Optimized Parameter Design of Linear Dispersion Codes in MIMO Channels

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***Abstract* - Among various techniques concerning MIMO system capabilities, Linear Dispersion Codes (LDC) conquer many deficiencies of earlier proposed schemes and achieve both the coding advantage and decoding simplicity. However, since no specific criterion is proposed to determine the coding length of LDC, the optimization of the symbol rate, which will essentially influence the system efficiency, is impossible. This paper studies the effect of these two key parameters on the LDC systems and puts forward a practical parameter decision and optimization criterion to lessen the coding complexity while retaining the codes' near-optimum performance simultaneously. Simulation results yield the trivial capacity loss and excellent BER over a wide range of SNR situations.**

I. INTRODUCTION

Due to the rapidly increasing demand for reliable high rate wireless services in recent years, various techniques for exploiting the capabilities of wireless channels have been proposed to counteract the harmful effect of fading and enhance the bandwidth efficiency. Among the multifold schemes, Multi Input Multi Output (MIMO) technique has exhibited its prominence in high transmission rate and extraordinary capacity compared with traditional single antenna systems, and the research on it has become a hot topic in the wireless communication field.

In recent years, some feasible techniques have been proposed to achieve the enormous capacity of MIMO channels [1], among which the most famous is the Bell-Lab Layered Space-Time (BLAST) architecture [2] that aims mainly at sufficient bandwidth efficiency and thereby results in relatively poor performance. Contrarily, other prevalent techniques such as the Space-Time Block Code (STBC) [3] and Space-Time Trellis Code (STTC) [4] that pursue both the coding gain and diversity gain inevitably suffer from transmission efficiency loss. Considering the deficiencies of existing schemes, B. Hassibi etc. constructed a novel technique named Linear Dispersion Codes (LDC) [5] to provide optimal mutual information between the transmit and receive signal as well as excellent diversity advantage compared with earlier proposed methods, and this novel technique can subsume both V-BLAST [2] and STBC as its special cases and generally outperform them while detected by simple algorithm such as serial interference cancellation (SIC) [2] or sphere decoding [6]. Therefore, it is a prospective technique to be utilized in realistic MIMO systems.

In the LDC scheme, the choice of two parameters T (the length of coding) and Q (the number of symbols transmitted per interval) crucially determines the code complexity and efficiency. Although [5] has recommended the value of T to satisfy $M \le T \le 2M$ in a LDC system with M transmit antennas and N receive antennas, and it conceived that when T is fully large the LDC structure can realize the ideal channel capacity, [5] did not carry through a detailed investigation into the effect of T on the coding scheme, which could restrict the value of Q and ultimately influence the unitary system quality.

In this paper, we focus our research on the effect of the different coding length T and corresponding Q on the LDC system, and eventually put forward a practical parameter decision and optimization criterion. Through our analytical derivation, we prove that LDC cannot achieve the ideal channel capacity when N>1 with full symbol rate. At the same time, our simulation demonstrates the trivial capacity penalty when using smaller but rational T value instead of fully large values. Hence in order to decrease the coding complexity for application and still implement the simple linear detection algorithms, we propound to adopt minimum integral T that satisfies $NT \ge M$, which is the minimal possible value of T, and still transmit the signal at full symbol rate. Consequently the complexity of LDC coding system can be lessened at the magnitude of T^2 and the simulation results show its satisfying performance over a wide range of SNR situations.

The rest of this paper is organized as follows. In section II, we will introduce the LDC structure and its channel capacity briefly. The details of our analysis on optimized parameter design are described in section III. Then, section IV will present our numerical simulation results. Finally, section V summarizes our conclusion remarks.

II. LDC STRUCTURE AND ITS CAPACITY

In this paper, we focus on a single-user, point to point LDC system with M transmit antennas and N receive antennas. We assume that the channel is frequency non-selective and remains constant in T transmit intervals. Denoting the transmit signals in T intervals by matrix $\mathbf{S}$ with dimension of $T \times M$ and the corresponding receive signals by matrix $\mathbf{Y}$ with

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dimension of $T \times N$, we can express the input-output relation of a LDC system with total transmit power P_t as

$$\mathbf{Y} = \sqrt{\frac{P_t}{M}}\mathbf{SH} + \mathbf{V} \quad (1)$$

where matrix $\mathbf{H} \in C^{M \times N}$ represents the status of the MIMO channel and its entries satisfy the $CN(0,1)$ (zero-mean, unit variance, complex-Gaussian) distribution, and the additive noise matrix $\mathbf{V}$ also has independent zero-mean complex-Gaussian distributed entries that are spatially and temporally white.

