Fairness Improves Throughput in Energy-Constrained Cooperative Ad-Hoc Networks

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Abstract—In ad-hoc networks, cooperative diversity is especially beneficial where the use of multiple antennas may be impractical. There has been a lot of work on improving the peer-to-peer link quality by using advanced coding or power and rate allocation between a single source node and its relays. However, how to fairly and efficiently allocate resources among multiple users and their relays is still unknown. In this paper, a multiuser cooperative protocol is proposed, where a power reward is adopted by each node to evaluate the power contributed to and by others. It will be shown that the proposed FAir cooperative Protocol (FAP) can significantly improve the fairness performance compared to full cooperation. It is further demonstrated that in energy-constrained cooperative ad-hoc networks, fairness can actually bring significant throughput gains. The tradeoff between fairness and throughput is analyzed and two price-aware protocols, FAP-R and FAP-S, will be further proposed to improve fairness. Simulation results will validate our analysis and show that compared to the direct transmission (i.e., without cooperation) and the full cooperation, our proposed FAP, FAP-R and FAP-S can achieve much better fairness performance along with substantial throughput gains.

Index Terms—Fairness, cooperative communications, multiuser diversity, energy-constrained networks, lifetime, ad-hoc networks.

I. INTRODUCTION

THE use of multiple antennas at both the transmitter and the receiver can provide significant capacity gains. Unfortunately, this could be impractical in ad-hoc wireless networks, in particular because of limitations on the size of a node or mobile unit. To address this problem, a new form of spatial diversity, in which diversity gains are achieved via cooperation among nodes, has been proposed. The main idea behind this approach, which is called *cooperative diversity*, is to form a virtual antenna array through the use of the relays' antennas to achieve diversity gain, without complicated signal design or requiring multiple antennas at each node.

Sendonaris *et al* first proposed the idea of cooperative diversity and applied it to CDMA cellular systems [1-2]. Laneman and Wornell extended this work and presented several cooperative protocols, including amplify-and-forward, decode-and-forward, selection relaying, and space-time-coded cooperation [3-4]. Coding is further introduced into the cooperation in [5-6]. Other work includes a cooperative-region analysis for the coded cooperative protocol [7], space-time code design criteria for amplify-and-forward relay channels [8], information-theoretic achievable rate regions and bounds [9], and symbol error rate analysis for Rayleigh-fading channels with K amplifying relays [10].

Most of the existing work in cooperative diversity focuses on improving the peer-to-peer link quality in the single-user scenario by using coding or power and rate allocation. In ad-hoc networks, how to efficiently and fairly allocate resources among multiple users and their relays is still unknown. In particular, fairness is an important issue for resource allocation that has not been well addressed. Usually, a user may regard itself as unfairly treated if its throughput is much lower than others. In cooperative ad-hoc networks, the issue is more complicated since unfairness would exist even if all the users achieve a similar throughput. For instance, if some node always acts as a relay but its own throughput is not improved accordingly, it may simply refuse to cooperate. In sensor networks, this means some nodes may consume their power very quickly, which could lead to routing failure and decreased network throughput.

Several cooperative protocols for medium-access control have been proposed in [4]. These symmetric and fixed protocols require that a group of users relay the signals for each other. In cellular networks, where all users transmit to the same destination (the base station), fairness and efficiency can be achieved simultaneously, for example, by carefully grouping the users with similar channel gains. However, in ad-hoc networks, each node may transmit to a different destination. So each node should have its own relay set in order to improve the spectral efficiency. As a result, there will probably be some nodes that have more opportunities to act as relays and an unfair situation could then occur.

In energy-constrained cooperative ad-hoc networks, each node is associated with an energy constraint, E. It is clear that the nodes that act as relays more often will run out of energy much faster, and therefore suffer from a shorter lifetime. To address the fairness issue in energy-constrained cooperative ad-hoc networks, in this paper, we define that fairness is

achieved if all the nodes have *equal lifetime*. This guarantees that the effort expended on each node is fair [11], i.e., each node is allocated equal energy and lasts for an equal period of time. The ratio of the minimum and maximum lifetimes of the nodes in the network is adopted as an indicator for the fairness performance. This ratio is desired to be one, corresponding to the case when all the nodes have equal lifetime. A small ratio indicates that severe unfairness occurs.

