Optimal Antenna Selection Based on Capacity Maximization for MIMO Systems in Correlated Channels

Lin Dai, *Member, IEEE*, Sana Sfar, *Associate Member, IEEE*, and Khaled Ben Letaief, *Fellow, IEEE*

Abstract—Recent work has shown that multiple-input multiple-output (MIMO) systems with multiple antennas at both the transmitter and receiver are able to achieve great capacity improvement. In such systems, it is desirable to select a subset of the available antennas so as to reduce the number of radio frequency (RF) chains. This paper addresses the problem of antenna selection in correlated channels. We consider a narrowband communication system with M transmit and N receive antennas. We present the criterion for selecting the optimal L_t out of M transmit and L_r out of N receive antennas in terms of capacity maximization, assuming that only the long-term channel statistics, instead of the instantaneous channel-state information, are known. Simulations will be used to validate our theoretical analysis and demonstrate that the number of required RF chains can be significantly decreased using our proposed selection strategy, while achieving even better performance than the conventional MIMO system without antenna selection.

Index Terms—Antenna selection, channel capacity, correlated channels, multiple-input multiple-output (MIMO) systems.

I. INTRODUCTION

MULTIPLE-input multiple-output (MIMO) systems have recently attracted tremendous interest due to their ability to provide great capacity improvements [1]–[2]. In particular, a new technology denoted by layered space–time for MIMO systems (BLAST) has been proposed in [3]. Such a scheme is shown to achieve unprecedented capacities that grow linearly with the number of transmit and receive antennas, when all signals undergo independent fading. However, the deployment of multiple antennas would require the implementation of multiple radio frequency (RF) chains that are typically very expensive. Dealing with this issue, [4] first proposed to select only the most useful antennas for further signal processing, namely, only L out of N antennas are effectively deployed, and only L RF

L. Dai was with The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. She is now with the University of Delaware, Newark, DE 19716 USA.

S. Sfar was with The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. She is now with Lehigh University, Bethlehem, PA 18015 USA.

K. B. Letaief is with the Center for Wireless Information Technology, Electrical and Electronic Engineering Department, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong (e-mail: eekhaled@ee.ust.hk).

chains are thus required. In [4]–[7], a system known as hybrid selection/maximum ratio combining is proposed, in which the antennas are selected to maximize the achieved diversity gain as well as minimize the obtained error rates. However, all of these previous studies are based on the multiple-input single-output (MISO) channel or the single-input multiple-output (SIMO) channel. Antenna selection is further applied to MIMO links, and it has been shown that in a multiple-antenna fading channel, antenna selection can also provide diversity advantage. Various criteria for receive-antenna selection or transmit-antenna selection were proposed, aiming at minimizing the error probability [9]–[14] or maximizing the capacity bounds [15]–[17]. For example, [10] presented some selection algorithms for spatial multiplexing systems when linear receivers are used. Orthogonal space–time block codes were further considered in [11]. [12] and [13] showed that with full-rank space–time trellis codes (STTCs) over quasi-static fading channels, the resulting diversity order with antenna selection can be maintained as that of the full-complexity system. [15] studied the effect of antenna selection from a channel capacity perspective, and showed that only a small loss in capacity is suffered when the receiver uses a good subset of the available receive antennas. [17] further proposed a low-complexity suboptimal selection algorithm that aims at maximizing capacity.

The above-stated results hold, unfortunately, only when the channel is rich enough. In such a case, the transmitted data is split into several streams and transmitted in parallel over individual and independent channel links so that spatial multiplexing gain can be obtained. In fact, such an assumption is generally not realistic, and channel links usually present spatial correlation[1] due to the lack of spacing between antennas, or to the existence of small angular spread. Both cases lead to a diminishing diversity and multiplexing gain, and this will significantly affect the capacity and error-probability performance [18], [29], [30]. Particularly, when BLAST is applied in correlated channels, the performance is severely degraded to an unacceptable level, since there are not enough independent dimensions supporting the simultaneously transmitted streams. Therefore, some processing at the transmitter must be done to combat this harmful effect. In this paper, we will show that in the correlated scenario, proper transmit-antenna selection can not only be used to decrease the number of RF chains, but also as an effective means to improve the performance.

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[1]Throughout this paper, the term "correlation" shall refer to "spatial correlation."

We consider a narrowband communication system with M transmit and N receive antennas over a slowly varying flat Rayleigh fading correlated channel. We propose to select only L_t out of M transmit and L_r out of N receive antennas for further signal processing, based on capacity maximization criterion assuming that only the long-term channel statistics (LTCS) are available. These statistics only vary with the antenna spacing and the signal-scattering angles resulting from the surrounding environment, and thus may change very slowly [19]. The proposed selection process is not, hence, updated for each channel instance, like those presented in [9]–[17], which are all based on the exact instantaneous channel-state information (ICSI). Moreover, we take the separable channel model proposed in [18], where the channel matrix can be written as the product of a receive correlation matrix, an independent and identically distributed (i.i.d.) complex Gaussian matrix, and a transmit correlation matrix. LTCS are then given by the correlation matrices at both ends, and transmit-antenna selection can be performed only based on the transmit correlation matrix, so that no feedback channel is needed. As a result, our proposed algorithm, which we shall refer to as the *correlated selection algorithm* (CSA), has the advantage of introducing significant complexity reduction over that using ICSI, which we refer to as the *instantaneous selection algorithm* (ISA).

Unlike the work in [20] and [21], which focus on the selection criterion for minimizing the average error probability, here we demonstrate that for capacity maximization, the transmit (receive) antenna subset should be chosen to maximize the determinant of the transmit (receive) correlation matrix. To do so, we derive the capacity upper and lower bounds, and show that they converge to the same limit. By maximizing both bounds, we obtain the selection criterion for capacity maximization. Although this criterion is proved to be optimal only in the high signal-to-noise ratio (SNR) regime, via extensive simulations, it is found that at low SNRs, the criterion also performs quite well. The fact that the selection results using CSA always coincide with those using exhaustive search seems to indicate that the proposed criterion is indeed optimal. Simulation results further show that with the proposed CSA, significant gains can be achieved over the random selection scheme. Besides, the comparison with ISA indicates only a slight capacity degradation, which implies that in correlated channels, antenna selection can be based on LTCS instead of ICSI with significant complexity reduction, while keeping capacity levels nearly unchanged. We also consider the case of the conventional system,[2] and show that optimal receive-antenna selection presents some capacity degradation, compared with the conventional system, while optimal transmit-antenna selection may enhance the capacity. This indicates that in correlated channels, proper antenna selection can not only be used to decrease the number of RF chains, but also as an effective means to improve the performance.

As for the joint antenna selection at both communication ends, we show that such a process can be decoupled with CSA, and significant complexity reduction is consequently obtained at satisfying levels. For a large M or N, however, CSA still involves high computational complexity levels, due to the required exhaustive search for the global optimal antenna set. To deal with this issue, we propose a low-complexity selection algorithm, denoted by L-CSA. It consists of iteratively searching the local optimal antenna subset at each stage. Significant complexity reduction, as well as very close performance to CSA, are shown to be achieved.

This paper is organized as follows. In Section II, we provide the system model and the various notations used throughout the paper. In Section III, we derive the antenna-selection criterion for capacity maximization, based on LTCS. Section IV presents the details of our selection algorithm, CSA, and the complexity analysis. L-CSA is also proposed. Section V shows the capacity performance of both CSA and L-CSA. Comparison results with ISA, as well as the conventional system, are also provided in this section. Finally, Section VI summarizes and concludes this paper.

II. SYSTEM MODEL

We consider the transmission of L_t signals through M antennas $(M \geq L_t)$, which undergo a flat slowly Rayleigh fading correlated channel to reach a receiver with N antennas. The received signals are next multiplexed into $L_r (L_r \leq N)$ RF chains so as to reduce the receiver cost and complexity.

Let $\mathbf{H}$ denote the $N \times M$ channel matrix and $\mathbf{x} = [x_1, x_2, \ldots, x_M]$ denote the transmitted signal vector, where x_i is the transmitted symbol from the ith antenna, and $(\cdot)$ refers to the transpose operator. Assuming perfect symbol synchronization at the receiver, as well as equal transmission power at the transmitter side, the discrete model of the received complex signal vector can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ denotes a complex Gaussian N-vector noise with covariance $\sigma^2 \mathbf{I_N}$.

Following the channel model provided in [18] and [26],[3] the channel matrix could be written as

$$\mathbf{H} = \mathbf{R}_r^{\frac{1}{2}} \mathbf{H_w} \mathbf{R}_t^{\frac{1}{2}} \tag{2}$$

where $\mathbf{H_w}$ is an $N \times M$ complex matrix of i.i.d. zero-mean, unit variance complex Gaussian entries. $\mathbf{R}_r$ and $\mathbf{R}_t$ denote the $N \times N$ and $M \times M$ antenna correlation matrices at the receiver and transmitter, respectively. It is also noted that $\mathbf{R}_r$ and $\mathbf{R}_t$ have unit diagonal entries.

