

A Novel Spectral Efficient Transmit Precoder Scheme Based on Channel Feedback

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Abstract—In this paper we propose a novel spectral efficient transmit precoder scheme in which channel state information is fully utilized to maximize channel capacity. It is shown that this new scheme provides striking performance that is 3.5-5 dB from the outage capacity at 10% frame error rate even in the uncoded case. Compared with the open-loop scheme V-BLAST, this new scheme can not only maintain the same high bandwidth efficiency, but also achieve much better performance thanks to more effective transmission power allocation and diversity gain. Nearly 7dB gain can be provided by this new scheme over V-BLAST in a (2, 2) system.

I INTRODUCTION

Deploying multiple antennas at the base station is an effective and promising solution to reliable communication in wireless channels. With multiple antennas, diversity gain can be provided to counteract the harmful effect of fading and thus improve the performance significantly. Previous studies focused on receive diversity since reverse-link capacity used to be considered as the bottleneck. However, as asymmetrical services are highly ongoing to be introduced into wireless communication, improvement of forward-link performance is becoming the main challenge. Transmit diversity is one of the key contributing technologies to solve this problem [1-4].

Numerous transmit diversity schemes have been proposed, such as delay diversity [1], orthogonal transmit diversity [2], phase sweeping transmit diversity [3] and selective transmit diversity [4]. Depending on whether the feedback information is utilized or not, these schemes can be categorized as closed-loop or open-loop ones. The performance of closed-loop schemes is usually considered better than that of open-loop ones, although higher system complexity is needed and their success depends greatly on the quality of the channel estimation, the feedback delay, the dynamics of the signal and etc.

In these traditional transmit diversity schemes, the

same information is transmitted through multiple antennas. Though partial or full spatial diversity can be achieved, either coherent receiver or additional resources (such as bandwidth, time or spreading codes) are required. Thus the overall efficiency of the system is very limited. Recently, a group of spectral efficient open-loop transmit diversity schemes in flat fading environment are proposed, known as Bell lab layered space-time architecture (BLAST) [5-7], including Diagonal BLAST (D-BLAST), Vertical BLAST (V-BLAST), and etc. In BLAST systems, parallel data streams are simultaneously transmitted through multiple antennas in the same frequency band, and separated at the receiver according to their distinct spatial signatures with interference rejection and cancellation techniques. It has gained great attention for its high spectral efficiency and reasonable complexity.

Nevertheless, as aforementioned, BLAST operates in an open-loop situation, that is, without channel knowledge at the transmitter. Better performance can be expected provided that channel state information (CSI) is utilized to shape the transmission waveforms, which is called transmit precoder. Motivated by this idea, we develop a spectral efficient transmit precoder scheme. It has the similar transmit structure as V-BLAST with m transmit antennas and n receive antennas. However, we add a linear transformation \mathbf{K} before transmission. The design of \mathbf{K} is based on water-filling principle that has been proved to maximize channel capacity when the channel is known at both the transmitter and the receiver [8]. Simulation results show that the performance of our new scheme is quite close to the closed-loop capacity limit when the optimal detection algorithm – Maximum Likelihood Detection (MLD) is adopted. With sub-optimal detection algorithm, its performance is still satisfactory especially when the number of receive antennas is large. Moreover, the performance comparison between our new scheme and V-BLAST shows that this new scheme can provide substantial gain over V-BLAST. With 2 transmit and 2 receive antennas, nearly 7 dB gain can be obtained by this new scheme over V-BLAST

at 0.1% frame error rate.

The rest of this paper is organized as follows. In Section II, we introduce the channel model. The details of the novel scheme are described in Section III. Section IV presents the numerical results. Finally, Section VI contains our concluding remarks.

II CHANNEL MODEL

We consider a single-user, point to point communication channel with m transmit and n receive antennas. We assume that the channel is flat quasi-static, namely, the channel is considered as constant over a frame, and varies from one frame to another. Each element of the channel gain matrix \mathbf{H} is modeled as an independent complex Gaussian random variable with zero mean and unit variance per dimension. Assume that \mathbf{H} is well known at both the receiver and transmitter through measurement and feedback. Thus transmission power allocation can be optimized according to the channel information to maximize channel capacity. The total transmission power is assumed to be P_t , regardless of m . The noise is assumed to be complex Gaussian distributed with zero mean and variance σ_z^2 per dimension.