To perform fair resource allocation in energy-constrained cooperative ad-hoc networks, a novel multiuser cooperative protocol, the FAir cooperative Protocol (FAP), is proposed in this paper, in which a *power reward* is adopted by each node to evaluate the power contributed to and by others. In particular, each node has to *pay* for cooperative transmission by subtracting the amount of transmission power contributed by its relays from its power reward. On the other hand, each node can also boost its power reward by helping others. Node cooperation can be performed only if the source node's power reward is large enough to cover the power required by its relays. By doing so, fairness can be approached in the following two aspects: 1) with the use of power reward, nodes cannot continuously employ relays. As a result, it is very unlikely that any node can occupy the channel for a long time; and 2) if some node frequently contributes to the other nodes' transmissions, it will have a larger power reward and as such it will have more chances to transmit using relays. As a result, it is very unlikely that any node will be over-utilized as a relay. Our analysis will show that the fairness indicator of the proposed FAP is close to 1. In contrast, for full cooperation (i.e., when cooperation is always adopted among nodes), the fairness indicator is much lower than 1, indicating a severely unfair condition.

Fairness and efficiency are two crucial issues in resource allocation. Spectral efficiency is evaluated in terms of the aggregate throughput, which is sometimes unfair to those users with poor channel conditions. On the other hand, absolute fairness requires resources to be allocated to those poor users, which may lead to low spectral efficiency. As a result, there is usually a tradeoff between efficiency and fairness. Somewhat surprisingly, as we will show in this paper, improved fairness may actually result in significant throughput gains in energy-constrained cooperative ad-hoc networks. With unfair protocols, some nodes will run out of energy rapidly. This implies that the number of available relay nodes will decrease quickly, which leads to lower throughput and higher transmission power for each node. Therefore, it is expected that a higher aggregate throughput may be achieved if all nodes run out of energy simultaneously.

We shall present an analytical framework in which the relationship between the fairness indicator and the aggregate throughput in energy-constrained cooperative networks is characterized. It will be demonstrated that an improvement in fairness achieved by the proposed FAP can lead to substantial throughput gains over the direct transmission and the full cooperation cases. Based on the tradeoff characterization, two price-aware cooperative protocols, namely, FAP-R and FAP-S, will be further proposed to illustrate how to steer the tradeoff between fairness and throughput. In these protocols, the residual energy information of each node is exploited to

reshape the relay set or to adjust the scheduling. It will be shown that although reshaping the relay set according to the residual energy information can achieve better improvement in fairness, it suffers from some throughput loss. This is because the aggregate throughput is more sensitive to the reduced cooperative diversity gain than to the multiuser diversity gain. Simulation results will validate our analysis and show that substantial throughput gains can be achieved by the proposed price-aware cooperative protocols over the direct transmission and the full cooperation.

A number of key assumptions are made in this paper: 1) Opportunistic transmission [12] is adopted to schedule the source-destination (s-d) pairs, i.e., the s-d pair with the highest throughput is selected for transmission at each time slot. The proposed framework, however, is applicable to other access schemes, such as random access; 2) Effort-based fairness is a central concern throughout the paper. Nevertheless, it will be demonstrated that with the proposed FAP protocol, most of the nodes can achieve throughput gains from cooperation in a fair way, indicating that the outcome-based fairness performance is also greatly improved.

Note that the use of pricing to stimulate cooperation in wireless ad-hoc networks has been extensively investigated in recent years (see [13-14] and references therein). A central focus of these studies is the optimization of the *price* charged by each node to reach the system equilibrium point. In contrast, this paper aims at the fairness performance analysis of various MAC protocols in energy-constrained cooperative ad-hoc networks. Here the price is set to be the transmission power contributed by the relays, and the node cooperation is performed at the physical layer, instead of the application layer.