We define the *selected transmit-antenna subset* and *selected receive-antenna subset* as $\mathbf{\Lambda_t}$ and $\mathbf{\Lambda_r}$, respectively, which are both unordered sets with L_t and L_r selected antennas. Let $\tilde{\mathbf{y}}$, $\tilde{\mathbf{x}}$, and $\tilde{\mathbf{n}}$ be the receive signal, transmit signal, and noise vector after selection, respectively. Let also $\mathbf{R}_{\mathbf{\Lambda_r}}$ and $\mathbf{R}_{\mathbf{\Lambda_t}}$ denote the crosscorrelation matrix of those L_r and L_t selected antennas, respectively. These matrices can be obtained by eliminating the columns and rows of the nondesired antennas from $\mathbf{R}_r$ and $\mathbf{R}_t$, respectively. We assume here that L_t and L_r are selected to

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[2]Throughout the paper, the "conventional system" shall refer to the system that uses all the M transmit and N receive antennas without selecting any antenna subset.

[3]In this paper, we adopt the separable correlated channel model proposed in [18], where the mutual coupling effect [27], [28] is not taken into account. Antenna selection based on a more comprehensive correlated channel model, where the mutual coupling is included, is an interesting open problem which can be further investigated.

guarantee that $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$ are both full rank. Now let $\tilde{\mathbf{H}}$ represent the $L_r \times L_t$ channel gain matrix between L_t selected transmit and L_r selected receive antennas. Then

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}} = \mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}^{\frac{1}{2}} \tilde{\mathbf{H}}_\mathbf{w} \mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}^{\frac{1}{2}} \tilde{\mathbf{x}} + \tilde{\mathbf{n}}. \tag{3}$$

Throughout this paper, we denote by $(\cdot)^*$, $\det(\cdot)$, $\operatorname{trace}(\cdot)$, and $\operatorname{rank}(\cdot)$ the complex conjugate transpose, the determinant, and the trace and the rank operators, respectively. $\mathbf{0}_{\mathbf{m}\times\mathbf{n}}$ represents an $m \times n$ zero matrix. For an arbitrary matrix $\mathbf{A}$, a_{ij} refers to its element at the ith row and the jth column, $\lambda_\mathbf{A}^{(i)}$ to its ith eigenvalue,[4] and $(\mathbf{A})_{n\times n}$ to its $n \times n$ principal submatrix. When $\mathbf{A}$ is a nonnegative definite matrix, we shall write it as $\mathbf{A} \geq \mathbf{0}$. We also write $\mathbf{A}$ as $\operatorname{diag}(a_{ii})$ when it is a diagonal matrix. Finally, a set $\mathbf{S}$ with elements s_i, $i = 1, 2, \ldots, n$, will be represented by $\{s_1, s_2, \ldots, s_n\}$, with its length denoted by $|\mathbf{S}|$.

III. ANTENNA-SELECTION CRITERION

In the following derivation, we highlight the effect of correlation on the capacity, so as to obtain a selection criterion that is related only to $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$. To do so, we start by applying eigenvalue decomposition to $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$. We then obtain

$$\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}} = \mathbf{U}_\mathbf{r}\mathbf{Q}_\mathbf{r}\mathbf{U}_\mathbf{r}^* \text{ and } \mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}} = \mathbf{U}_\mathbf{t}\mathbf{Q}_\mathbf{t}\mathbf{U}_\mathbf{t}^* \tag{4}$$

where $\mathbf{U}_\mathbf{r}$ and $\mathbf{U}_\mathbf{t}$ are both unitary matrices whose columns are the eigenvectors of $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$, respectively. $\mathbf{Q}_\mathbf{r}$ and $\mathbf{Q}_\mathbf{t}$ are both diagonal matrices whose diagonal entries are the eigenvalues of $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$, respectively. The channel matrix $\tilde{\mathbf{H}}$ is thus rewritten as

$$\tilde{\mathbf{H}} = \mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}^{\frac{1}{2}} \tilde{\mathbf{H}}_\mathbf{w} \mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}^{\frac{1}{2}} = \mathbf{U}_\mathbf{r}\mathbf{Q}_\mathbf{r}^{\frac{1}{2}}\mathbf{U}_\mathbf{r}^*\tilde{\mathbf{H}}_\mathbf{w}\mathbf{U}_\mathbf{t}\mathbf{Q}_\mathbf{t}^{\frac{1}{2}}\mathbf{U}_\mathbf{t}^*. \tag{5}$$

From [22], we know that the capacity of a flat slow-fading channel without CSI at the transmitter satisfies[5]

$$C = \log_2 \det\left[\mathbf{I}_{\mathbf{L}_\mathbf{r}} + \frac{\rho}{L_t}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^*\right]. \tag{6}$$

Hence, letting $\hat{\mathbf{H}}_\mathbf{w} = \mathbf{U}_\mathbf{r}^*\tilde{\mathbf{H}}_\mathbf{w}\mathbf{U}_\mathbf{t}$, we have

$$C = \log_2 \det\left[\mathbf{I}_{\mathbf{L}_\mathbf{r}} + \frac{\rho}{L_t}\mathbf{U}_\mathbf{r}\mathbf{Q}_\mathbf{r}^{\frac{1}{2}}\hat{\mathbf{H}}_\mathbf{w}\mathbf{Q}_\mathbf{t}\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}^{\frac{1}{2}}\mathbf{U}_\mathbf{r}^*\right]$$
$$= \log_2 \det\left[\mathbf{I}_{\mathbf{L}_\mathbf{r}} + \frac{\rho}{L_t}\hat{\mathbf{H}}_\mathbf{w}\mathbf{Q}_\mathbf{t}\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}\right] \tag{7}$$

where ρ is the mean SNR per receive branch. Obviously, $\hat{\mathbf{H}}_\mathbf{w}^*\hat{\mathbf{H}}_\mathbf{w}$ has the same eigenvalues as $\tilde{\mathbf{H}}_\mathbf{w}^*\tilde{\mathbf{H}}_\mathbf{w}$.

In order to select the optimal set of antennas that maximizes the above capacity expression, we distinguish three cases, $L_r = L_t$, $L_r > L_t$, and $L_r < L_t$. Actually, the $L_r < L_t$ case is similar to $L_r > L_t$, since $\mathbf{Q}_\mathbf{r}$ and $\mathbf{Q}_\mathbf{t}$ can be swapped with no effect on capacity, as (7) shows. Therefore, in the following, we will focus on the first two cases.

[4]The eigenvalues are all assumed to be sorted in a descending order.

[5]The capacity for fading channels can be defined in a number of ways, depending on many factors, such as the statistical nature of the channel and the amount of channel knowledge. In this paper, we mainly consider the outage characteristics of MIMO capacity. Equal power transmission is adopted, since no CSI is available at the transmitter.

In the first case, $L_r = L_t$ and the capacity is found to be equivalent to

$$C \approx \log_2 \det\left[\frac{\rho}{L_t}\hat{\mathbf{H}}_\mathbf{w}\mathbf{Q}_\mathbf{t}\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}\right]$$
$$= L_t \log_2\left(\frac{\rho}{L_t}\right) + \log_2 \det\left[\hat{\mathbf{H}}_\mathbf{w}\hat{\mathbf{H}}_\mathbf{w}^*\right]$$
$$+ \log_2 \det[\mathbf{Q}_\mathbf{t}] + \log_2 \det[\mathbf{Q}_\mathbf{r}] \tag{8}$$

at high values of ρ. Consequently, it is clear that to maximize the capacity, we should maximize the determinants of $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$. In other words, the optimal transmit (receive) antenna set $\boldsymbol{\Lambda}_\mathbf{t}$ ($\boldsymbol{\Lambda}_\mathbf{r}$), in terms of capacity maximization, should be selected to maximize the determinant of the corresponding correlation matrix $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$ ($\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$). In the second scenario, however, it is difficult to obtain a closed form of the exact capacity expression. Thus, we propose to derive both a lower and an upper bound for the capacity to obtain the optimal selection criterion.