Specifically, the following discrete-time equivalent model is used:

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{z}$$

where $\mathbf{x} = [x_0, x_1, \dots, x_{m-1}]^T$ is an $m \times 1$ vector whose the j -th component represents the signal transmitted from the j -th antenna. The received signal and noise vector are both $n \times 1$ vectors which are denoted by \mathbf{y} and \mathbf{z} , respectively. The complex channel gain between the i -th transmitter and the j -th receiver is h_{ji} , for $i = 0, 1, \dots, m-1$ and $j = 0, 1, \dots, n-1$.

The following notations will be used throughout this paper: $*$ for transpose conjugate, $'$ for transpose, \mathbf{I}_n for the $n \times n$ identity matrix, $E[\cdot]$ for expectation, bold lowercase letters for vector, and bold uppercase letters for matrix.

III THE NEW TRANSMIT PRECODER SCHEME

Fig.1 shows the block diagram of our new transmit precoder scheme.

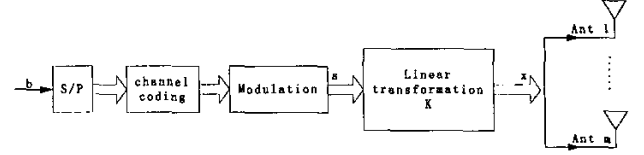


Fig.1 Block diagram of our new scheme

As shown in Fig.1, the information is split into m parallel data streams and encoded separately. After being modulated, these streams are multiplied by a linear transformation matrix $\mathbf{K} \in \mathbb{C}^{m \times m}$ and then transmitted through m antennas. CSI is utilized to design \mathbf{K} so as to maximize channel capacity.

In particular, let $\mathbf{s} = [s_0, s_1, \dots, s_{m-1}]^T$ represents the modulated symbol vector, where each s_i is independent symbol with equal power P_t/m . Then the transmitted symbol vector is $\mathbf{x} = \mathbf{K} \cdot \mathbf{s}$, and its covariance matrix is

$$\mathbf{Q}_1 = \mathbf{K} \mathbf{Q} \mathbf{K}^*, \text{ where } \mathbf{Q} = E[\mathbf{s} \mathbf{s}^*] = \text{diag}(P_t/m). \quad (1)$$

\mathbf{Q}_1 should also satisfy the average power constraint. Therefore \mathbf{K} is requested not to change the trace of \mathbf{Q}_1 .

By the singular value decomposition theorem [10], $\mathbf{H} \in \mathbb{C}^{n \times m}$ can be written as $\mathbf{H} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^*$ where the columns of \mathbf{U} and \mathbf{V} are eigenvectors of $\mathbf{H} \mathbf{H}^*$ and $\mathbf{H}^* \mathbf{H}$, respectively. The diagonal entries of $\mathbf{\Lambda}$ are square roots of the eigenvalues of $\mathbf{H} \mathbf{H}^*$. Thus we have

$$\tilde{C} = \log \det(\mathbf{I}_n + \frac{1}{\sigma_z^2} \mathbf{H} \mathbf{Q}_1 \mathbf{H}^*) = \log \det(\mathbf{I}_n + \mathbf{\Lambda} \tilde{\mathbf{Q}} \mathbf{\Lambda}^*)$$

$$\text{where } \tilde{\mathbf{Q}} = \frac{1}{\sigma_z^2} \mathbf{V}^* \mathbf{K} \mathbf{Q} \mathbf{K}^* \mathbf{V}. \quad (2)$$

From [8] we know that,

$$\text{when } \tilde{\mathbf{Q}} = \text{diag}(\mu - \lambda_i^{-1})^+ \text{ and } \sum_{i=1}^m \tilde{q}_{ii} = P_t / \sigma_z^2 = \rho, \quad (3)$$

$$\max(C) = \sum_{i=0}^{r-1} (\log(\mu \lambda_i))^+. \quad (4)$$

where a^+ denotes $\max\{0, a\}$. r is the rank of \mathbf{H} . λ_i 's are eigenvalues of $\mathbf{H} \mathbf{H}^*$.

