

Reverse-link Capacity with MRC-based Macrodiversity in DS-CDMA Cellular systems

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Abstract—In this paper, the effect of maximal ratio combining (MRC)-based macrodiversity on CDMA reverse-link capacity is studied. We present an analytical model and derive the exact outage probability expression. Both fast fading and log-normal shadowing fading are considered. The numerical results show that with MRC-based macrodiversity the reverse-link capacity increases rapidly with the number of involved base stations. Besides, we make a comparison between the capacity with microdiversity and that with MRC-based macrodiversity and find that with the same amount of antennas, more reverse-link capacity can be obtained with MRC-based macrodiversity.

I INTRODUCTION

Macrodiversity is an effective means to counteract the harmful effect of large-scale fading. In CDMA systems, combining signals from widely separated base station (BS) antennas allows exploitation of macrodiversity gains. Numerous papers studied the reverse-link capacity improvement with macrodiversity in CDMA systems. However, most of them focused on switched-selection macrodiversity [2,3]. As we know, maximal ratio combining (MRC) maximizes the output signal-to-noise ratio (SNR) and is optimum in a noise-limited environment [4,5]. Obviously MRC-based macrodiversity outperforms switched-selection macrodiversity [7]. Thus it's necessary to investigate its effect on the reverse-link in CDMA systems.

MRC-based macrodiversity has been proven to be an effective way of improving the reverse-link capacity [6,7]. In [6], S.V.Hanly proved that using MRC-based macrodiversity the capacity can be unaffected by outside interference with certain power control scheme. In [7], the authors constructed a simple proof showing that the reverse-link capacity is close to an isolated-cell capacity and simulated the capacity with MRC-based macrodiversity and switched-selection macrodiversity. However, in [6] Hanly only derived the upper bound of the reverse-link capacity using a mathematical method and the analysis in [7] was based on a simulation model with 3 BS's instead of an

analytical model.

In this paper, we present an analytical CDMA reverse link model with MRC-based macrodiversity and derive the outage probability expression which depends on many factors, such as the number of involved BS's and fading parameters. Fast fading, slow fading and path loss are considered, while the effect of multipath is not included. Our numerical results show that with MRC-based macrodiversity, the received signal-to-interference ratio (SIR) can be improved greatly and thus substantial capacity benefits can be brought.

Microdiversity is another type of diversity that can be obtained usually by placing an antenna array at each BS [8]. It has been proved that with microdiversity the CDMA reverse-link capacity can also be increased greatly [8]-[11]. Thus we further made a comparison between the effect of MRC-based macrodiversity and that of microdiversity. We found that with the same amount of antennas, more reverse-link capacity can be obtained with MRC-based macrodiversity.

In next section, a system model is established and the outage probability expression is derived. Numerical results are presented in Section III. Finally, some concluding remarks are given in Section IV.

II SYSTEM MODEL

Consider a CDMA reverse-link system with L cells. We assume that totally K mobiles are uniformly distributed in the system and every mobile communicates with adjacent H BS's wherever it is (We call these H BS's "Active Set"). The conventional matched filter receiver is adopted at each BS and the signals received by an Active Set are combined with an MRC combiner. The effect of thermal noise is ignored in the interference-limited environment and the channel is assumed to be frequency non-selective. We assume that with perfect instantaneous power control the output desired signal power of the MRC combiner of each

mobile's Active Set is equal to P^* .