This paper is organized as follows. In Section II, we provide our system model. FAP is proposed in Section III and an analytical framework is presented in Section IV where the fairness performance of FAP is compared to that of the direct transmission and the Full Cooperative Protocol. The tradeoff between throughput and fairness is analyzed and two price-aware cooperative protocols are further proposed. Simulation results are given in Section V. Finally, Section VI summarizes and concludes this paper.

II. SYSTEM MODEL AND FULL COOPERATIVE PROTOCOL

We consider an ad-hoc network with K stationary nodes and assume that each node is equipped with only one antenna. All nodes are associated with an energy constraint, denoted by E. In this paper we assume that the energy consumed in the transmission mode is the dominant source of energy consumption. We also assume that the channel is time-invariant over one time slot but changes over different time slots. Let T denote the length of one time slot. Assume a flat fading channel between any node i and node j with the channel gain q_{ij} and the variance of the additive white Gaussian noise is N_0, where q_{ij} is assumed to be a complex Gaussian random variable with zero-mean and variance σ_{ij}^2. σ_{ij}^2 accounts for the effect of large-scale path loss and shadowing. In this paper, we neglect the effect of shadowing and hence $\sigma_{ij}^2 = u d_{ij}^{-\alpha}$, where d_{ij} is the distance between node i and node j and α is

the path loss exponent. The constant u accounts for all of the other attenuation factors and u is set to be 1 here without loss of generality. For any s-d pair $(k, D(k))$, assume that power control is available at the source node k so that the effect of path loss can be overcome by letting the transmission power $P = P_0 d_{k,D(k)}^{\alpha}$, where $D(k)$ denotes the destination node and P_0 is the required average received power at the destination node in each time slot.

With user cooperation, each source node may employ some nodes to serve as relays. Each cooperative transmission will be assumed to occur over two sub timeslots, where the source node transmits the data packet to the relays in the first sub timeslot and the relays decode and forward the packet with the source node to the destination node in the second sub timeslot. The source-relay (s-r) channels should be good enough compared to the s-d channel so as to avoid severe error propagation. In ad-hoc networks, each node may have different relay sets when it transmits to different destinations. Therefore, the fixed multiuser cooperative protocols proposed in [4] will not work in this case. In this paper, we define a *relay region* $\mathcal{R}_k$ with a radius of R_k for any s-d pair $(k, D(k))$. As shown in Fig. 1, the nodes located inside the relay region $\mathcal{R}_k$ can be regarded as the relays for source node k, i.e., k has a relay set $\mathbf{R}_k = \{j: d_{kj} \leq R_k\}$. In particular, we assume that the distance between the source and destination for pair $(k, D(k))$ is $d_{k,D(k)}$. Then, the ratio of R_k and $d_{k,D(k)}$ should satisfy

$$\varphi = \frac{R_k}{d_{k,D(k)}} = \left(\frac{\eta}{\beta}\right)^{1/\alpha}, \tag{1}$$

where $\eta = P_{sr}/P_{sd}$ is the ratio of the transmission power for the first sub timeslot to the transmission power for the second sub timeslot, with $(P_{sr}+P_{sd})/2 = P$. In this paper, equal transmission power is assumed to be allocated in the two sub timeslots, i.e., $\eta = 1$. β is the required average error probability ratio of the s-d channel to the s-r channel. For a large β ($\beta = 100$, for instance), the s-r channels will have a much lower error probability than the s-d channel so that they can be approximately regarded as error-free relative to the s-d channel (most of the errors come from the s-d channel). Therefore, the relay region $\mathcal{R}_k$ of the s-d pair $(k, D(k))$ should be a circular area with a radius $R_k = d_{k,D(k)}(1/\beta)^{1/\alpha}$. It is clear that the relay region for an s-d pair with a large distance, $d_{k,D(k)}$, will be large. Therefore, more relays are available to contribute to the transmission.

