

## Performance Improvement of V-BLAST through an Iterative Approach

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**Abstract**—This paper proposes an iterative V-BLAST detection algorithm that improves the error performance over error propagation. Traditional detection algorithm cannot alleviate error propagation because the decision feedbacks from low-diversity substreams are used to decode high-diversity substreams. In our algorithm, we iteratively suppress the interference towards low-diversity substreams by using decisions from high-diversity substreams, and the system performance is highly improved over the traditional one, which is demonstrated via simulation results. Besides, existing algorithms combating error propagation operate in high complexity, while the complexity of our algorithm is proportional to the loop times, providing a tradeoff between performance and complexity.

### I INTRODUCTION

Multi-input multi-output (MIMO) communication systems have already been proved in theory and practice to have enormous spectral efficiency [1] [7]. It is a possible solution to the increasing demand of high wireless communication rate and high quality wireless multimedia services. The Vertical Bell Laboratories Layered Space-Time (V-BLAST) system, which was proposed by P. W. Wolniansky et al. [2], is a scheme that attains very high spectral efficiency while maintains low implementation complexity. The enormous transmission rate provided by V-BLAST has drawn lots of attention and different studies have been conducted to achieve better performance [3][4] or low complexity [8]. V-BLAST has been considered as one of the promising techniques for next generation wireless communications.

However the error performance of the traditional V-BLAST system is not very satisfactory even in an ideal case where no error propagation is considered. Furthermore in practical V-BLAST system with successive interference cancellation (SIC) detector its performance suffers great

degradation due to the error propagation in its decision feedbacks, which has been proposed and analyzed in [3] and [4]. To solve this problem, [3] proposed a method to improve the performance through using Space-Time Block Codes at the transmitter and applying an iterative Turbo decoding at the receiver. In an uncoded case, [4] have proposed a detection algorithm combining ML and DFE decoding. However all these algorithms have a common problem of operating in a high complexity, which is inevitable for the characteristics of Turbo or ML decoding.

In the paper, we are motivated to propose an iterative approach to suppress error propagation and hence improve the performance in an uncoded case. This approach exhibits considerable performance gains over the traditional V-BLAST detection algorithm. At the same time its complexity is proportional to the loop times, which is more flexible and provides a tradeoff between complexity and performance.

The remaining parts of the paper are organized as follows. In Section II, we describe the system model. In Section III, we first describe an equivalent procedure of the traditional detection scheme in [2], and then analyze why error propagation is responsible for performance degradation from the diversity point of view. Section IV gives our new algorithm. In particular, we first give a detailed illustration to our algorithm in a (4, 4) system, and then generalize it for an (m, n) system where  $m \leq n$ . In Section V, we back up our results by providing simulation results. We also discuss the issue of algorithm complexity. Finally we conclude in Section VI.

### II SYSTEM MODEL

#### A. Notations

Vectors are denoted by bold lowercase letters, and bold uppercase letters denotes matrices. Other notations used in

the paper are as follows.

- $(\cdot)'$  transpose of  $(\cdot)$ .
- $(\cdot)^*$  transpose conjugate of  $(\cdot)$ .
- $\mathbf{I}$  the identity matrix.
- $N^k(\mathbf{a}, \mathbf{B})$   $k$ -dimensional complex Gaussian random vector with mean  $\mathbf{a}$  and covariance matrix  $\mathbf{B}$ .
- $\chi_k^2$  chi-squared distribution with  $k$  degree of freedom.

### B. System Model

The system model used in our research is similar to that in [2]. For an  $(m, n)$  system, a single data stream is separated into  $m$  parallel substreams and then transmitted simultaneously, one on its corresponding transmit antenna. At the receiver, each antenna receives signals transmitted from all the  $m$  transmit antennas, and the optimal-ordered detection for each substream with interference nulling and SIC are performed.