Substituting (1) and (3) into (2) yields

$$\mathbf{K} \mathbf{K}^* = \mathbf{V} \mathbf{D} \mathbf{V}^*, \quad (5)$$

$$\text{where } \mathbf{D} = \text{diag}(m(\mu - \lambda_i^{-1}) / \rho)^+. \quad (6)$$

It should be pointed out that the solution to \mathbf{K} of (5) is

not unique. Let \mathbf{W} be an arbitrary unitary matrix, then \mathbf{K} can be written as

$$\mathbf{K} = \mathbf{V}\mathbf{D}^{1/2}\mathbf{W}. \quad (7)$$

It can be proved that (7) satisfies the constraint condition $\text{tr}(\mathbf{Q}_1) = \text{tr}(\mathbf{Q})$ (see Appendix), which indicates that the transmitted streams also satisfy the average power constraint. Moreover, although the channel capacity is maximized if only \mathbf{W} is a unitary matrix, \mathbf{W} should be carefully selected for particular coding and modulation since the error performance depends on \mathbf{W} . In this paper, we search the optimal \mathbf{W} to minimize the pairwise error probability, namely, to maximize the minimum distance between received vectors:

$$\max_{\mathbf{W}} \left\{ \min_{\mathbf{s}_1, \mathbf{s}_2} d^2(\mathbf{s}_1, \mathbf{s}_2) \right\},$$

where \mathbf{s}_1 and \mathbf{s}_2 are two arbitrary possible modulated symbol vectors, and $d^2(\mathbf{s}_1, \mathbf{s}_2) = \|\mathbf{H}\mathbf{K} \cdot (\mathbf{s}_1 - \mathbf{s}_2)\|^2$.

Searching the optimal \mathbf{W} exhaustively will lead to intolerable complexity. Because \mathbf{W} is an $m \times m$ unitary matrix, this problem will become an NP problem as m increases, and the optimal solution can hardly be achieved. So we consider reducing searching complexity by searching the local optimal \mathbf{W} , not the global optimal one. In our scheme we only search the \mathbf{W} that is a symmetric orthogonal matrix, and hence the number of variables will reduce significantly. Simulation results show that satisfying results can be achieved if \mathbf{W} is searched under this criterion.

Finally, according to (7) we can design the linear transformation \mathbf{K} to maximize channel capacity. Obviously in this new scheme high bandwidth efficiency can be achieved since m symbols are transmitted simultaneously and in the same frequency band. Besides, from (6) it is shown that in this new scheme, more transmission power is effectively allocated to those antennas with higher gain, which is based on water-filling principle. Therefore, remarkable performance can be expected.

IV NUMERICAL RESULTS AND DISCUSSIONS

In this section, we evaluated the performance of our new scheme and compared it with V-BLAST. Maximum likelihood detection (MLD) and interference nulling and cancellation [6] algorithms are assumed to be adopted. However, minimum mean-squared error (MMSE) criterion is used instead of zero-forcing so as not to enhance the noise¹. For simplicity, we limit our results to the case of

uncoded QPSK modulation with $m=2$, although our new scheme applies to R-ary PSK with m transmit antennas and n receive antennas in general. Obviously our new scheme achieves the same spectral efficiency as V-BLAST, namely, 4 bits/s/Hz. As aforementioned, \mathbf{H} is considered as constant over a frame, and varies from one frame to another. Each frame consists of 130 transmissions out of each transmit antenna [9].