A. Capacity Lower Bound

From (7), we have

$$C = \log_2 \det\left[\mathbf{I}_{\mathbf{L}_\mathbf{t}} + \frac{\rho}{L_t}\mathbf{Q}_\mathbf{t}\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}\hat{\mathbf{H}}_\mathbf{w}\right] \tag{9}$$

given that $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$. Let $\mathbf{U}_\mathbf{H}$ denote an $L_r \times L_r$ unitary matrix whose columns are the eigenvectors of $\hat{\mathbf{H}}_\mathbf{w}^*\hat{\mathbf{H}}_\mathbf{w}$, and $\hat{\mathbf{Q}}_\mathbf{r}$ the $L_t \times L_t$ leading principal submatrix of $\mathbf{U}_\mathbf{H}^*\mathbf{Q}_\mathbf{r}\mathbf{U}_\mathbf{H}$, i.e., $\hat{\mathbf{Q}}_\mathbf{r}$ is composed of the intersection elements of rows from 1 to L_t and columns from 1 to L_t of $\mathbf{U}_\mathbf{H}^*\mathbf{Q}_\mathbf{r}\mathbf{U}_\mathbf{H}$. It follows that:

$$C > \log_2 \det\left[\frac{\rho}{L_t}\mathbf{Q}_\mathbf{t}\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}\hat{\mathbf{H}}_\mathbf{w}\right]$$
$$= L_t \log_2\left(\frac{\rho}{L_t}\right) + \log_2 \det[\mathbf{Q}_\mathbf{t}] + \log_2 \det\left[\hat{\mathbf{H}}_\mathbf{w}^*\mathbf{Q}_\mathbf{r}\hat{\mathbf{H}}_\mathbf{w}\right]$$
$$= L_t \log_2\left(\frac{\rho}{L_t}\right) + \log_2 \det[\mathbf{Q}_\mathbf{t}] + \log_2 \det\left[\hat{\mathbf{H}}_\mathbf{w}^*\hat{\mathbf{H}}_\mathbf{w}\right]$$
$$+ \log_2 \det[\hat{\mathbf{Q}}_\mathbf{r}]. \tag{10}$$

Next, let $\{q_r^{(j)}\}_{j\in\{1,\ldots,L_r\}}$ denote the sorted eigenvalues in descending order of $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{r}}$. Since $\mathbf{U}_\mathbf{H}^*\mathbf{Q}_\mathbf{r}\mathbf{U}_\mathbf{H}$ is Hermitian, from [23], we know that $\lambda_{\hat{\mathbf{Q}}_\mathbf{r}}^{(i)} \geq \lambda_{\mathbf{U}_\mathbf{H}^*\mathbf{Q}_\mathbf{r}\mathbf{U}_\mathbf{H}}^{(L_r-L_t+i)}$, $i = 1, \ldots, L_t$. As a result, we have

$$\log_2 \det[\hat{\mathbf{Q}}_\mathbf{r}] > \log_2 \prod_{i=L_r-L_t+1}^{L_r} q_r^{(i)}. \tag{11}$$

By substituting (11) into (10), we have

$$\log_2 \det[\mathbf{Q}_\mathbf{t}] + \log_2 \det\left[\hat{\mathbf{H}}_\mathbf{w}^*\hat{\mathbf{H}}_\mathbf{w}\right] + \log_2 \det[\hat{\mathbf{Q}}_\mathbf{r}]$$
$$> \log_2 \prod_{i=1}^{L_t} q_t^{(i)} + \log_2 \prod_{i=1}^{L_t} d_i + \log_2 \prod_{i=L_r-L_t+1}^{L_r} q_r^{(i)} \tag{12}$$

where $\{q_t^{(i)}\}_{i\in\{1,\ldots,L_t\}}$ and $\{d_i\}_{i\in\{1,\ldots,L_t\}}$ are the sorted eigenvalues in descending order of $\mathbf{R}_{\boldsymbol{\Lambda}_\mathbf{t}}$ and $\hat{\mathbf{H}}_\mathbf{w}^*\hat{\mathbf{H}}_\mathbf{w}$, respectively. Furthermore, it is easy to note that

$$\log_2\left(\prod_{i=L_r-L_t+1}^{L_r} q_r^{(i)}\right) = \log_2\left(\prod_{i=1}^{L_r} q_r^{(i)}\right) - \log_2\left(\prod_{i=1}^{L_r-L_t} q_r^{(i)}\right). \tag{13}$$

Recall that $\mathbf{R}_{\mathbf{\Lambda_r}}$ has unit diagonal entries. Therefore, we have $\sum_{i=1}^{L_r} q_r^{(i)} = L_r$, and obviously, $\sum_{i=1}^{L_r-L_t} q_r^{(i)} < L_r$. By applying the arithmetic–geometric inequality, we further obtain

$$
\prod_{i=1}^{L_r-L_t} q_r^{(i)} \leq \left(\frac{\sum_{i=1}^{L_r-L_t} q_r^{(i)}}{L_r - L_t} \right)^{L_r - L_t} < \left(\frac{L_r}{L_r - L_t} \right)^{L_r - L_t}.
$$
(14)

Using (12)–(14), we have

$$
C = \log_2 \det \left[\mathbf{I_{L_t}} + \frac{\rho}{L_t} \mathbf{Q_t} \hat{\mathbf{H}}_{\mathbf{w}}^* \mathbf{Q_r} \hat{\mathbf{H}}_{\mathbf{w}} \right]
$$
$$
> L_t \log_2 \left(\frac{\rho}{L_t} \right) + \log_2 \left(\prod_{i=1}^{L_t} d_i \right) + \log_2 \left(\prod_{i=1}^{L_t} q_t^{(i)} \right)
$$
$$
+ \log_2 \left(\prod_{i=1}^{L_r} q_r^{(i)} \right) - (L_r - L_t) \log_2 \left(\frac{L_r}{L_r - L_t} \right). \quad (15)
$$

We thus obtain a lower bound C_L for the capacity that is given by

$$
C_L = L_t \log_2 \left(\frac{\rho}{L_t} \right) + (L_r - L_t) \log_2 \left(\frac{L_r - L_t}{L_r} \right)
$$
$$
+ \log_2 \left(\prod_{i=1}^{L_t} d_i \right) + \log_2 \left(\prod_{i=1}^{L_t} q_t^{(i)} \right) + \log_2 \left(\prod_{i=1}^{L_r} q_r^{(i)} \right). \quad (16)
$$

B. Capacity Upper Bound

We begin by presenting the following lemma.

Lemma 1: Assume that $\mathbf{A}$ is an arbitrary $n \times m$ matrix, such that $\mathbf{AA}^*$ is nonnegative definite, and $\mathbf{B}$ is an $m \times m$ nonnegative definite matrix with $m \leq n$. Then, we have $\lambda_{\mathbf{ABA}^*}^{(i)} \leq \lambda_{\mathbf{A}^*\mathbf{A}}^{(1)} \lambda_{\mathbf{B}}^{(i)}$, $i = 1, \ldots, m$.

Proof: See Appendix I.

Next, we note that

$$
\det \left[\mathbf{I_{L_t}} + \frac{\rho}{L_t} \mathbf{Q_t} \hat{\mathbf{H}}_{\mathbf{w}}^* \mathbf{Q_r} \hat{\mathbf{H}}_{\mathbf{w}} \right]
$$
$$
= \det \left[\mathbf{I_{L_r}} + \frac{\rho}{L_t} \mathbf{Q_r} \hat{\mathbf{H}}_{\mathbf{w}} \mathbf{Q_t} \hat{\mathbf{H}}_{\mathbf{w}}^* \right]
$$
$$
= \det \left[\mathbf{I_{L_r}} + \mathbf{Q_r W} \right] \quad (17)
$$

where $\mathbf{W} = (\rho/L_t) \hat{\mathbf{H}}_{\mathbf{w}} \mathbf{Q_t} \hat{\mathbf{H}}_{\mathbf{w}}^*$. Then, by applying the above lemma to $\mathbf{W}$, we have $\lambda_{\mathbf{W}}^{(i)} \leq (\rho/L_t) d_1 q_t^{(i)}$, $i = 1, \ldots, L_t$.

Now consider the $L_r \times L_r$ diagonal matrix $\mathbf{Z} = \mathrm{diag}(z_{ii})$ with

$$
z_{ii} = \begin{cases} \frac{\rho}{L_t} d_1 q_t^{(i)}, & i = 1, \ldots, L_t \\ 1, & i = L_t + 1, \ldots, L_r. \end{cases} \quad (18)
$$

Clearly, we have $z_{ii} \geq \lambda_{\mathbf{W}}^{(i)} \, \forall i = 1, \ldots, L_r$. Then, we can write the following:

$$
\det [\mathbf{I_{L_r}} + \mathbf{Q_r W}] \leq \det [\mathbf{I_{L_r}} + \mathbf{Q_r Z}] = \prod_{i=1}^{L_r} \left(1 + q_r^{(i)} z_{ii} \right). \quad (19)
$$

Finally, since $\mathbf{R}_{\mathbf{\Lambda_r}}$ is of full rank, we have, at high values of ρ

$$
\prod_{i=1}^{L_r} \left(1 + q_r^{(i)} z_{ii} \right) \approx \left(\frac{\rho}{L_t} \right)^{L_t} d_1^{L_t} \prod_{i=1}^{L_t} q_t^{(i)} \prod_{i=1}^{L_r} q_r^{(i)}. \quad (20)
$$

The capacity is thus upper bounded by C_U, where

$$
C_U = L_t \log_2 \left(\frac{\rho}{L_t} \right) + L_t \log_2(d_1)
$$
$$
+ \log_2 \left(\prod_{i=1}^{L_t} q_t^{(i)} \right) + \log_2 \left(\prod_{i=1}^{L_r} q_r^{(i)} \right). \quad (21)
$$

C. Lower and Upper Bound Convergence

We will now show that the lower and upper bounds, C_L and C_U, converge to the same limit. To do so, we assume that L_t and L_r can increase without bound at the same rate, such that $L_r \to \infty$ and $L_t \to \infty$ with $L_t/L_r \to r$, as in [24]. Obviously, r satisfies $0 \leq r < 1$. We then show that $\lim_{L_r \to \infty} (1/L_r)(C_U - C_L) \xrightarrow{r \to 0} 0$.