Opportunistic transmission is adopted in this paper to schedule different s-d pairs [12]. Suppose that node cooperation is always adopted, and the source node and its relays use beamforming to transmit to the destination node. With decode-and-forward, the throughput of s-d pair $(k, D(k))$ is given by

$$c_{k,D(k)} = \frac{1}{2}\log_2\left(1 + \rho(|h_{k,D(k)}|^2 + \sum_{i \in \mathbf{R}_k}|h_{i,D(k)}|^2)\right), \tag{2}$$

where $\rho = P_0/N_0$ is the required receive SNR and $h_{i,j}$ represents the small-scale channel gain between node i and j. The s-d pair with the highest throughput is selected for
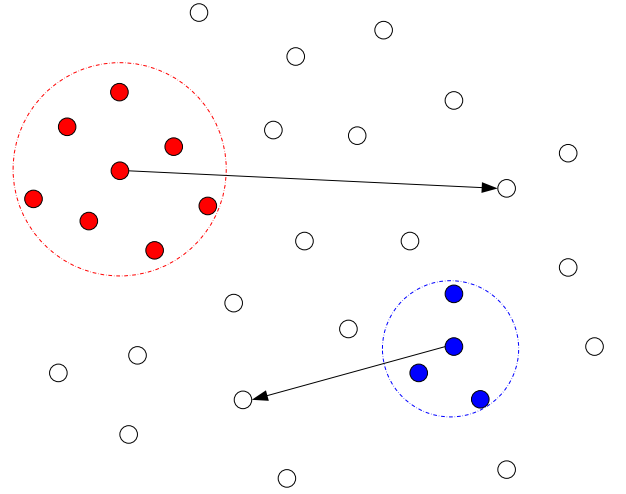

Fig. 1. Each node should have its own relay region in cooperative ad-hoc networks.

transmission. This is referred to as *Full Cooperative Protocol* and is described below.

Algorithm 1 Full Cooperative Protocol

1: For each s-d pair $(k, D(k))$, compute its transmission rate request according to (2).
2: Compare the rate requests of all the s-d pairs and select the one with the maximum rate: $(k^*, D(k^*)) = \operatorname{argmax}_{(k,D(k))} c_{k,D(k)}$.

III. FAIR COOPERATIVE PROTOCOL

In energy-constrained cooperative ad-hoc networks, nodes may have quite disparate lifetimes if node cooperation is always adopted. In particular, with the Full Cooperative Protocol, nodes with more relays will have higher access probability. As a result, some nodes may keep occupying the channel and run out of energy very quickly. In addition, there are always some nodes that have greater chances to act as relays (those that are located in the central area of the network, for instance). Their power will then be used up much faster than the others.

To improve the fairness performance, a novel cooperative protocol will be proposed in this section. We define a *Power Reward* W_k for any node k, $k=1,\ldots, K$. At each transmission, W_k will increase if node k acts as a relay. That is,

$$W_k \rightarrow W_k + P_k^j, \tag{3}$$

where P_k^j is the transmission power of node k when node k acts as a relay for node j. W_k will decrease if node k employs the other nodes as relays:

$$W_k \rightarrow W_k - \Psi_k, \tag{4}$$

where $\Psi_k = \sum_{j \in \mathbf{R}_k} P_j^k$, and P_j^k is the transmission power of node j if j acts as a relay for node k.

For each s-d pair $(k, D(k))$, source node k will compute its transmission rate request according to its power reward W_k before competing for the time slot. W_k indicates whether node k should use cooperation or not. Node cooperation is adopted only if W_k is larger than the total required power of node k's relays. This cooperative protocol shall be referred to as the *FAir cooperative Protocol* (*FAP*) and is described as follows.