Assuming that Q constellation symbols $s_1, \ldots, s_Q$ are transmitted in T intervals, then the transmit matrix $\mathbf{S}$ of a LDC system can be expressed as

$$\mathbf{S} = \sum_{q=1}^{Q}(\alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q) \quad (2)$$

where α_q and β_q respectively represent the real part and imaginary part of s_q. The dispersion matrix $\{\mathbf{A}_q, \mathbf{B}_q\}$ with dimension of $T \times M$ decide the distribution of the Q symbols in the transmit matrix, and the pivotal idea of LDC is to design the optimal dispersion matrix to provide the maximum mutual information between the transmit and receive signal, which equally means to maximize the channel capacity of the system.

It has been demonstrated in [5] that the channel capacity of a LDC system is

$$C(\rho, T, M, N) = \max_{A_q, B_q, q=1,\ldots,Q} \frac{1}{2T} E \log\det(\mathbf{I}_{2NT} + \frac{\rho}{M}\mathcal{H}\mathcal{H}^*) \quad (3)$$

where ρ is the transmit signal-to-noise ratio of the system, * means conjugate transpose, and $\mathcal{H} \in R^{2NT \times 2Q}$ is the equivalent channel matrix which can be expressed as

$$\mathcal{H} = \begin{pmatrix} A_1 h_1 & B_1 h_1 & \ldots & A_Q h_1 & B_Q h_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_1 h_N & B_1 h_N & \ldots & A_Q h_N & B_Q h_N \end{pmatrix} \quad (4)$$

in which

$$A_q = \begin{pmatrix} \mathbf{A}_{R,q} & -\mathbf{A}_{I,q} \\ \mathbf{A}_{I,q} & \mathbf{A}_{R,q} \end{pmatrix}, B = \begin{pmatrix} -\mathbf{B}_{I,q} & -\mathbf{B}_{R,q} \\ \mathbf{B}_{R,q} & -\mathbf{B}_{I,q} \end{pmatrix}, h_n = \begin{pmatrix} h_{R,n} \\ h_{I,n} \end{pmatrix} (5)$$

In (5), $\mathbf{A}_{R,q}, \mathbf{A}_{I,q}, \mathbf{B}_{R,q}, \mathbf{B}_{I,q}$ respectively represent the real and imaginary part of dispersion matrix $\{\mathbf{A}_q, \mathbf{B}_q\}$, and $h_{R,n}, h_{I,n}$ respectively represent the real and imaginary part of the n th column vector in the channel matrix $\mathbf{H}$. Based on (3), (4) and (5), some local optimal codes have been calculated in [5] for fixed T and Q, and [5] conjectured that when T is fully large the LDC system may realize the ideal channel capacity in MIMO environment. However, our derivation in the following section will demonstrate the contrary conclusion and as a result motivate the optimized parameter design for more practical coding in realistic application.

III. Optimized Parameter Design of LDC

In order to search out a practical parameter design strategy to reduce the coding complexity and retain the near-optimum performance simultaneously, we investigate the ideal channel capacity of the LDC system and its relation to the coding parameters. According to this research result, we then propose our parameter scheme for simple system realization.

A. Ideal Channel Capacity of LDC System

Since the channel capacity of a LDC system is defined as (3) in which $\mathcal{H}$ has the expression shown in (4), we firstly denote that

$$a_i = \begin{pmatrix} A_i h_1 \\ \vdots \\ A_i h_N \end{pmatrix}, b_i = \begin{pmatrix} B_i h_1 \\ \vdots \\ B_i h_N \end{pmatrix}, i = 1, \ldots, Q \quad (6)$$

then $\mathcal{H}$ can be written as

$$\mathcal{H} = (a_1 \quad b_1 \quad \ldots \quad a_Q \quad b_Q) \quad (7)$$

Thus, we will get

$$\mathcal{H}^*\mathcal{H} = \begin{pmatrix} a_1^* a_1 & a_1^* b_1 & \ldots & a_1^* a_Q & a_1^* b_Q \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b_Q^* a_1 & b_Q^* b_1 & \cdots & b_Q^* a_Q & b_Q^* b_Q \end{pmatrix} \quad (8)$$

whose diagonal entries are $a_i^* a_i$ and $b_i^* b_i$, $1 \le i \le Q$. Considering that it has been pointed out in [5] that the optimal LDC codes should obey the strict power constraint