Using the expressions of C_L and C_U given in (16) and (21), respectively, we have

$$
\lim_{L_r \to \infty} \frac{1}{L_r}(C_U - C_L)
$$
$$
= \lim_{L_r \to \infty} \left(\frac{L_r - L_t}{L_r} \log_2 \frac{L_r}{L_r - L_t} \right.
$$
$$
\left. + r \log_2 d_1 - \frac{1}{L_r} \sum_{i=1}^{L_t} \log_2 d_i \right)
$$
$$
= \lim_{L_r \to \infty} \left(-(1 - r) \log_2(1 - r) \right)
$$
$$
+ \lim_{L_r \to \infty} \left(r \log_2 d_1 - \frac{1}{L_r} \sum_{i=1}^{L_t} \log_2 d_i \right). \quad (22)
$$

Now consider the limit of $\left(r \log_2 d_1 - (1/L_r) \sum_{i=1}^{L_t} \log_2 d_i \right)$ in (22) when $L_r \to \infty$. To do so, let $\mathbf{S} = (1/L_t) \tilde{\mathbf{H}}_{\mathbf{w}} \tilde{\mathbf{H}}_{\mathbf{w}}^*$, whose ith eigenvalue satisfies $\lambda_{\mathbf{S}}^{(i)} = (1/L_t) \lambda_{\tilde{\mathbf{H}}_{\mathbf{w}}^* \tilde{\mathbf{H}}_{\mathbf{w}}}^{(i)} = (1/L_t) d_i$, for any $i = 1, \ldots, L_t$. According to [25], these eigenvalues satisfy

$$
\lim_{L_r \to \infty} \max_{i \in \{1 \ldots L_t\}} \lambda_{\mathbf{S}}^{(i)} \to (1 + \sqrt{r})^2. \quad (23)
$$

In addition, they have an empirical distribution function, denoted by $F_{L_r}(x)$, given by

$$
\lim_{L_r \to \infty} F_{L_r}(x) \to F_r(x) \quad (24)
$$

where we have (25), shown at the bottom of the next page. Using the above results, we obtain

$$
\lim_{L_r \to \infty} \left(r \log_2 d_1 - \frac{1}{L_r} \sum_{i=1}^{L_t} \log_2 d_i \right)
$$
$$
= r \log_2 \left[L_t(1 + \sqrt{r})^2 \right]
$$
$$
- \lim_{L_r \to \infty} \left[r \frac{1}{L_t} \sum_{i=1}^{L_t} \log_2 \left(\frac{d_i}{L_t} \cdot L_t \right) \right]
$$
$$
= r \log_2(1 + \sqrt{r})^2 - \lim_{L_r \to \infty} \left(r \frac{1}{L_t} \sum_{i=1}^{L_t} \log_2 \frac{d_i}{L_t} \right)
$$
$$
= r \log_2(1 + \sqrt{r})^2 - r \int_{(1-\sqrt{r})^2}^{(1+\sqrt{r})^2} \log_2 x \cdot f_r(x) dx
$$
$$
= r \log_2(1 + \sqrt{r})^2 - r\eta(r) \quad (26)
$$

where $\eta(r) = \int_{(1-\sqrt{r})^2}^{(1+\sqrt{r})^2} \log_2 x \cdot f_r(x) dx$. We note that $\eta(r) < \infty$ for $0 \leq r < 1$.

Finally, by substituting (26) into (22), we get

$$\lim_{L_r \to \infty} \frac{C_U - C_L}{L_r} = -(1 - r) \log_2(1 - r)$$
$$+ r \left(\log_2(1 + \sqrt{r})^2 - \eta(r) \right). \quad (27)$$

Clearly, when $r \to 0$, we have $\lim_{L_r \to \infty}((C_U - C_L)/L_r) \to 0$, implying that when $L_r \gg L_t$, the capacity lower and upper bounds converge to the same limit.

D. Optimal Antenna Selection

From the closed-form expressions of C_L and C_U given in (16) and (21), respectively, we clearly distinguish the channel-correlation effect on the capacity from the instantaneous channel effect given by $\tilde{\mathbf{H}}_{\mathbf{w}}$. We can then proceed with antenna selection based on the LTCS, given by $\mathbf{R}_{\boldsymbol{\Lambda}_t}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_r}$. Obviously, the antenna-selection criterion that maximizes C_L has to maximize the term $\log_2(\prod_{i=1}^{L_t} q_t^{(i)}) + \log_2(\prod_{i=1}^{L_r} q_r^{(i)})$ in (16). The resulting antenna set thus needs to be selected so as to maximize the determinant of $\mathbf{R}_{\boldsymbol{\Lambda}_t}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_r}$ at the transmitter and receiver, respectively. Furthermore, a closer look at (21) indicates that C_U is maximized according to the same selection criterion. It follows that the selected antenna subset that maximizes the upper bound also maximizes the lower bound simultaneously. Moreover, according to Section III-C, C_L and C_U converge to the same limit when $L_r \gg L_t$. This implies that the antenna subset that maximizes both bounds also maximizes the capacity, and is, hence, asymptotically optimal.

So far, we have shown that to maximize the capacity, the antennas should be selected to maximize the determinant of the corresponding correlation matrix in both scenarios, $L_r = L_t$ and $L_r > L_t$. Therefore, the capacity maximization criterion for joint transmit and receive antenna selection can be described as follows.

Proposition 1: For a given L_t and L_r, the optimal selected transmit-antenna subset $\boldsymbol{\Lambda}_t^*$ and receive-antenna subset $\boldsymbol{\Lambda}_r^*$ that maximize the capacity are given by

$$\boldsymbol{\Lambda}_r^* = \arg \max_{\boldsymbol{\Lambda}_r} \det(\mathbf{R}_{\boldsymbol{\Lambda}_r}) \text{ and } \boldsymbol{\Lambda}_t^* = \arg \max_{\boldsymbol{\Lambda}_t} \det(\mathbf{R}_{\boldsymbol{\Lambda}_t}).$$
$$(28)$$

Note that the upper bound C_U is derived based on the high-SNR assumption. This implies that the proposed antenna-selection criterion is optimal only in the high-SNR regime. At low SNRs, the effect of $\mathbf{R}_{\boldsymbol{\Lambda}_t}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_r}$ cannot be separated from that of $\tilde{\mathbf{H}}_{\mathbf{w}}$, and therefore, the optimality of the selection criterion cannot be proved. Nevertheless, we checked the low-SNR cases via extensive simulations, and found that the selection results using CSA always coincide with those using exhaustive search. Therefore, it appears that the proposed criterion is indeed optimal for all SNR values. It must be also recalled that the above derivation is based on the assumption that both $\mathbf{R}_{\boldsymbol{\Lambda}_t}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_r}$ are of full rank (see Section II). Actually, this assumption can be relaxed. By conducting extensive

simulations, we found that even when $\mathbf{R}_{\boldsymbol{\Lambda}_t}$ and $\mathbf{R}_{\boldsymbol{\Lambda}_r}$ are singular, the criterion is also applicable if we substitute L_t and L_r in (28) by $\text{rank}(\mathbf{R}_{\boldsymbol{\Lambda}_t})$ and $\text{rank}(\mathbf{R}_{\boldsymbol{\Lambda}_r})$, respectively. In other words, when the number of antennas to be selected L_t is larger than $\text{rank}(\mathbf{R}_{\boldsymbol{\Lambda}_t})$, the selected L_t set of antennas should include those who maximize $\prod_{i=1}^{\text{rank}(\mathbf{R}_{\boldsymbol{\Lambda}_t})} q_t^{(i)}$. Selection at the receiver side is similar.

IV. ANTENNA-SELECTION ALGORITHMS FOR CORRELATED CHANNELS

We describe here a selection process according to *Proposition 1*, which we shall denote CSA. This algorithm consists of creating all possible $C_M^{L_t}$ $(C_N^{L_r})$ antennas sets $\boldsymbol{\Lambda}_t$ $(\boldsymbol{\Lambda}_r)$ with L_t (L_r) out of M transmit (N receive) antennas. The corresponding $\det(\mathbf{R}_{\boldsymbol{\Lambda}_r})(\det(\mathbf{R}_{\boldsymbol{\Lambda}_t}))$ are computed, and the one with the best measure, as described in *Proposition 1*, is selected. For simplicity, we only take the example of transmit selection. The description of CSA for receive selection is similar.

ALGORITHM I: CSA
Given L_t, let $K = C_M^{L_t}$, and generate all possible sets $\boldsymbol{\Lambda}$:
$\boldsymbol{\Lambda}_t^{(1)}, \boldsymbol{\Lambda}_t^{(2)}, \dots, \boldsymbol{\Lambda}_t^{(K)}$
Initialization:

$$\xi^C = 0, \boldsymbol{\Lambda}_t^* = \mathbf{0}_{\mathbf{L_t} \times 1}$$

Recursion:
  For $m = 1 : K$
    Compute $\det(\mathbf{R}_{\boldsymbol{\Lambda}_t^{(m)}})$ for each $\boldsymbol{\Lambda}_t^{(m)}$.
    If $\det(\mathbf{R}_{\boldsymbol{\Lambda}_t^{(m)}}) \geq \xi^C$ Then
      $\boldsymbol{\Lambda}_t^* = \boldsymbol{\Lambda}_t^{(m)}, \xi^C = \det(\mathbf{R}_{\boldsymbol{\Lambda}_t^{(m)}})$
    End if.
  End loop
Output $\boldsymbol{\Lambda}_t^*$.