Algorithm 2 FAP

1: For each pair $(k, D(k))$, compare W_k and the total required power of relays Ψ_k:
   If $W_k > \Psi_k$, compute the transmission rate request according to (2).
   Else, compute the transmission rate request as:

$$c_{k,D(k)} = \frac{1}{2}\log_2(1 + \rho\|h_{k,D(k)}\|^2). \qquad (5)$$

2: Compare the rate requests of all the s-d pairs and select the optimal one $(k^*, D(k^*))$.
3: Update the power reward of source node k^* and its relays $i \in \mathbf{R}_{k^*}$, using (3) and (4).

It is clear that nodes cannot continuously employ relays with the use of power reward. In addition, if a node frequently contributes to other nodes' transmissions, it will have a larger power reward so that it can afford more transmissions using relays. Note that the transmission rate request of a source node with cooperation is usually much higher than that without cooperation. As a result, with the proposed FAP, it is very unlikely that any node would occupy the channel, or, act as a relay, for a long time. The energy of all the nodes would decrease at a similar rate, indicating that, as we will show in Section IV, the fairness indicator of FAP is close to 1.

It can be also seen from (3-4) that for any node k, $k=1,\ldots,$ K, we should have

$$\sum_{j=1,\ldots,K,j\neq k} P_j^{(k)} \leq \sum_{j=1,\ldots,K,j\neq k} P_k^{(j)}, \qquad (6)$$

where $P_j^{(k)}$ is the total power that node j contributes to node k during node k's lifetime, i.e., the total amount of transmission power of node j when it acts as node k's relays. It is clear that the left side of (6) is the total amount of power that the other nodes contribute to node k's transmission, and the right side of (6) is the total amount of power that node k contributes to the other nodes' transmission. "$\leq$" comes from the fact that the power reward W_k is always non-negative. (6) implies that with the proposed FAP, the benefit that a node enjoys from cooperation is bounded by the contributions of this node. In this way, no one would boost its throughput by *exploiting* the other nodes, or suffer from great throughput loss due to relaying. As we will show in Section V, compared to direct transmission, most of the nodes can achieve throughput gains from cooperation in a fair way, indicating that FAP can also greatly improve the fairness performance from the outcome aspect.

Note that the power reward is computed in a distributed way. Each node only needs to collect the transmission power information of its relay set when it transmits as a source node.

Otherwise, it updates its power reward according to its own transmission power contributed to relaying. By introducing a slight overhead, the fairness, however, can be improved significantly compared to the Full Cooperative Protocol. Moreover, in spite of the assumption of opportunistic transmission in this paper (an access point is assumed to be available for scheduling, which is feasible in Wireless Mesh Networks, for example), the idea of power reward can be easily applied to other MAC protocols such as random access. For instance, the average back-off window size can be adjusted according to the value of the power reward, so that the node with a high power reward will have a shorter back-off window size and then obtain a higher access probability.

IV. TRADEOFF BETWEEN FAIRNESS AND THROUGHPUT

In this section, we will present an analytical framework where fairness is evaluated by the ratio of the maximum lifetime and the minimum lifetime of the nodes in the network. The relationship between fairness and aggregate throughput will also be characterized, from which it can be clearly seen how the improvement in fairness turns into a throughput gain. Two additional protocols will be further proposed to illustrate how to achieve a good tradeoff between fairness and throughput in energy-constrained ad-hoc networks.

A. Fairness Indicator

Let T_k be the lifetime of node k, $k=1,\ldots,$ K. That is, node k runs out of energy at the T_k-th time slot. In this paper, we define that fairness is achieved if all the nodes have *equal lifetime*, i.e., $T_k = T_0$, $k=1,\ldots,$ K. The ratio of the maximum and minimum lifetimes of the nodes in the network is adopted as an indicator for the fairness performance:

$$\xi = T_{\min}/T_{\max}, \qquad (7)$$

where $T_{\max} = \max\{T_1, T_2, \ldots, T_K\}$ and $T_{\min} = \min\{T_1, T_2, \ldots, T_K\}$. ξ is desired to be one, corresponding to the case when the nodes have equal lifetimes. A small ξ indicates that severe unfairness occurs, as some nodes run out of energy very quickly.