Then the received signal of the Active Set of mobile 0¹ is

$$\mathbf{X}(t) = \sum_{i=0}^{K-1} \psi_i \sqrt{P_i} \mathbf{S}_i(t) \gamma_i + \mathbf{n}(t),$$

where P_i is the transmit power of mobile i . ψ_i is a Bernoulli variable with probability success λ that models the voice activity of mobile i . $\mathbf{S}_i(t)$ is an $H \times H$ diagonal matrix with the diagonal element

$$s_{i,j}(t) = b_i \left(\left\lfloor \frac{t - \tau_{i,j}}{T} \right\rfloor \right) c_i(t - \tau_{i,j}), j = 0, 1, \dots, H-1,$$

where $b_i(\cdot)$ is the transmitted bit of mobile i in duration T and $c_i(\cdot)$ is the spreading code used by mobile i . $\tau_{i,j}$ is the propagation delay from mobile i to the j th BS of Active Set 0. γ_i is an $H \times 1$ vector which represents the channel gain between mobile i and each BS of Active Set 0. $\gamma_i = (\gamma_{i,0}, \gamma_{i,1}, \dots, \gamma_{i,H-1})^T$, where $\gamma_{i,j} = \sqrt{r_{i,j}^{-\alpha}} \cdot \beta_{i,j} \cdot r_{i,j}$ is the distance from mobile i to the j th BS of Active Set 0 and α is the path-loss exponent. $\beta_{i,j}$ is a complex random variable that represents the corresponding amplitude fade along the path and combines both the fast fading and log-normal shadowing effects, that is, $|\beta_{i,j}|$ has a Rayleigh or Rice distribution and its mean square value $E\{|\beta_{i,j}|^2\}$ is a log-normal random variable with zero mean and variance σ_s^2 [9]. $\mathbf{n}(t)$ is the thermal noise vector.

The output vector of the matched filters is $\mathbf{Y}(t) = (Y_0(t), Y_1(t), \dots, Y_{H-1}(t))^T$, where

$$Y_j(t) = \int_{t_{1j}}^{t_{2j}} c_0(t - \tau_{0,j}) X_j(t) dt = E_j(t) + \sum_{i=0}^{K-1} \psi_i I_{ij}(t) + n_{\eta_j}(t),$$

$j = 0, 1, \dots, H-1$, $t_{1j} = (l-1)T - \tau_{0,j}$, $t_{2j} = lT - \tau_{0,j}$, and

$$E_j(t) = \int_{t_{1j}}^{t_{2j}} \sqrt{P_0} b_0 \left(\left\lfloor \frac{t - \tau_{0,j}}{T} \right\rfloor \right) c_0(t - \tau_{0,j}) c_0(t - \tau_{0,j}) \gamma_{0,j} dt,$$

$$I_{ij}(t) = \int_{t_{1j}}^{t_{2j}} \sqrt{P_i} b_i \left(\left\lfloor \frac{t - \tau_{i,j}}{T} \right\rfloor \right) c_i(t - \tau_{i,j}) c_0(t - \tau_{0,j}) \gamma_{i,j} dt.$$

All the output branches are combined using MRC. Then we have

$$\begin{aligned} z(t) &= \gamma_0^H \mathbf{Y}(t) = N \sqrt{P_0} b_0(t) \gamma_0^H \gamma_0 + \sum_{i=1}^{K-1} \psi_i \gamma_0^H \mathbf{I}_i(t) + \gamma_0^H \mathbf{n}_T(t) \\ &= z_0(t) + z_1(t) + z_2(t) \end{aligned}$$

where N is the spreading factor (processing gain), and

$$\mathbf{I}_i(t) = (I_{i0}(t), I_{i1}(t), \dots, I_{iH-1}(t))^T.$$

Similar to [9], it can be derived that

$$\text{Var}(z_0(t)) = N^2 P_0 \|\gamma_0^H \gamma_0\|^2, \quad \text{Var}(z_i(t)) = N \sum_{i=1}^{K-1} \psi_i P_i \|\gamma_0^H \gamma_i\|^2.$$

Then ignoring the thermal noise, the bit energy-to-interference density ratio can be written as

$$\frac{E_b}{N_0 + I_0} \approx \frac{E_b}{I_0} = \frac{N^2 P_0 \|\gamma_0^H \gamma_0\|^2}{N \sum_{i=1}^{K-1} \psi_i P_i \|\gamma_0^H \gamma_i\|^2} \quad (1)$$