Clearly, transmit and receive antenna selection are decoupled with CSA. Therefore, only $C_M^{L_t} + C_N^{L_r}$ comparisons are needed with CSA, instead of $C_M^{L_t} \times C_N^{L_r}$ with the full exhaustive search algorithm. The latter consists of considering all possible antenna sets at both ends simultaneously in couples $(\boldsymbol{\Lambda}_t, \boldsymbol{\Lambda}_r)$. Nevertheless, at either communication end, CSA employs an exhaustive search for the optimal antenna set, which will incur prohibitive computational complexity for a large M (or N). For instance, assume that $M = 20$ and $L_t = 8$, then a total of $C_M^{L_t} = 125\,970$ comparisons are needed, which is still considerably computationally high.

To further reduce the complexity, we propose to apply a suboptimal sequential-selection approach instead of the exhaustive one used in CSA. This is briefly described as follows, in the case

$$\frac{dF_r(x)}{dx} = f_r(x)$$
$$= \begin{cases} \frac{1}{2\pi r x} \sqrt{(x - (1 - \sqrt{r})^2)((1 + \sqrt{r})^2 - x)}, & (1 - \sqrt{r})^2 < x < (1 + \sqrt{r})^2 \\ 0, & \text{otherwise} \end{cases} \quad (25)$$
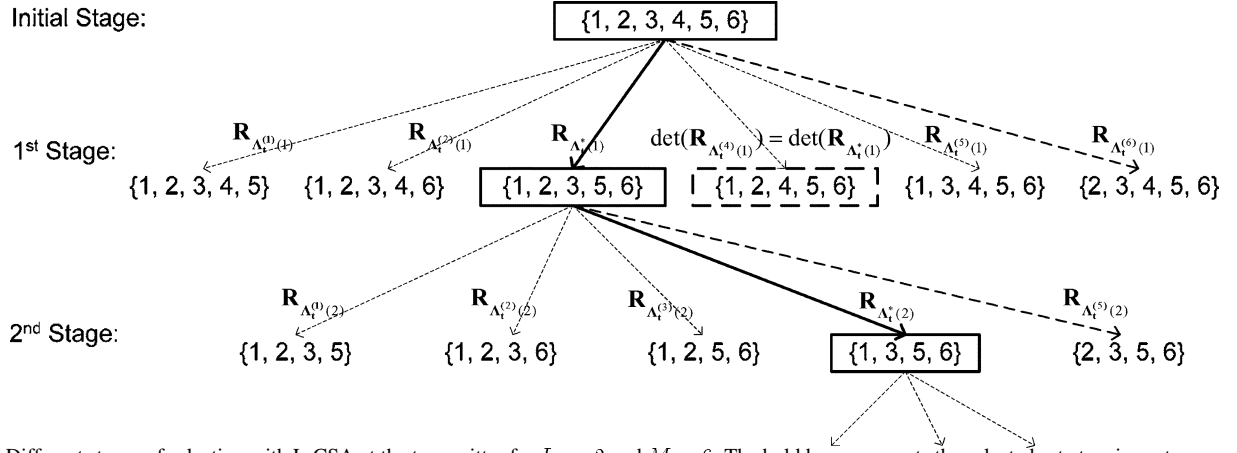
Fig. 1. Different stages of selection with L-CSA at the transmitter for $L_t = 2$ and $M = 6$. The bold box represents the selected set at a given stage.

TABLE I
COMPLEXITY COMPARISON BETWEEN CSA AND L-CSA

Algorithm	Complexity	$M=6, N=6$	$M=10, N=15$		$M=20, N=30$	
		$L_t=3, L_r=3$	$L_t=2, L_r=4$	$L_t=5, L_r=8$	$L_t=3, L_r=6$	$L_t=8, L_r=12$
CSA	$C_M^{L_t} + C_N^{L_r}$	40	1410	6687	594910	86619000
L-CSA	$\sum_{i=L_t+1}^{M} i + \sum_{j=L_r+1}^{N} j$	30	162	124	648	561

of transmit selection. We begin by considering all M transmit antennas. Antenna selection is then performed in stages. In the ith stage, all possible antenna subsets $\mathbf{\Lambda_t}^{(j)}(i)$ are obtained by removing only one antenna each time from the original set $\mathbf{\Lambda_t}^*(i-1)$ of the previous $(i-1)$th stage. For each subset $\mathbf{\Lambda_t}^{(j)}(i)$, we compute its corresponding determinant, given by

$$\det\left(\mathbf{R}_{\mathbf{\Lambda_t}}^{(j)}(i)\right) = \prod_{k=1}^{\left|\mathbf{\Lambda_t}^{(j)}(i)\right|} q_t^{(k)}. \qquad (29)$$

For ease of reference, we will refer to the above determinant as the metric of each antenna subset $\mathbf{\Lambda_t}^{(j)}(i)$. Then, the optimal antenna subset with the highest metric, $\mathbf{\Lambda_t}^*(i)$, is selected. All other subsets are then removed, and the search will continue as described above until only L_t antennas are left. For instance, consider transmit selection with $L_t = 2$ and $M = 6$, as shown in Fig. 1. We label the transmit antennas as $1, 2, \ldots, 6$. In particular, the notation $i, j, \ldots, k$ represents the selection of antenna i, j, and k from the set of M transmit antennas. It can be seen that four stages of search are required. In the first stage, five transmit antennas are selected according to the metric as described in (29). In the second stage, the best four antennas are chosen out of the already-selected optimal antenna subset in the first stage. This process is continued until we obtain the best selected transmit-antenna set with $L_t = 2$ antennas. This sequential search algorithm shall be referred to as *low-complexity CSA* (L-CSA), and is described below in the case of transmit selection.

ALGORITHM II: L-CSA

Let $\mathbf{\Lambda_t}^*(i)$ denote the optimal selected-antenna set at the ith stage, and label the transmit antennas $1, 2, \ldots, M$.

Initialization:
   $L = M, i = 0, \mathbf{\Lambda_t}^*(0) = \{1, 2, \ldots, M\}$
Recursion:
   While $(L > L_t)$
   (a) $i = i + 1$;
   (b) For $j = 1$:
      $\mathbf{\Lambda_t}^{(j)}(i) = \mathbf{\Lambda_t}^*(i-1) - \{j\}, \det \mathbf{R}_{\mathbf{\Lambda_t}^{(j)}(i)} = \prod_{i=1}^{|\mathbf{\Lambda_t}^{(j)}(i)|} q_t^{(i)}$
    End
   (c) $\mathbf{\Lambda_t}^*(i) = \arg \max_{\mathbf{\Lambda_t}^{(j)}(i)} \det(\mathbf{R}_{\mathbf{\Lambda_t}^{(j)}(i)}), L = L - 1$.
   End While.
Output the optimal antenna subset $\mathbf{\Lambda_t}^*(i)$.

The optimal selected receive-antenna set $\mathbf{\Lambda_r}^*$ can be obtained in a similar way. Clearly, L-CSA requires much fewer comparisons than CSA. Specifically, only $\sum_{i=L_t+1}^{M} i + \sum_{j=L_r+1}^{N} j$ comparisons are needed, instead of $C_M^{L_t} + C_N^{L_r}$ with CSA. To further demonstrate the amount of complexity reduction obtained with L-CSA, we provide in Table I the number of required comparisons to find the optimal antenna set using both CSA and L-CSA. For instance, to select 8 out of 20 transmit and 12 out of 30 receive antennas, CSA needs about 8×10^7 comparisons, while with L-CSA, only 561 comparisons are needed. Nevertheless, despite its low complexity, L-CSA may not provide the optimal set that maximizes the capacity. As shown in Fig. 1, two sets in the first stage have the same value of the largest metric. That is, the sets inside the solid- and dashed-line boxes. In this case, L-CSA chooses the first found set. However, note that a better capacity may be obtained if the other set is considered. In this paper, we consider the simplest form of L-CSA, which always selects the first found set that maximizes the metric as

described in (29) at any given stage, to provide the lowest possible complexity.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present simulation results that validate the selection criterion derived previously. We also compare the performance of CSA with both ISA and the conventional system. Performance is evaluated in terms of 10% outage capacity[6] averaged over 50 000 frames. We consider uplink transmission and adopt the correlated channel model described in [18] and [26]. Linear arrangement of the antenna array is assumed at both the receiver (basestation side) and transmitter (mobile side) with the antenna separation being 4 and 1/2 wavelengths, respectively. We also assume the "broadside" case as defined in [18], and that the incoming waves are uniformly distributed in the angular spread $\Delta_r(\Delta_t)$ [26].