In the following, we will evaluate the fairness performance of Direct Transmission (where no cooperation is adopted among the nodes), the Full Cooperative Protocol and the proposed FAP.

1) Direct Transmission: Assume that the distance of the source-destination pair is fixed to be d_0. The transmission power is then $P_0 d_0^\alpha$. Therefore, the total time slots in which node k, $k=1,\ldots,$ K, can actively transmit are given by[1]

$$N_d = \lceil P_t/(P_0 d_0^\alpha) \rceil. \qquad (8)$$

where $P_t = E/T$ is the total power of each node. The maximum lifetime is clearly given by $T_{\max}^d = k N_d$. The minimum lifetime, $T_{\min}^d$, is presented in the following theorem.

Theorem 1: The minimum lifetime of Direct Transmission is given by

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[1] P_0 is the required received power at the destination node. When the residual power is lower than $P_0 d_0^\alpha$, an outage event occurs. The throughput in this time slot will not be counted accordingly.

$$T_{\min}^d = \sum_{x=N_d}^{K(N_d-1)+1} x \cdot P_r\left[X = x\right], \qquad (9)$$

where

$$P_r\left[X = x\right] = K \begin{pmatrix} x-1 \\ N_d-1 \end{pmatrix} \left[\frac{1}{K}\right]^x \sum_{l=1}^{L} \prod_{i=1}^{z_l}$$

$$\left[K - 1 - \sum_{j=1}^{i-1} k_j^l\right] \begin{pmatrix} x - N_d - \sum_{j=1}^{i-1} b_j^l \\ b_i^l \end{pmatrix} / \left(k_i^l!\right), \quad (10)$$

with $\{k_j^l\}$ and $\{b_j^l\}$ denoting the possible positive integers which satisfy $\sum_{j=1}^{z_l} k_j^l b_j^l = x - N_d$ and $b_1^l < b_2^l < ... < b_{z_l}^l$.

Proof: See Appendix I.

Theorem 1 provides the exact expression of $T_{\min}^d$. However, the computational complexity increases exponentially with the total number of nodes *K*. Fortunately, with a large *K*, *X* can be approximated by $X= \min\{X_1, X_2, ..., X_K\}$, where X_i is an i.i.d. negative binomial distributed random variable with *probability density function* (pdf)

$$p_{N-bi}(x - N_d, N_d, 1/K) = \begin{pmatrix} x-1 \\ N_d-1 \end{pmatrix} \left(\frac{1}{K}\right)^{N_d}$$
$$\left(1 - \frac{1}{K}\right)^{x-N_d}. \quad (11)$$

As a result,

$$P_r\left[X = x\right] = Kp_{N-bi}(x - N_d, N_d, 1/K)\cdot$$
$$(1 - P_{N-bi}(x - N_d, N_d, 1/K))^{K-1}, \qquad (12)$$

where $P_{N-bi}(.)$ is the *cumulative distribution function* (cdf) of a negative binomial distributed variable. Fig. 2 shows the values of the minimum lifetime $T_{\min}^d$ computed via Theorem 1, as well as the approximation. A perfect match can be observed when the number of nodes *K* is large.

From (11-12), we have $E[X_i]/K < T_{\min}^d < E[X_i]$, where $E[X_i]$ is the expected value of X_i and is given by $E[X_i]=N_d(K-1)$. In energy-constrained systems, N_d is usually small and $T_{\min}^d$ can be approximated by $T_{\min}^d \approx \Theta(K^v), v<1$.[2] As shown in Fig. 2, when N_d =2, $T_{\min}^d$ increases with *K* at a rate of $K^{0.55}$. The lifetime ratio of Direct Transmission, ξ_d, then varies as $(1/K)^{0.45}$, which will be quite small when there is a large number of nodes in the network. This indicates that Direct Transmission will lead to severe unfairness as the number of nodes increases. Nevertheless, *v* approaches 1 with a large N_d, implying that the fairness performance of Direct Transmission can be improved by enhancing the battery of each node.