Specifically, the following discrete-time model is used:

$$\mathbf{r} = \mathbf{H} \cdot \mathbf{s} + \mathbf{n} \quad (1)$$

where  $\mathbf{s} = [s_0, s_1, \dots, s_{m-1}]$  is an  $m \times 1$  vector whose  $j$ -th component represents the signal transmitted from the  $j$ -th antenna. The received signal and noise vector are both  $n \times 1$  vectors, which are denoted by  $\mathbf{r}$  and  $\mathbf{n}$ , respectively. The complex channel gain between the  $i$ -th transmitter and the  $j$ -th receiver is  $h_{ij}$ , for  $i = 0, 1, \dots, m-1$  and  $j = 0, 1, \dots, n-1$ . We assume that the channel is flat quasi-static, namely, the channel is considered as constant over a frame, but varies from one frame to another. Each element of the channel gain matrix  $\mathbf{H}$  is modeled as an independent complex Gaussian random variable with zero mean and unit variance per dimension. The noise is assumed to be complex Gaussian distributed with zero mean and variance  $\sigma_n^2$  per dimension.

## III. PROBLEM ANALYSIS

### A. The Original Detection of V-BLAST

The detection algorithm in the original works of V-BLAST [2] consists of interference nulling, SIC and optimal ordering. Here we give a matrix-decomposition illustration to this detection procedure that will be proved equivalent to the original one. The equivalent illustration makes it more explicit for us to understand the variation of diversity degrees

in V-BLAST detection. This will be analyzed in details in Section III. We consider here the case of  $m = n$ , but all the analysis and results can be easily extended to the case when  $m \leq n$ .

First we perform the optimal ordering, which is totally determined by  $\mathbf{H}$ . After this we take for granted that  $m, m-1, \dots, 1$  is the optimal detection order, i.e., we decode the  $m$ -th substream first and the first substream last. We also update  $\mathbf{s}$ ,  $\mathbf{H}$ ,  $\mathbf{r}$  and  $\mathbf{n}$  to fulfill this assumption, namely, we need to exchange elements in these matrices according to the optimal ordering.

Write  $\mathbf{H} = (h_1, \dots, h_n)$  where  $h_j$  denotes the  $j$ -th column of matrix  $\mathbf{H}$ . The QR decomposition [5] of  $\mathbf{H}$ , which is implemented by Gram-Schmidt orthogonalization of  $(h_1, \dots, h_n)$ , is

$$\mathbf{H} = \mathbf{Q}\mathbf{R} = (q_1, \dots, q_n) \begin{pmatrix} r_{1,1} & \dots & \dots & \dots \\ 0 & \ddots & & \\ \vdots & 0 & r_{n-1,n-1} & r_{n-1,n} \\ 0 & \dots & 0 & r_{n,n} \end{pmatrix} \quad (2)$$

As stated in [6], entries in upper triangular matrix  $\mathbf{R}$  have the following distributions.

$$|r_{i,i}|^2 \sim \chi_{2(n+1-i)}^2, 1 \leq i \leq n \quad (3)$$

$$|r_{i,j}|^2 \sim \chi_2^2, 1 \leq i < j \leq n \quad (4)$$

which indicates that the diversity degree of the  $j$ -th detected stream is  $j$ .

We multiply  $\mathbf{r}$  by  $\mathbf{Q}^*$ , and then get

$$\mathbf{y} = \mathbf{Q}^* \mathbf{r} = \mathbf{R} \mathbf{s} + \mathbf{w} \quad (5)$$

where  $\mathbf{w} = \mathbf{Q}^* \mathbf{n} \sim N^n(0, \mathbf{I})$  is still an i.i.d Gaussian random vector with the same mean and variance as  $\mathbf{n}$ . Because  $\mathbf{R}$  is an upper triangular matrix, signals with larger indices will be subtracted to decode signals with small indices, which is just what the traditional detection algorithm performs. The unitary transformation of the received signal actually performs all the interference nulling process in one step, and the interference cancellation operation is performed on decoding feedbacks. Obviously it is an equivalent procedure with the traditional one.

