups, G, on the optimized tradeoff perofrmance with optimal rate allocation. N=K=8.*

ing the gap between BLAST [5] and the optimal scheme [2] while offering lower levels of complexity.

VII. CONCLUSIONS

In this paper, we considered a MIMO system where group detection is employed at the receiver. For each of the groups, we evaluated its outage probability and diversity-multiplexing tradeoff. We provided an expression of the overall system outage probability as a function of the group outage ones, and showed that the overall system tradeoff is given by the minimum group tradeoff performances. The optimal rate allocation scheme that maximizes the system tradeoff performance has been proposed. We have also provided the closed form expression of the tradeoff in this case. Comparison results with the equal rate allocation scheme have demonstrated significant diversity, as well as, multiplexing rate gains. We had further optimized the obtained tradeoff with both schemes over all possible partitions for a given *G*. Simulation results put in evidence unprecedented diversity gains with the optimal scheme at low data rates. On the other hand, when high data rates are required, we showed that the equal rate allocation scheme might be used to provide the same level of performance as the optimal one. Finally, we have demonstrated that the GZF optimized performance with the optimal rate allocation approaches the ultimate optimal performance [2] when *G* is reduced. By doing so, we can bridge the gap between the optimal receiver [2] and BLAST.

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