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[2]$f(x) = \Theta(g(x))$ means that $0 < \lim_{x\to\infty} \inf |f(x)/g(x)| \leq \lim_{x\to\infty} \sup |f(x)/g(x)| < \infty$.
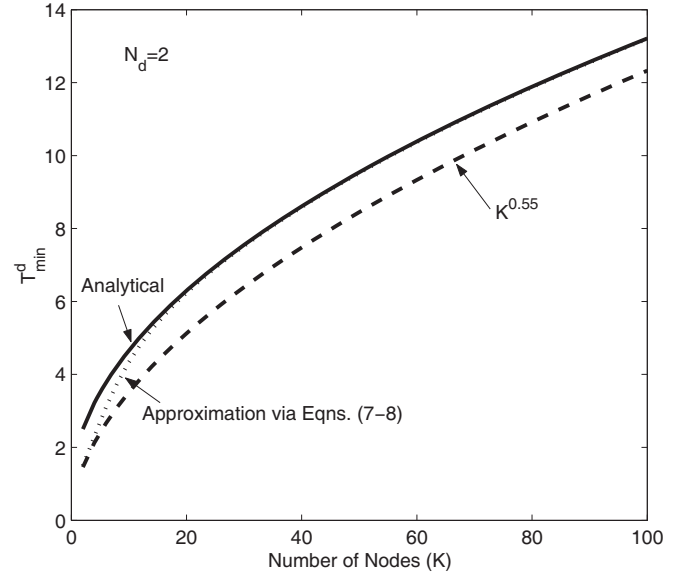

Fig. 2. The minimum lifetime $T_{\min}^d$ vs. the number of nodes *K* with $N_d = 2$. A perfect match can be observed between the analytical value (solid line) and the approximation (dotted line). Besides, $T_{\min}^d$ increases as $K^{0.55}$ (dashed line)

2) Full Cooperative Protocol: Under this protocol, node cooperation is always adopted. In particular, for any s-d pair (*i*, *D*(*i*)), M_i relay nodes located in the relay region will assist the transmission. The total transmission power at each time slot is still $P_0 d_0^\alpha$. Therefore, the maximum lifetime $T_{\max}^f$ is given by

$$T_{\max}^f = \lceil KP_t/(P_0 d_0^\alpha) \rceil. \qquad (13)$$

To obtain the minimum lifetime $T_{\min}^f$, we focus on the symmetrical case where all the nodes have the same number of relays. That is, $M_k = M = K \left(\frac{d_0}{\beta^{1/\alpha} R_0}\right)^2$, k=1,..., *K*, where R_0 is the radius of the network. If we represent $T_{\min}^d$ as a function, $g(K, N_d)$, of *K* and N_d, then the upper and lower bounds of $T_{\min}^f$ are given by Theorem 2.

Theorem 2: The minimum lifetime of the Full Cooperative Protocol is bounded by

$$g\left(K, MN_d\right)/M \leq T_{\min}^f \leq g\left(K, 2N_d\right). \qquad (14)$$

Proof: See Appendix II.

It is found that $T_{\min}^d$ is a convex function of N_d when *K* is given. Therefore, we have $g\left(K, MN_d\right)/M \geq g\left(K, N_d\right) = T_{\min}^d$. In other words, $T_{\min}^f$ is always larger than $T_{\min}^d$. Hence, we can conclude that compared to Direct Transmission, the Full Cooperative Protocol can always achieve a higher ξ_f which indicates better fairness. Nevertheless, ξ_f is upper bounded by $g(K,2N_d)/KN_d$ which is scaled by K^{v-1}, *v* <1, with a small N_d. Therefore, it is still quite low when the total number of nodes *K* is large.