According to the power control scheme, we have

$$\frac{P_0}{P_i} = \frac{P / \|\gamma_0\|^2}{P / \|\gamma_i\|^2} = \frac{\|\gamma_i\|^2}{\|\gamma_0\|^2} \quad (2)$$

where γ_i represents the channel gain between mobile i and each BS of its own Active Set. Substitute (2) into (1), finally we have

$$\frac{E_b}{I_0} = \frac{N^2 P_0 \|\gamma_0^H \gamma_0\|^2}{N \sum_{i=1}^{K-1} \psi_i P_i \|\gamma_0^H \gamma_i\|^2} = \frac{N}{\sum_{i=1}^{K-1} \psi_i \frac{\|\gamma_0^H \gamma_i\|^2}{\|\gamma_0\|^2 \|\gamma_i\|^2}}$$

Let δ be the E_b / I_0 value required to achieve the level of performance, then the outage probability is

$$P_{out} = P_r \left(\frac{E_b}{I_0} < \delta \right) = P_r \left(I > \frac{N}{\delta} \right),$$

$$\text{where } I = \sum_{i=1}^{K-1} \psi_i \zeta_i, \quad \zeta_i = \frac{\|\gamma_0^H \gamma_i\|^2}{\|\gamma_0\|^2 \|\gamma_i\|^2}.$$

As we know, the total interference largely depends on a small amount of users whose Active Set is the same as mobile 0's. Those users have much larger ζ_i and should be distinguished from others. Assume that K_0 is the number of mobiles with the same Active Set as mobile 0, and S represents the distribution zone of those mobiles. Then Fig. 1 shows S in 2 BS's macrodiversity case, 3 BS's macrodiversity case and 4 BS's macrodiversity case.

¹ The Active Set of mobile 0 is called "Active Set 0" for short in the following text.

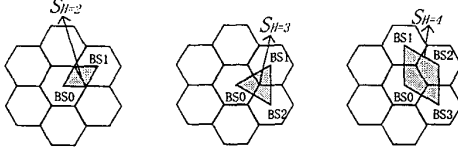


Fig.1 S in 2 BS's, 3 BS's and 4 BS's macrodiversity case

Let S_p be the area per cell and K_p be the number of mobiles per cell. Then from Fig. 1, it can be derived that

$$S_{H=2} = S_p / 3, \quad S_{H=3} = S_p / 2, \quad S_{H=4} = S_p.$$

Therefore we have

$$K_{0_{H=2}} = K_p / 3, \quad K_{0_{H=3}} = K_p / 2, \quad K_{0_{H=4}} = K_p,$$

and $K = LK_p$, where L is the number of BS's considered.

Rewrite I as $I = I_1 + I_2 = \sum_{i=1}^{K_0-1} \beta_i + \sum_{i=K_0}^{K-1} \psi_i \xi_i$, where

$$\beta_i = \psi_i \frac{\|\gamma_0^H \gamma_i\|^2}{\|\gamma_0\|^2 \|\gamma_i\|^2}, \quad \xi_i = \frac{\|\gamma_0^H \gamma_i\|^2}{\|\gamma_0\|^2 \|\gamma_i\|^2}. \text{ It can be seen that when}$$

$H=1$, I_1 represents the intracellular interference and I_2 represents the intercellular interference.

For a large number of mobiles, I_2 (interference due to $K - K_0$ users) can be approximated by a Gaussian random variable with the mean

$$\mu_{I_2} = (K - K_0) \cdot E(\psi_i \xi_i) = (K - K_0) \lambda \mu_{\xi} \quad \text{and} \quad \text{variance}$$

$$\sigma_{I_2}^2 = (K - K_0) \cdot \text{Var}(\psi_i \xi_i) = (K - K_0) [\lambda (\mu_{\xi}^2 + \sigma_{\xi}^2) - \lambda^2 \mu_{\xi}^2],$$

where μ_{ξ} is the mean of ξ_i and σ_{ξ}^2 is the variance of ξ_i . However, for I_1 , sometimes K_0 is not large enough to regard I_1 as a Gaussian variable, especially when $H=2$. Therefore we calculate the distribution of I_1 in the method of characteristic function.