$$\mathbf{A}_q^* \mathbf{A}_q = \mathbf{B}_q^* \mathbf{B}_q = \frac{T}{Q}\mathbf{I}_M, q = 1, \ldots, Q \quad (9)$$

provided substituting (6) and (9) into (8) and through some simple calculation, we can observe that all the diagonal entries in (8) are constant for each particular channel matrix $\mathbf{H}$, that is

$$a_i^* a_i = b_i^* b_i = \frac{T}{Q}\sum_{n=1}^{N}\sum_{m=1}^{M}(|h_{R,mn}|^2 + |h_{I,mn}|^2) = con \quad (10)$$

where $h_{R,mn}$ and $h_{I,mn}$ respectively denote the real and imaginary part of $\mathbf{H}$'s entry at its m th row and n th column. Because the ideal channel capacity of a LDC system should be the possible maximal value of (3), then each $\det(\mathbf{I}_{2NT}+\rho/M\bullet\mathcal{H}\mathcal{H}^*)$ should reach its maximum to realize this capacity. Due to that all the diagonal entries of matrix $\mathbf{I}_{2NT}+\rho/M\bullet\mathcal{H}\mathcal{H}^*$ equal to a constant $1+con$, it is obvious that only when all the non-diagonal entries of matrix $\mathbf{I}_{2NT}+\rho/M\bullet\mathcal{H}\mathcal{H}^*$ are zero can the LDC system reach its ideal capacity.

Unfortunately, this condition cannot be satisfied while N>1 at full transmit symbol rate (Q=N • T). For example, in order to realize $a_i^* a_j = 0, \forall i \neq j, 1 \leq i,j \leq Q$, we can prove that the dispersion matrixes must obey the following constraint condition: $\forall 1 \leq i,j \leq Q$ and $i \neq j$, all the diagonal entries of matrix $A_i^* A_j$ are zero, which means that at least Q vectors with $T\times 1$ dimension must be orthogonal. Therefore, T should be no smaller than Q, that is $N \leq 1$. On account of N is larger than 1 in most MIMO systems, we conclude that the LDC scheme can hardly realize the ideal channel capacity in realistic application no matter how large coding length T is adopted.

B. Optimized Parameter Scheme of LDC

Owing to the tremendous $2NT\times 2Q$ size of the equivalent channel matrix $\mathcal{H}$ in LDC systems, we deem that the complexity of LDC which mostly depends on the complexity of the dispersion matrixes is the magnitude of T^2 at full symbol rate. Since no T value can realize the ideal channel capacity when N>1, for the sake of the least realization complexity, we propose to adopt the minimal T value in practical LDC systems. Limited by the requirement of no fewer receive antennas than transmit antennas for the implementation of linear detection method such as SIC, the coding length T must satisfy $NT \geq M$ thanks to the equivalent NT dimension of receive antennas in LDC systems. Thus, the coding length is designed as the minimum integral T that satisfies $NT \geq M$ in our optimized parameter scheme, and as a result the system complexity is reduced dramatically.

In addition, we still transmit signal with full symbol rate in our scheme for the sake of sufficient efficiency, which means that Q is equal to NT at our code design. Our simulation results have revealed the trivial capacity loss and near-optimal performance of the codes designed by this criterion, which proved the validity of our parameter criterion. The details of these numerical results will be illustrated in section IV.

IV. Simulation Results

In this section, we present some examples of the performance comparison between our codes and the codes designed in [5]. According to our parameter design criterion, we find the optimal LDC code by constraint gradient search for M=3, N=1 system with T=3, Q=3, which is expressed by (11) at the bottom of this page, and calculate out that the capacity loss is only 0.3bit/channel relative to the ideal channel capacity for such LDC system. Fig 1 shows its BER performance under various SNR values. For comparison, the performance curves of T=4, Q=4 LDC system [5] and T=4, Q=3 orthogonal design system with the same antenna number are also shown in this figure. It should be noticed that though the orthogonal design with insufficient symbol rate can provide better diversity gain, it needs to implement more sophisticated modulation compared with our scheme so as to obtain the same transmission rate. Therefore, in Fig 1 the two LDC systems are modulated by 64QAM while the orthogonal design system is modulated by 256QAM, and all of them are detected by SIC method. Through this figure we can observe that our parameter design incurs only about 1 dB SNR penalty and still excels the performance of orthogonal design system over varied SNR situations.