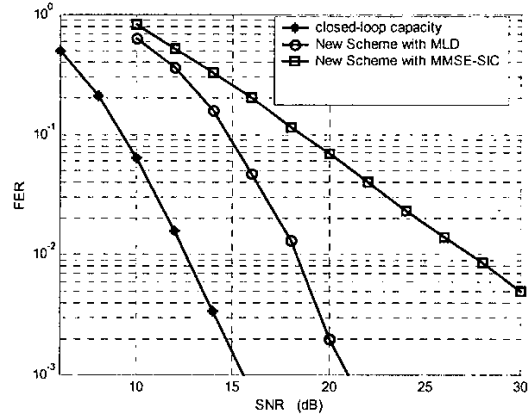


Fig. 2 FER performance of the new scheme, $m=2$, $n=2$

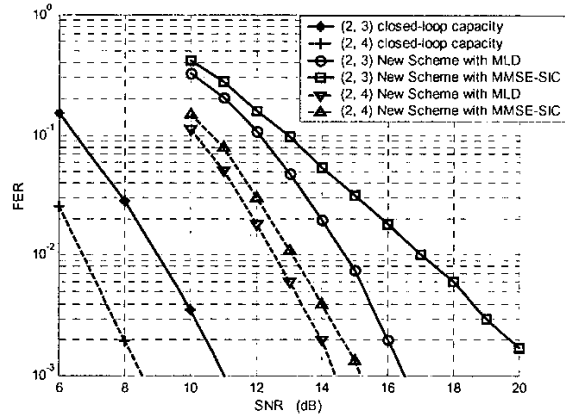


Fig. 3 FER performance of the new scheme, $m=2$, $n=3$ or 4

Fig. 2 shows the frame error rate (FER) performance of our new scheme for the case of $m=2$ and $n=2$. For comparison, we also provide its corresponding results of capacity limit, i.e. the curves of outage probability vs. SNR for a capacity of 4 bits/s/Hz. It can be observed that with MLD our new scheme performs within 5 dB of the closed-loop capacity limit. It is rather striking since this is

¹ We call it MMSE-SIC in the following text.

obtained without coding. Even better performance can be expected if powerful code is adopted in the new scheme, such as turbo code. However, with MMSE-SIC some performance degradation can be observed. At the FER of 10%, MLD outperforms MMSE-SIC by about 3.5 dB. The performance loss increases to 9 dB at 1% FER.

Fig. 3 presents the results of 3 and 4 receive antennas. Obviously with the number of receive antennas increasing, both the performance of MLD and MMSE-SIC are improved greatly. At the FER of 1%, the performance of MLD still degrades by only 5 dB with the reference to the capacity limit curves. Moreover, It can be also seen that the gap between the performance of MLD and MMSE-SIC becomes narrow with increasing n . For the case of $n = 4$, the performance loss of MMSE-SIC decreases to less than 1 dB compared with MLD.

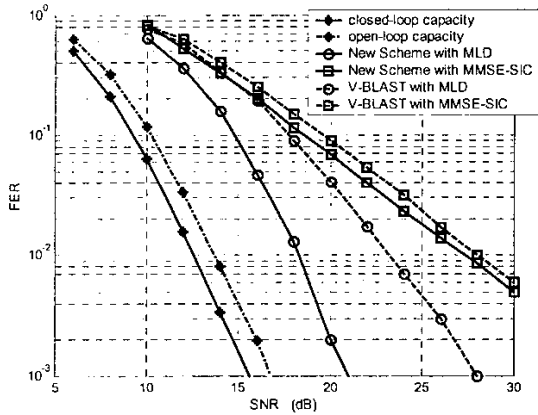


Fig. 4 Comparison of FER performance of the new scheme and V-BLAST, $m=n=2$

Thus far, these results have clearly reveals that the performance of our new scheme is quite remarkable since it is only 3.5-5 dB from the outage capacity at 10% FER even in the uncoded case. We further provide the comparison of our new scheme with V-BLAST. From Fig. 4 it can be observed that when $m = n = 2$, even using the optimal detection algorithm – MLD, V-BLAST degrades by about 7.5 dB from the open-loop capacity at 10% FER and the degradation increases to 10dB at 1% FER, which is much worse than our new scheme. As might be expected, with MLD our new scheme outperforms V-BLAST by about 3.2 dB and the performance difference increases with increasing SNR. At 0.1% FER, the performance of our new scheme is much better than that of V-BLAST by about 7 dB. Nevertheless, note that when MMSE-SIC is adopted, the performance of V-BLAST is rather close to that of our new

scheme. This is somewhat pessimistic which seems to imply that our new scheme has no significant advantage over V-BLAST when sub-optimal detection algorithm is adopted.