Finally, if $P(I_1 = i) = p_i$, $i = 0, 1, \dots, K_0 - 1$, then

$$P_{out} = \sum_{i=0}^{K_0-1} p_i \cdot Q\left(\frac{N/\delta - i - \mu_{I_2}}{\sigma_{I_2}}\right) \quad (3),$$

$$\text{where } Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy.$$

This equation gives the outage probability as a function of the number of mobiles that can be supported per cell.

III NUMERICAL RESULTS AND DISCUSSIONS

Consider a 3-tier model, which means that $L=37$. We assume that the voice activity factor λ is 0.375 and the spreading factor N is 127. The standard variance σ_s of the log-normal shadowing variable is assumed to be 8 dB. As noted in [1], adequate performance ($\text{BER} < 10^{-3}$) is achieved with $E_b/I_0 > 7\text{dB}$. Thus δ is assumed to be 7 dB.

The outage probability can be calculated using (3). We simulated the capacity with 2 BS's macrodiversity ($H=2$), 3 BS's macrodiversity ($H=3$) and 4 BS's macrodiversity ($H=4$) in shadowed-Rayleigh, shadowed-Rice and shadowed-Rice/shadowed-Rayleigh² environments. The results are summarized from Fig. 2 to Fig. 4. From the figures, it is shown that with MRC-based macrodiversity, the reverse-link capacity can be increased greatly. For example, in Fig. 2, for 10^{-2} outage probability, the capacity goes up from 30 users per cell for non-macrodiversity case ($H=1$) to 41 users per cell if 2 BS's macrodiversity is used ($H=2$). With 3 BS's and 4 BS's macrodiversity, even higher capacity can be achieved. However, it can be also seen that the increase of capacity is convergent, that is, the growth rate of capacity decreases as H increases.

The effect of path-loss exponent α on reverse-link capacity in shadowed-Rayleigh environment is shown in Fig. 5. It can be seen that when α decreases from 4 to 3, the curves shift left, which means that the reverse-link capacity decreases with α . Fig. 6 shows the outage probability in shadowed-Rice environment when Rice factor is 2 and 4. Obviously the capacity increases with Rice factor.

In order to highlight the effect of MRC-macrodiversity on CDMA reverse-link capacity, we made a comparison of capacity with microdiversity and MRC-based macrodiversity. Similar to [9], it is assumed that each BS uses a multi-element antenna array and the number of antenna elements is H . The other assumptions are the same

² Here the shadowed Rice/Rayleigh model is similar to that in [12], namely, we assume that the signals sent within a radius of $2 \times R$ are Rician with the path loss exponent $\alpha = 3$, while those outside are Rayleigh with the path loss exponent $\alpha = 4$. R is the radius of a cell.

as above. We simulate the outage probability when H is 2, 3 and 4 in shadowed-Rayleigh environment, as shown in Fig.7. It can be seen that with antenna array the number of mobiles per cell increases greatly. However, it should be pointed out that here the number of antennas per cell is H times that with macrodiversity. In order to make a fair comparison, we plot the capacity curves of microdiversity and macrodiversity with the same amount of antennas per cell, as shown in Fig.8. Obviously MRC-based macrodiversity can bring more benefits to reverse-link capacity than microdiversity.

IV CONCLUSIONS

We have investigated the capacity improvement for CDMA reverse-link with MRC-based macrodiversity. The exact outage probability expression is derived as a function of cell loading, the number of involved BS's, fading parameters, and etc. Our analytical and simulation results show that with MRC-based macrodiversity the reverse-link capacity can be increased greatly in shadowed-Rayleigh, shadowed-Rice and shadowed-Rice/Rayleigh environments. Besides, the comparison of capacity with macrodiversity and microdiversity indicates that with the same amount of antennas more reverse-link capacity can be obtained with MRC-based macrodiversity.