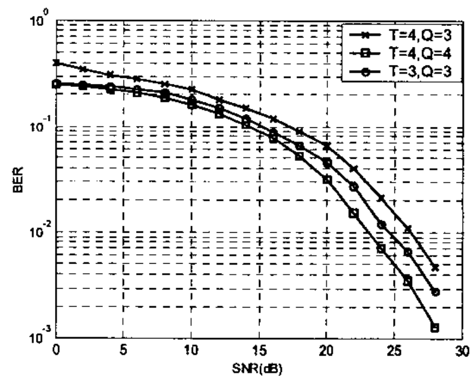

Fig 1 BER performance of M=3,N=1 LDC systems

$$\mathbf{S}=\begin{bmatrix} -(\frac{\alpha_1}{2}+\frac{\alpha_2+\alpha_3}{\sqrt{2}})+j(\frac{\beta_1}{\sqrt{2}}-\frac{\beta_3}{2}) & \frac{\alpha_1}{2}+\frac{\alpha_3}{\sqrt{2}}+j(\frac{\beta_2}{\sqrt{2}}-\frac{\beta_3}{2}) & \frac{-\alpha_1+\alpha_2}{\sqrt{2}}-j(\frac{\beta_1+\beta_2+\beta_3}{\sqrt{2}}) \\ -\frac{\alpha_1}{\sqrt{2}}+j(\beta_2-\frac{\beta_3}{\sqrt{2}}) & -(\frac{\alpha_1}{\sqrt{2}}+\alpha_2)-j(\beta_1-\frac{\beta_3}{\sqrt{2}}) & \alpha_3 \\ -(\frac{\alpha_1}{2}+\frac{\alpha_2-\alpha_3}{\sqrt{2}})-j(\frac{\beta_1}{\sqrt{2}}+\frac{\beta_3}{2}) & \frac{\alpha_1}{2}+\frac{\alpha_3}{\sqrt{2}}+j(\frac{\beta_2}{\sqrt{2}}-\frac{\beta_3}{2}) & \frac{\alpha_1-\alpha_2}{\sqrt{2}}-j(\frac{\beta_1-\beta_2-\beta_3}{\sqrt{2}}) \end{bmatrix} \quad (11)$$

Similarly, Fig 2 shows the performance comparison between our T=2,Q=4 LDC and the T=4,Q=8 LDC presented in [5] with M=3,N=2 antennas, which are still modulated by 64QAM and detected by SIC. It can be noticed that these two codes have nearly identical performance over different SNR, but our codes yield less complexity with much smaller dispersion matrixes, which are only 8 matrixes with dimension of 2×3. Comparatively, the code when T=4 contains 16 dispersion matrixes with dimension of 4×3, and its equivalent channel matrix is much larger than our system.

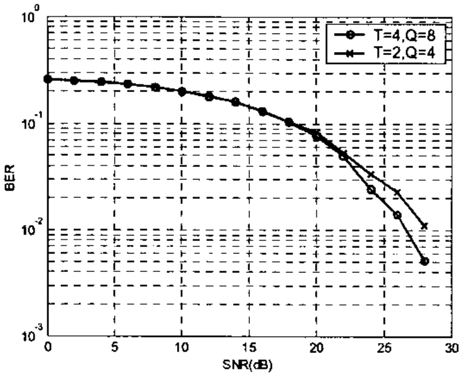

Fig 2 BER performance of M=3,N=2 LDC systems

From these detailed examples we notice that the codes deigned by our optimized parameter scheme can provide similar performance to existent optimal codes but with less realization complexity, both in N=1 system and the system with multi-receive antennas. Thus, it is a valuable criterion to be implemented in realistic systems.

V. CONCLUSION

In this paper we propose an optimized parameter scheme to design the coding length T and symbol rate Q of Linear Dispersion Code in MIMO environment. Since our analysis has proved that no codes with any T value can realize the ideal channel capacity of LDC systems with more than one receive antennas, we adopt the minimum integral T that satisfies $NT \geq M$ in our coding scheme on account of the need of less complexity in realistic application. In addition, in order to make full use of the bandwidth resource, we still transmit the signal at its possible full symbol rate that is Q=NT, and as a result our system can obtain the maximal efficiency with the lowest level modulation method which also reduces the detection complexity.

Our simulation reveals that the codes design by this scheme incur only trivial capacity loss and performance penalty, and they still outperform the classical orthogonal codes in a wide range of SNR situations. At the same time, the complexity of the dispersion matrixes and equivalent channel is lessened at the magnitude of T^2, which is most beneficial for practical realization. However, the code calculation in our scheme is still numerical gradient search, and the more effective analytical method is under research.

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