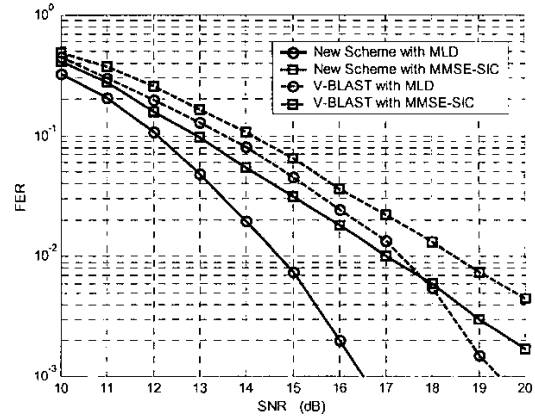


Fig. 5 Comparison of FER performance of the new scheme and V-BLAST, $m=2, n=3$

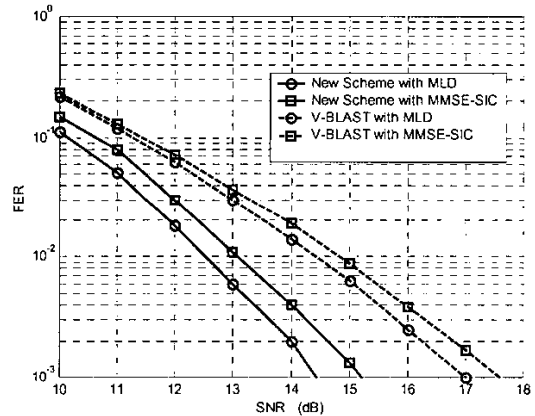


Fig. 6 Comparison of FER performance of the new scheme and V-BLAST, $m=2, n=4$

However, more gain can be achieved using MMSE-SIC in our new scheme with more receive antennas. This fact is clearly illustrated in Fig. 5 and Fig. 6. As can be seen, with increasing n , the performance gap between MLD and MMSE-SIC becomes slighter and slighter. With 4 receive antennas, the performance of our new scheme with MMSE-SIC is even better than V-BLAST with MLD. Nearly 2dB gain can be obtained at 0.1% FER. Besides, it can be also observed when MLD is adopted, with more receive antennas, less benefit can be obtained with our new

scheme. For the case of $n = 4$, at 0.1 % FER only 2.5 dB can be gained over V-BLAST, while when $n = 2$ this value is 7dB.

V CONCLUSIONS

We have presented a novel spectral efficient transmit precoder scheme, in which channel capacity is maximized. It is shown that our new scheme can not only achieve high spectral efficiency, but also provide remarkable error performance that is 3.5—5 dB from the closed-loop outage capacity at 10% frame error rate when MLD is adopted. With 2 transmit and 4 receive antennas, our new scheme can perform within 4 dB of the capacity limit even if sub-optimal detection algorithm MMSE-SIC is adopted.

We also made a performance comparison between our new scheme and V-BLAST. It is shown that our scheme always outperforms V-BLAST and the performance gain increases with increasing SNR. Moreover, with sufficient receive antennas, our scheme with MMSE-SIC is even superior to V-BLAST with MLD. It indicates that using our transmit precoder scheme, the receiver complexity can be reduced greatly, while excellent performance can still be achieved.

Our scheme is quite general and can be applied to arbitrary channel coding and R-ary PSK modulation with m transmit antennas and n receive antennas. However, with increasing m and n , the complexity of designing the optimal \mathbf{W} increases greatly. Researches on finding a simplified algorithm are being pursued.

APPENDIX

Lemma: When $\mathbf{K} = \mathbf{V}\mathbf{D}^{1/2}\mathbf{W}$, $tr(\mathbf{Q}_1) = tr(\mathbf{Q})$.

Proof: Since $\mathbf{Q}_1 = \mathbf{K}\mathbf{Q}\mathbf{K}^*$ and $\mathbf{K} = \mathbf{V}\mathbf{D}^{1/2}\mathbf{W}$, we get

$$tr(\mathbf{Q}_1) = tr(\mathbf{K}^*\mathbf{K}\mathbf{Q}) = tr(\mathbf{W}^*\mathbf{D}\mathbf{W}\mathbf{Q}) = tr\left(\mathbf{W}^*\left(\frac{P_t}{m}\mathbf{D}\right)\mathbf{W}\right).$$

Also we have $\mathbf{D} = diag(m(\mu - \lambda_i^{-1})/\rho)^+$. So we get

$$tr\left(\mathbf{W}^*\left(\frac{P_t}{m}\mathbf{D}\right)\mathbf{W}\right) = tr\left(\frac{P_t}{m}\mathbf{D}\right) = P_t = tr(\mathbf{Q}).$$

Hence $tr(\mathbf{Q}_1) = tr(\mathbf{Q})$ \square

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