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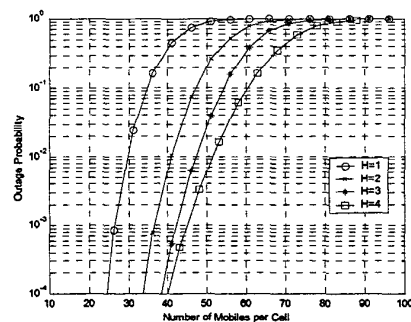


Fig.2 outage probability vs. the number of mobiles per cell in shadowed-Rayleigh environment, $\alpha=4$

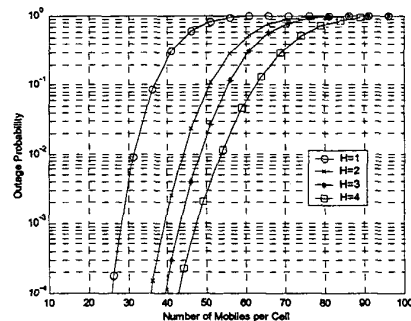


Fig.3 outage probability vs. the number of mobiles per cell in shadowed-Rice environment, $\alpha=4$, Rice factor is 4

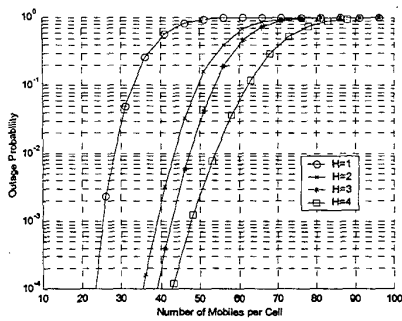


Fig. 4 outage probability vs. the number of mobiles per cell in shadowed-Rice/Rayleigh environment

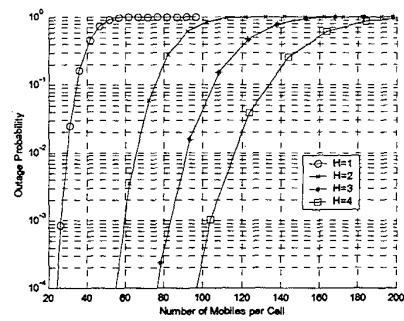


Fig. 7 outage probability vs. the number of mobiles per cell with antenna array

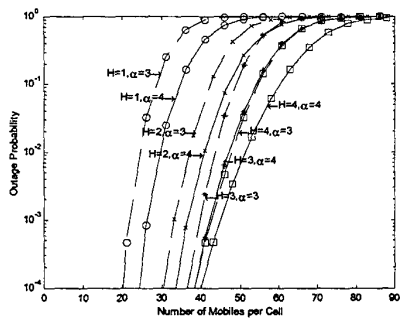


Fig. 5 outage probability in shadowed-Rayleigh environment when α is 3 and 4

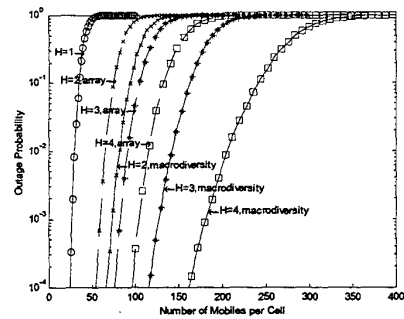


Fig. 8 comparison of capacity with antenna array and macrodiversity in shadowed-Rayleigh environment

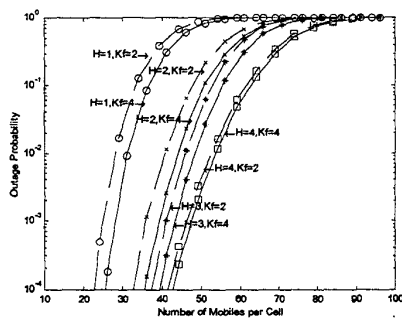


Fig. 6 outage probability in shadowed-Rice environment when Rice factor (K_f) is 2 and 4