A. Theoretical Results Validation

To confirm the optimality of our criterion, we compare the CSA selection to the exhaustive search one. The latter is obtained by using Monte Carlo simulations to find the best $L_t(L_r)$ antennas that should be selected on the transmitter (receiver) side. To do so, we compute the corresponding capacity for each possible antenna subset $\mathbf{\Lambda_t}(\mathbf{\Lambda_r})$, and choose the one with the highest capacity. Since the antenna selection at both ends can be decoupled, we take the example of receive-antenna selection for illustration. Assume that the correlation is existent only at the receiver. We compare the selection results using CSA and those using exhaustive search under different receive angular spreads Δ_r, different number of selected receive antennas L_r, and different SNRs. It is found that the CSA selection results always coincide with the optimal ones obtained using exhaustive search. For example, with $\Delta_r = 10°$, $L_r = 3$, and SNR $= 20$ dB, the optimal antenna subsets with CSA and the exhaustive search are both given by $\mathbf{\Lambda_r^*} = \{1, 2, 6\}$. When $\Delta_r = 50°$, $L_r = 2$, and SNR $= 5$ dB, it is found that the optimal antenna subset is given by $\mathbf{\Lambda_r^*} = \{1, 6\}$. As a result, this confirms that our selection criterion, as derived in Section III, is indeed optimal.

B. Selection Gain

We compare the capacity of our proposed CSA and the random-selection algorithm (RSA) so as to see how much gain can be obtained with the optimal selection. In particular, with RSA, we randomly select L_r receive antennas or L_t transmit antennas for each channel realization. Fig. 2 shows the comparison capacity cumulative distribution function (cdf) curve results under different values of L_r, L_t, Δ_r, and Δ_t. We first consider the capacity gain at the receiver side. Particularly, we assume that correlation only exists at the receiver and for a system of $N = 6$ and $M = 2$, we select three receive antennas randomly or according to our proposed CSA. From Fig. 2, it can be seen that the optimal receive-antenna selection using CSA can achieve a gain of at least 1 b/s/Hz for 10% outage capacity when $\Delta_r = 35°$ and SNR $= 20$ dB. As for the transmit-antenna
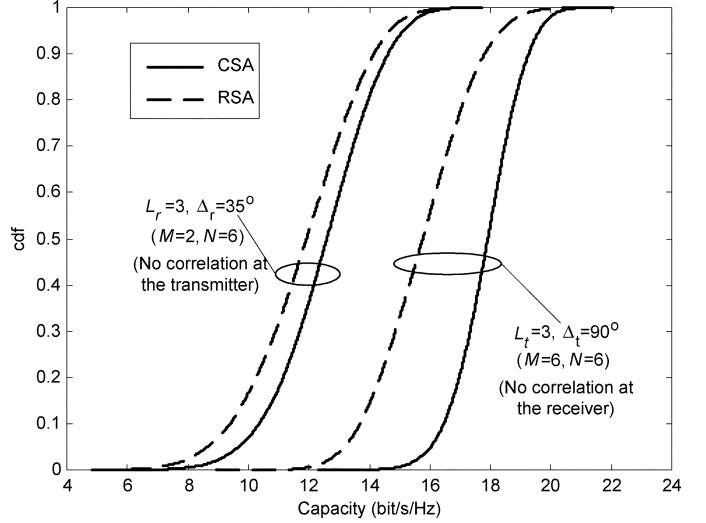
[6]The $x\%$ outage capacity $C_{\mathrm{out},x}$ is defined as the information rate that is guaranteed for $(100-x)\%$ of the channel realizations, i.e., $P_r(C \geq C_{\mathrm{out},x}) = 1 - x/100$.

Fig. 2. Capacity cdf curves of CSA and RSA for different values of L_r, L_t, Δ_r, and Δ_t for SNR $= 20$ dB.
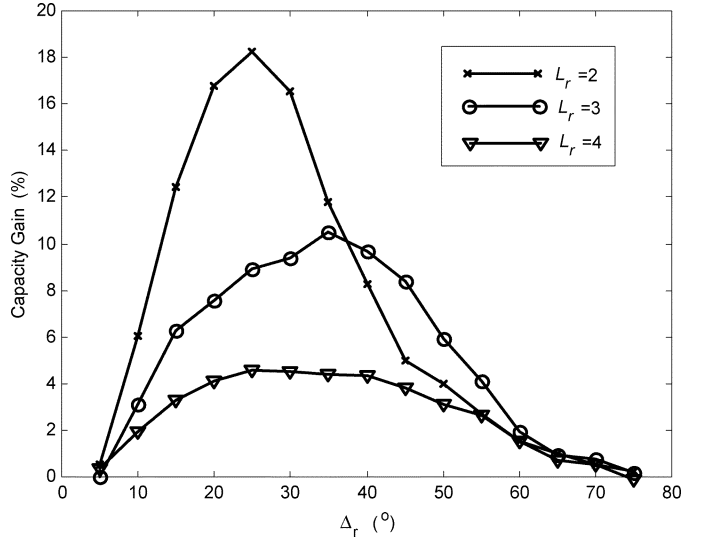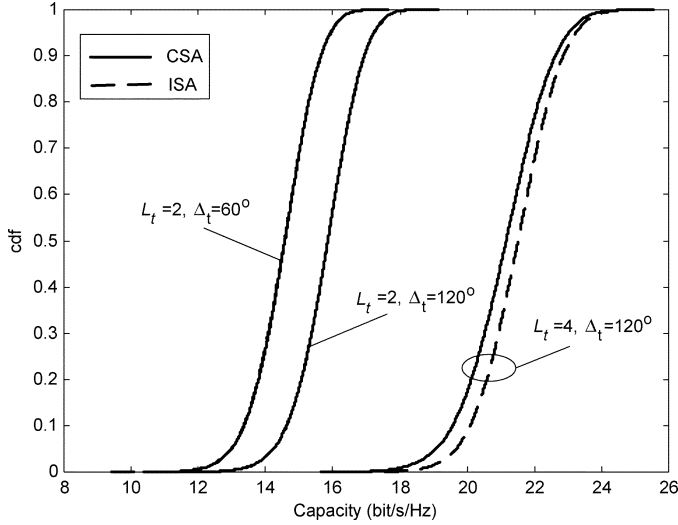

Fig. 3. Capacity gain of CSA over RSA versus Δ_r for different values of L_r, $N = 6$, $M = 2$, and SNR $= 20$ dB. No correlation exists at the transmitter.

selection, we consider a system of $M = N = 6$, and assume that correlation only exists at the transmitter side. A gain of 3 b/s/Hz at 10% outage capacity is observed with CSA when $\Delta_t = 90°$ and SNR $= 20$ dB.

We further investigate in what follows the capacity gain achieved with CSA compared with RSA with different values of L_r and Δ_r. To do so, we evaluate here such gain as $(C_{\mathrm{CSA}} - C_{\mathrm{RSA}})/C_{\mathrm{RSA}} \times 100\%$, where C_{CSA} and C_{RSA} denote the 10% outage capacity obtained by CSA and RSA, respectively. Fig. 3 provides the gain results for $\Delta_r \in [5°, 75°]$ and $L_r = 2, 3, 4$. A closer observation of this figure indicates that CSA exhibits significant gain compared with RSA only for an angular spread, i.e., $\Delta_r \in [10°, 60°]$. In fact, when Δ_r is relatively small, i.e., $\Delta_r < 10°$, the correlation matrix tends to be singular with a unit rank. No matter what antennas are selected, there is only one independent dimension for the whole channel, and thus, the capacity gain obtained with CSA is very low. A similar observation is also noticed for a relatively large angular

Fig. 4.   Capacity cdf curves of CSA and ISA with different values of L_t and Δ_t when $N = M = 6$ and SNR $= 20$ dB. No correlation exists at the receiver.

Fig. 5.   Capacity cdf curves of CSA and the conventional system (with $L_t = M = 6$, $L_r = N = 6$) for different values of Δ_t, Δ_r, L_t, and L_r. SNR $= 20$ dB. The capacity cdf curves of the conventional system are drawn in dash-dot lines.

spread, i.e., $\Delta_r > 60°$. In this case, however, the correlation between different antennas tends to be zero, implying that $\mathbf{R_r I_N}$. Optimal antenna selection does not, hence, achieve a significant gain over the random antenna selection. As a result, we can conclude that CSA performs much better than RSA when the channel links are neither independent nor severely correlated. Besides, Fig. 3 indicates that such gain decreases with an increasing L_r. This is because the larger the antenna set to be selected, the higher the chances that RSA would select the same antenna set as CSA. Similar results can be obtained for transmit selection. However, we do not include them here due to space limitations.