### B. Error Propagation Analysis

In practical V-BLAST systems, error propagation is

inevitable and it is responsible for performance degradation [3][4]. During the detection procedure, later-detected substreams are decoded through decision feedbacks from prior-detected ones. It is interference increasing from prior substreams to the lately detected substreams that limits the overall performance.

From our equivalent illustration of the traditional detection we can find that under the assumption of no error propagation, the diversity degree of the next detected substream is supposed to increase by one after each cancellation. This means if no error propagation is considered, the next detected substream should get great benefit from the diversity increase, and hence its performance should be improved significantly, which is obviously demonstrated in Fig. 1. However in practice error propagation significantly counteracts the benefit offered by diversity increase, which is also demonstrated in Fig. 2 compared with Fig. 1.

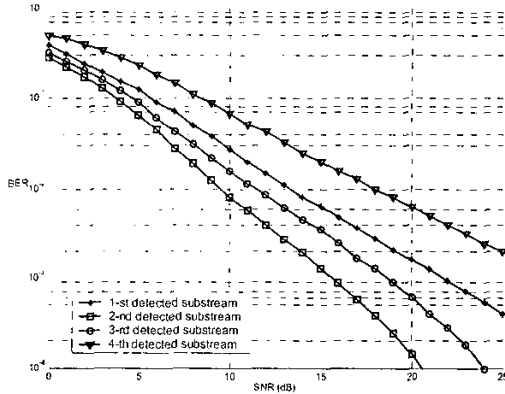


Fig. 1 BER performance of each substream without error propagation, (4, 4) system

In Fig. 1 and Fig. 2 we plot both the ideal and the practical BER performance for all substreams in a (4, 4) system. Comparing ideal performance with actual simulation result, we can find that imperfect decision feedbacks severely compromise the performance of substreams with large diversity degree. Moreover, as evident from the figure, the overall system performance is limited by the worst substream, which is the first detected one. Therefore, the worst substream is the bottleneck, and the significant gap between ideal and actual performance of substreams with high diversity degree promises significant room for improvement. It is utilized in the following new algorithm.

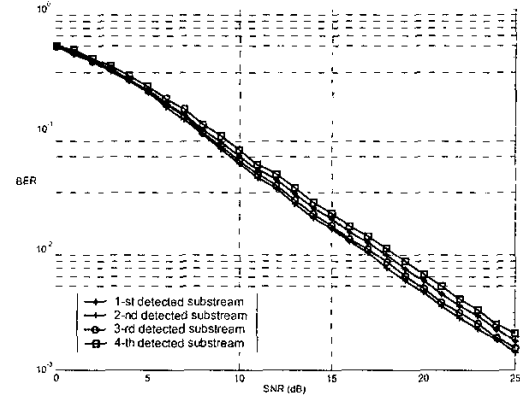


Fig. 2 BER performance of each substream with error propagation, (4, 4) system

#### IV ITERATIVE DETECTION ALGORITHM

Since the first detection step is crucial for the overall performance, the diversity degree of this substream should be increased. Adding extra receive antennas will increase the diversity degree of the worst substream. However, in practice the number of receive antennas is limited by the size and feasible complexity of the mobile unit. Therefore it is not a practical solution. We present an iterative detection algorithm to effectively improve the diversity degree of the substreams detected antecedently, and hence improve the performance without adding more receive antennas. This algorithm suppresses interference towards low-diversity substreams using decisions from high-diversity substreams. In this section we give a detailed analysis to it.