C. Performance Comparison With Instantaneously Selected System

We compare here the capacity performance of CSA and ISA. Recall that antenna selection in ISA is performed according to the exact channel-state information (CSI) per channel instance. For every realization of the channel matrix $\mathbf{H}$, a complete set of all the possible matrices $\mathcal{H}$ is created by eliminating all possible permutations of $N - L_r$ rows (and/or $M - L_t$ columns) from the matrix. Capacity is then computed for each possible $\mathcal{H}$, and the antenna set corresponding to $\mathcal{H}$ that maximizes the capacity is selected.[7]

In Fig. 4, we consider antenna selection at the transmitter side. Capacity cdf results of both CSA and ISA for $L_t = 2, 4$, $\Delta_t = 60°$, and $120°$ with an SNR of 20 dB are provided in this figure. The number of transmit and receive antennas is assumed to be six, and correlation is assumed to only exist at the transmitter. It can be seen that CSA can always achieve nearly the same capacity as ISA, even for large angular spreads. For example, when $L_t = 2$, the gap between the capacity of ISA and CSA is so slight that the two curves overlap. Recall that our proposed CSA is based on LTCS provided by the correlation matrices.
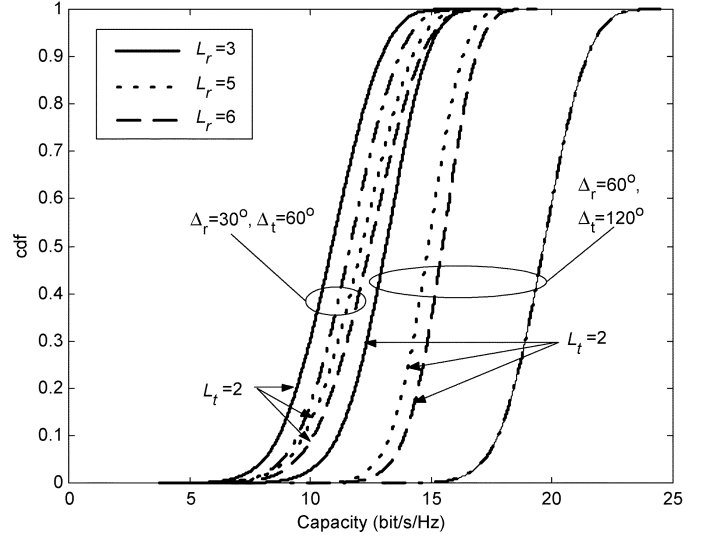
With an increasing angular spread, which implies a more and more uncorrelated channel, CSA will incur some performance degradation compared with ISA, since capacity in this case is affected by ICSI rather than by LTCS. Nevertheless, from Fig. 4, it can be observed that this degradation is rather slight. Even for an angular spread of $120°$, 10% outage capacity of CSA is only 0.4 b/s/Hz less than that of ISA with $L_t = 4$. As a result, we conclude that in correlated channels, antenna selection can be based on LTCS instead of ICSI, with very slight capacity loss and significant complexity reduction. Similar results are also obtained for the receiver side, but we do not present them here due to limited space.

D. Performance Comparison With the Conventional System

We compare the capacity achieved with CSA and the conventional system without selection (i.e., $L_t = M$ and $L_r = N$) to highlight the importance of antenna selection in correlated channels. Fig. 5 shows the capacity cdf curves of CSA and the conventional system. It can be observed that using fewer receive antennas will lead to a decrease of capacity. The capacity cdf curve of CSA with $L_t = 2, L_r = 6$ is always on the right side of those of CSA with $L_t = 2, L_r = 3, 5$. However, using fewer transmit antennas may actually increase the capacity in some high correlated scenario. As Fig. 5 shows, when $\Delta_t = 60°$ and $\Delta_r = 30°$, the capacity of CSA with $L_t = 2, L_r = 6$ (or 5) is larger than that of the conventional system ($L_t = 6$, $L_r = 6$)! While with increasing Δ_t and Δ_r, the conventional system can achieve more capacity than CSA. For instance, with a $\Delta_t = 120°$ and $\Delta_r = 60°$, the conventional system gains about 4 b/s/Hz more than CSA with $L_t = 2, L_r = 6$.

To show the effect of transmit selection and receive selection more clearly, Figs. 6 and 7 present the capacity of receive selection and transmit selection, respectively. In Fig. 6, we assume that no correlation exists at the transmitter, and the number of transmit antennas is fixed at six. Clearly, the conventional

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[7]In this paper, we assume equal power allocation in both ISA and CSA. However, note that in ISA with the CSI at the transmitter, waterfilling power allocation may be adopted instead of the equal power allocation. This can bring more capacity gains at the cost of higher complexity cost.
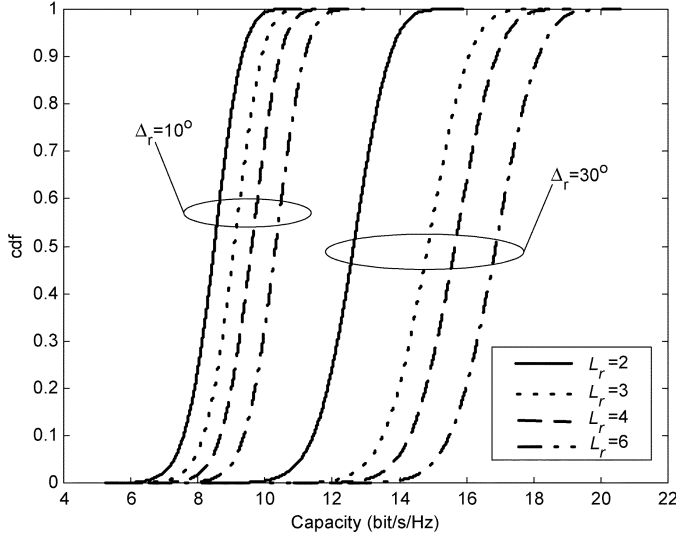
Fig. 6. Capacity cdf curves of CSA for receive selection and the conventional system. $N = M = 6$ and SNR $= 20$ dB. No correlation exists at the transmitter. The capacity curves of the conventional system are drawn in dash-dot lines.
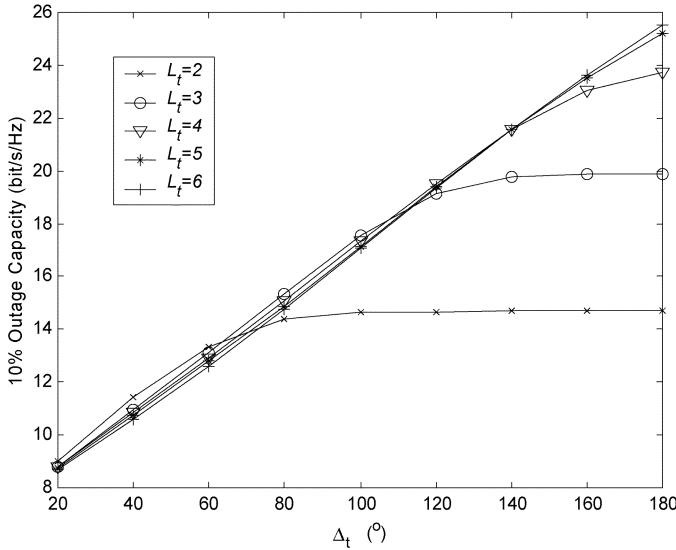

Fig. 8. 10% outage capacity versus Δ_r curves of CSA and L-CSA for receive selection with different values of L_r, for $N = 6$, $M = 2$, SNR $= 20$ dB. No correlation exists at the transmitter.

Fig. 7. 10% outage capacity versus Δ_t curves of CSA for transmit selection and the conventional system, for $N = M = 6$ and SNR $= 20$ dB. No correlation exists at the receiver. The curve of the conventional system is marked with "+".

system always outperforms CSA with receive-antenna selection. Moreover, the capacity loss increases with a decreasing L_r. This is explained by the fact that when fewer receive antennas are selected, the overall diversity order and multiplexing gain decrease, and so does the system capacity. Such a loss is also observed to decrease with a decreasing Δ_r. In fact, a closer observation of Fig. 6 indicates that when $\Delta_r = 30°$ and $L_r = 2$, CSA presents around a 4 b/s/Hz loss compared with the conventional system at 10% outage capacity. With the same selected number of receive antennas, only a loss on the order of 1 b/s/Hz is observed when $\Delta_r = 10°$. This is explained by the fact that when Δ_r decreases, the channel links becomes severely correlated, so that the effective degrees of freedom of the channel (or the
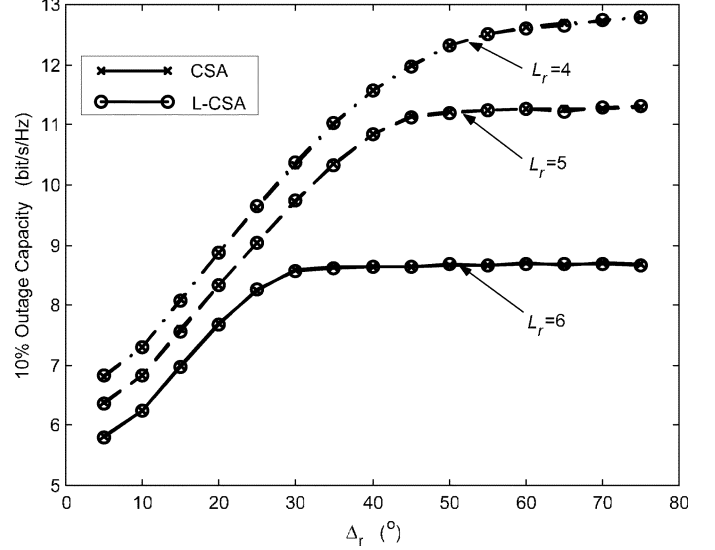
rank of the channel) decrease. Hence, the capacity of the conventional system does not present much improvement compared with CSA. As a result, we conclude that CSA with receive-antenna selection presents capacity degradation, compared with the conventional system. Such degradation will increase with a decrease in the number of selected receive antennas and an increase of angular spread.

On the other hand, using fewer transmit antennas may boost the capacity with a small angular spread. In order to show the effect of L_t and angular spreads on capacity more clearly, we plot the curves of 10% outage capacity versus Δ_t with different L_t. Similarly, here no correlation is assumed to be at the receiver, and the number of receive antennas is fixed at six. As Fig. 7 shows, under a highly correlated channel, i.e., $\Delta_t \leq 60°$, CSA with $L_t = 2$ provides the highest capacity. In such case, the effective degrees of freedom of the channel are rather low. Therefore, by allocating the transmission power only to the "good" subchannels, transmit-antenna selection can bring capacity gains over the conventional system. As the angular spread increases, the effective degrees of freedom also increase. That is why the capacity with $L_t = 2$ will converge to a constant quickly, while the capacity with more selected transmit antennas still goes on climbing. Finally, when Δ_t increases to 180°, the conventional system $(L_t = M = 6)$ achieves the best capacity performance. Actually, if the number of transmit antennas exceeds the rank of the channel matrix, capacity will decrease with an increase of L_t. The highest capacity is always achieved when the number of transmit antennas is equal to the rank of the channel matrix. From Fig. 7, it is clear that only for high values of Δ_t (i.e., $\Delta_t \geq 160°$), which implies a nearly uncorrelated channel), the conventional system outperforms CSA. As a result, we conclude that in correlated channels, the conventional system does not always provide the best capacity performance. In particular, optimal transmit-antenna selection can bring capacity gain.

So far, we have shown that the optimal receive-antenna selection presents some capacity degradation compared with the

conventional system. The optimal transmit-antenna selection, on the other hand, may enhance the capacity. Therefore, in correlated channels, CSA with a proper L_t and L_r can actually achieve more capacity than the conventional system. Proper antenna selection can not only be used to decrease the number of RF chains, but also as an effective means to improve the performance.

E. Performance Comparison of CSA With L-CSA

In Section IV, it has been shown that with L-CSA, the complexity can be reduced dramatically. We further present the performance comparison of L-CSA and CSA in the case of receive selection. Their 10% outage capacity versus Δ_r curves for different values of L_r are plotted in Fig. 8. It can be seen that L-CSA provides nearly the same capacity as CSA. In particular, it was found that the selection results using L-CSA are usually the same as those using CSA. As a result, we conclude that L-CSA can perform closely to CSA, but with much lower complexity.

VI. CONCLUSIONS

In this paper, we derived the capacity maximization criterion for transmit- and receive-antenna selection according to LTCS. We showed that in correlated channels, our algorithm, which we refer to as the CSA scheme, can achieve nearly the same capacity as the ISA scheme while dramatically decreasing the complexity, since only the correlation matrix is needed for selection instead of the ICSI. It was also shown that with optimal transmit-antenna selection, CSA can even achieve performance gain over the conventional system. Finally, we proposed an L-CSA which can achieve very close performance to CSA, but with much lower complexity.

APPENDIX I
PROOF OF *LEMMA 1*

Assume that $\mathbf{X}$ and $\mathbf{Y}$ are both $n \times n$ nonnegative definite matrices. Then, from [23], we know that

$$\lambda_{\mathbf{XYX}}^{(i)} \le \lambda_{\mathbf{Y}}^{(1)} \lambda_{\mathbf{X^2}}^{(i)}, \quad i = 1, \ldots, n. \tag{30}$$

Applying (30) with $\mathbf{X} = \mathbf{B}^{1/2}$ and $\mathbf{Y} = \mathbf{A}^*\mathbf{A}$, we have

$$\lambda_{\mathbf{B}^{\frac{1}{2}}\mathbf{A}^*\mathbf{AB}^{\frac{1}{2}}}^{(i)} \le \lambda_{\mathbf{A}^*\mathbf{A}}^{(1)} \lambda_{\mathbf{B}}^{(i)}, \quad i = 1, \ldots, n. \tag{31}$$

Furthermore, using the fact that $\lambda_{\mathbf{AA}^*}^{(i)} = \lambda_{\mathbf{A}^*\mathbf{A}}^{(i)}$, $i = 1, \ldots, m$, we know that

$$\lambda_{\mathbf{ABA}^*}^{(i)} = \lambda_{\mathbf{B}^{\frac{1}{2}}\mathbf{A}^*\mathbf{AB}^{\frac{1}{2}}}^{(i)}, \quad i = 1, \ldots, m. \tag{32}$$

As a result, we get

$$\lambda_{\mathbf{ABA}^*}^{(i)} \le \lambda_{\mathbf{A}^*\mathbf{A}}^{(1)} \lambda_{\mathbf{B}}^{(i)}, \quad i = 1, \ldots, m.$$

■

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Lin Dai (S'00-M'03) was born in Wuhan, China, in 1979. She received the B.S. degree in electronic engineering from Huazhong University of Science and Technology, Wuhan, China, in 1998, and the M.S. and Ph.D. degrees in electrical and electronic engineering from Tsinghua University, Beijing, China, in 2000 and 2002, respectively.

She was with Alcatel Shanghai Bell Co., Ltd. as a Visiting Researcher and worked on the design and implementation of a WCDMA commercial prototype from April to July, 2001. Since 2003, she has been with the Hong Kong University of Science and Technology, Hong Kong, where she is a Postdoctoral Fellow. Her research interests include wireless communications, and information and communication theory.

Sana Sfar (S'01–A'05) received the Ph.D. and MPhil. degrees in 2005 and 2001, respectively, both in electrical and electronic engineering from The Hong Kong University of Science and Technology (HKUST), Hong Kong, with a focus on wireless communications. She received the Engineering Diploma in telecommunications (with excellence) in 1999 from the École Supérieure des Communications de Tunis, Tunis, Tunisia, (SupCom) with a focus on multimedia and networking.

She is currently holding a Postdoctoral position at Lehigh University, Bethlehem, PA, in collaboration with Bell Labs, Holmdel, NJ. Her current research interests are in the area of wireless communication systems and communication theory. These include wireless ad-hoc networks, broadband wireless access, MIMO techniques, space–time coding, channel coding, OFDM, CDMA, and Beyond 3G systems.

Dr. Sfar is the recipient of the HKTIIT Post-Graduate Excellence Award in 2005, the IEEE Communications Society Student Travel Grant Award in 2001, 2003, and 2004; the Tunisian National Postgraduate Fellowship Award in 1999, and the Tunisie Telecom Excellency Grant in 1999.

Khaled Ben Letaief (S'85–M'86–SM'97–F'03) received the B.S. degree (with distinction) and M.S. and Ph.D. degrees from Purdue University, West Lafayette, IN, in 1984, 1986, and 1990, respectively, all in electrical engineering.

Since January 1985 and as a Graduate Instructor in the School of Electrical Engineering at Purdue University, he has taught courses in communications and electronics. From 1990 to 1993, he was a faculty member with the University of Melbourne, Melbourne, Australia. Since 1993, he has been with the Hong Kong University of Science and Technology, Kowloon, where he is currently a Chair Professor and Head of the Electrical and Electronic Engineering Department. He is also the Director of the Hong Kong Telecom Institute of Information Technology as well as the Director of the Center for Wireless Information Technology. His current research interests include wireless and mobile networks, broadband wireless access, space–time processing for wireless systems, wideband OFDM, and CDMA systems.

Dr. Letaief served as consultant for different organizations and is currently the founding Editor-in-Chief of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He has served on the Editorial Board of other journals, including the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS Wireless Series (as Editor-in-Chief). He served as the Technical Program Chair of the 1998 IEEE Globecom Mini-Conference on Communications Theory, held in Sydney, Australia. He was also the Co-Chair of the 2001 IEEE ICC Communications Theory Symposium, held in Helsinki, Finland. He was the Co-Chair of the 2004 IEEE Wireless Communications, Networks, and Systems Symposium, held in Dallas, TX, and was the Vice-Chair of the International Conference on Wireless and Mobile Computing, Networking, and Communications, WiMob'05, to be held in Montreal, QC, Canada. He is currently serving as the Chair of the IEEE Communications Society Technical Committee on Personal Communications. In addition to his active research activities, he has also been a dedicated teacher committed to excellence in teaching and scholarship. He was the recipient of the Mangoon Teaching Award from Purdue University in 1990, the Teaching Excellence Appreciation Award by the School of Engineering at HKUST (four times), and the Michael G. Gale Medal for Distinguished Teaching (the highest university-wide teaching award and only one recipient/year is honored for his/her contributions). He is a Distinguished Lecturer of the IEEE Communications Society.