We use a (4, 4) system as an example to explain our algorithm. There are 4 substreams, denoted by  $c_1, c_2, c_3, c_4$ . The traditional V-BLAST decoding order [2] is  $c_4 \rightarrow c_3 \rightarrow c_2 \rightarrow c_1$  (taking for granted that it is the optimal detection order), respectively. We iteratively perform the traditional algorithm to achieve high diversity degree for each substream. First, we perform the detection algorithm to get initial decisions  $\hat{s}_4^0, \hat{s}_3^0, \hat{s}_2^0$  and  $\hat{s}_1^0$  corresponding to substreams  $c_4, c_3, c_2, c_1$ , respectively. Then we begin the iteration. In the first loop, we subtract  $\hat{s}_1^0$ , which has the highest diversity degree 4, from the total receive signal, and then we decode the remaining (3, 4) system using traditional algorithm. This completes the first loop, and we get the first updated decisions  $\hat{s}_4^1, \hat{s}_3^1, \hat{s}_2^1$  and  $\hat{s}_1^0$ . In the second loop, we subtract both  $\hat{s}_1^0$  and  $\hat{s}_2^1$  from the total receive signal,

then get the second decisions  $\hat{s}_4^2$  and  $\hat{s}_3^2$ . In the last loop, we subtract  $\hat{s}_1^0$ ,  $\hat{s}_2^1$  and  $\hat{s}_3^2$  from the total receive signal and get the third decision  $\hat{s}_4^3$ . Finally, decisions  $\hat{s}_4^3$ ,  $\hat{s}_3^2$ ,  $\hat{s}_2^1$  and  $\hat{s}_1^0$  are decoding results.

It may be helpful to understand the performance improvement by analyzing the diversity degree change of each substream in each loop of our algorithm, which is shown in Table. 1.

Detection State	Detection Order in each State	Diversity degree of $c_4, c_3, c_2, c_1$ respectively
Initialization	$\hat{s}_4^0 \rightarrow \hat{s}_3^0 \rightarrow \hat{s}_2^0 \rightarrow \hat{s}_1^0$	1, 2, 3, 4
Loop 1	$\hat{s}_1^0 \rightarrow \hat{s}_4^1 \rightarrow \hat{s}_3^1 \rightarrow \hat{s}_2^1$	2, 3, 4, 4
Loop 2	$\hat{s}_1^0, \hat{s}_2^1 \rightarrow \hat{s}_4^2 \rightarrow \hat{s}_3^2$	3, 4, 4, 4
Loop 3	$\hat{s}_1^0, \hat{s}_2^1, \hat{s}_3^2 \rightarrow \hat{s}_4^3$	4, 4, 4, 4

Table 1 Diversity degree change in each loop

In general, we give the following algorithm for an  $(m, n)$  MIMO system where  $m \leq n$ .

*Initialization:*

- (1) Perform the optimal detection order operation put forward in [2]. After this we assume that the order  $m, m-1, \dots, 1$  is the optimal order.
- (2) Decode each substream of the  $(m, n)$  system by using traditional algorithm.

*Recursion:*

FOR  $i=1$  to  $m-1$  do:

- (3) Subtract signals from substreams 1 to  $i$  ( $\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_i$ ) as

$$\tilde{r} = r - \sum_{j=1}^i h_j \tilde{s}_j.$$

- (4) Apply the traditional algorithm to the remaining  $(m-i, n)$  system to generate ( $\tilde{s}_{i+1}, \tilde{s}_{i+2}, \dots, \tilde{s}_m$ ).

## V NUMERICAL RESULTS AND DISCUSSIONS

In this section we demonstrate the performance improvement of our new algorithm via simulations. The results of the  $(4, 4)$  system are shown in Fig. 3. At the transmitter bits are modulated with 16QAM at each transmit antenna. The performance of the actual BER performance with error propagation is also plotted for comparison. As evident from the figure, a large performance gain is achieved by using our iterative algorithm. For example, at the BER of 0.1% our new algorithm with three loop times outperforms the traditional algorithm by about 3.7 dB.

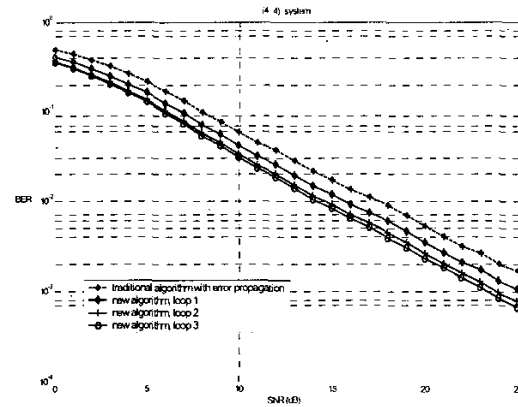


Fig. 3 BER performance of the new algorithm in each loop,  $(4, 4)$  system

It is also worth noting that the better performance achieved in our algorithm is at the expense of a reasonable complexity, which makes our algorithm more practical for implementation. The complexity of our algorithm equals to that of the traditional algorithm multiplying the loop times. From our simulation results it is also clear that the performance gain between two conjoint loops degrades as the loop times increase. It is also clear from the simulation results that if we only loop two times, the performance gain offered by iteration has been mostly achieved. More than two loop times does little to the performance improvement, which makes it possible to reduce the complexity by decreasing the loop times, with relatively very little performance loss. This indicates that our algorithm is feasible to make a tradeoff between performance and complexity.

Fig. 4 and Fig. 5 give the performance analysis of the  $(4, 5)$  and  $(4, 6)$  system, respectively. As aforementioned, as the number of receive antennas increase, the overall system

performance will be improved over error propagation, which is obvious in both figures compared with Fig. 3. From the two figures we also observe that as the number of receive antennas increases, the performance gain offered by our new algorithm decreases. For example, at the BER of 0.1% our new algorithm with three loop times outperforms the traditional algorithm by about 1.9 dB in the (4, 5) system, while in the (4, 6) system it decreases to around 0.7 dB, compared with that of about 3.7 dB in the (4, 4) system as aforementioned. The reason for this behavior is that the additional receive antennas have already provided major degrees of diversity, so that the additional diversity of our new algorithm becomes less important.

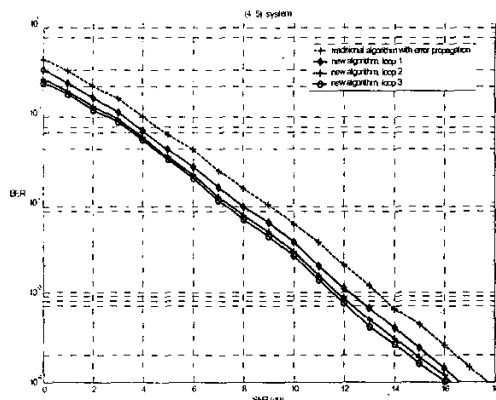


Fig. 4 BER performance of the new algorithm in each loop, (4, 5) system

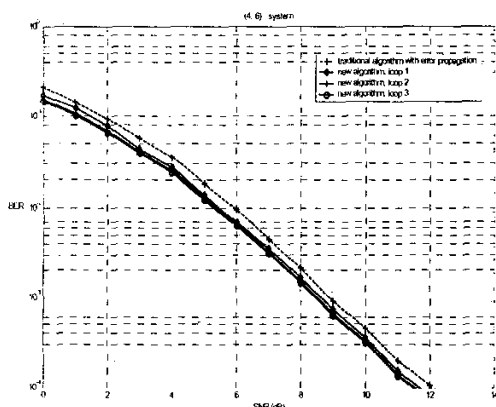


Fig. 5 BER performance of the new algorithm in each loop, (4, 6) system

## VI CONCLUSIONS

In the paper we have proposed an iterative V-BLAST detection algorithm that can efficiently alleviate error propagation and hence improve the performance. We first gave an equivalent explanation to the traditional algorithm, and then illustrated the occurrence of error propagation. It is just because relatively unreliable decision feedbacks from substreams with lower diversity degrees are used to decode substreams with higher diversity degrees that the significant room for performance improvement offered by diversity increase cannot be effectively utilized. Then we proposed a new algorithm to iteratively suppress the interference towards low-diversity substreams by using decisions from high-diversity substreams. Simulation results have demonstrated that the system performance is highly improved over the traditional one. We also considered reducing the complexity of our algorithm by decreasing the loop times, which makes a tradeoff between performance and complexity.

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