3) FAP: Similar to the Full Cooperative Protocol, the maximal lifetime of FAP, $T_{\max}^a$, is given by $T_{\max}^a = \lceil KP_t/(P_0 d_0^\alpha) \rceil$. It is quite difficult to obtain the exact expression of $T_{\min}^a$. Therefore, we again resort to upper and lower bounds. First consider the worst case, where we assume that

the network can be decoupled into K/M independent clusters and that in each cluster the nodes act as relays for each other. The time slots are always allocated to Cluster 1 until all the nodes in Cluster 1 run out of energy. By using the power reward, the nodes in Cluster 1 will alternately access the channel until they run out of energy at the MN_d th time slot.[3] This is the worst case since, in each time slot, every node of Cluster 1 always transmits either as the source node or as a relay node. Therefore, MN_d is a lower bound on $T_{\min}^a$. When $K=M$, $T_{\min}^a = KN_d$, will be the upper bound and thus

$$MN_d \le T_{\min}^a \le KN_d. \qquad (15)$$

It can be seen from (15) that in contrast to ξ_d and ξ_f which are approximately scaled by K^{v-1}, ξ_a is lower bounded by $M/K = \left(\frac{d_0}{\beta^{1/\alpha}R_0}\right)^2$. It is clear that ξ_a can be improved significantly with a larger s-d distance d_0 or a decrease in β (both of which allow more relays), irrespective of whether the number of nodes K is large or not. This also implies that compared to Direct Transmission and the Full Cooperative Protocol (where ξ_d and ξ_f decreases as K increases), much better fairness performance can be achieved by FAP with a large number of nodes K.

It should be noted that (15) is a rough bound which is based on the observation that on average a node always requires M time slots to accumulate enough power reward. Due to the effect of small-scale fading, a node with a good channel may need less than M time slots to be able to afford cooperation again. Nevertheless, (15) does shed some light on how $T_{\min}^a$ can be improved.

B. Aggregate Throughput

Let $\mathcal{K}(t)$ denote the total number of nodes at time slot t. According to the definitions of $T_{\max}$ and $T_{\min}$, we know that $\mathcal{K}(t)$ remains to be K at time slot $t<T_{\min}$, and then starts dropping and finally becomes 0 at time slot $t=T_{\max}$. Suppose that $\mathcal{K}(t)$ linearly decreases with t during $t \in [T_{\min}, T_{\max}]$. $\mathcal{K}(t)$ can be then written as

$$\mathcal{K}(t) = \begin{cases} K, & t < \xi T_{\max}, \\ K(T_{\max}-t)/(T_{\max}(1-\xi)), & \xi T_{\max} \le t \le T_{\max}. \end{cases} \qquad (16)$$

It can be clearly seen from (16) that the fairness indicator ξ determines the dropping rate of $\mathcal{K}(t)$. At any give time slot t, a smaller fairness indicator ξ implies a smaller number of nodes. Since opportunistic transmission is adopted, the aggregate throughput is dependent on the multiuser diversity gain which will decrease with the number of nodes. In addition, the reduction in the number of available relay nodes will further lead to a lower throughput and a higher transmission power for each node. As a result, it can be expected that a higher aggregate throughput can be achieved with a larger ξ, which corresponds to better fairness performance. Nevertheless, the improvement in fairness is usually obtained at the cost of sacrificing the performance of users with good channels which, on the other hand, will impair the aggregate throughput. In

what follows, we show how these positive and negative factors affect the throughput performance.

Consider a symmetrical network topology where, at any time slot t, the number of relay nodes for each source node is equal to $\mathcal{M}(t)$, which is given by

$$\mathcal{M}(t) = b(t) \left(\frac{d_0}{\beta^{1/\alpha}R_0}\right) \mathcal{K}(t). \qquad (17)$$

Here $0 \le b(t) \le 1$ is determined by the relaying strategy. For the Full Cooperative Protocol and the proposed FAP, all the nodes located in the relay region are included in the relay set, which means that $b(t)=1$, $t=1,\dots, T_{\max}$. However, in some cases we may only choose a sub-set of the nodes in the relay region as relays. For instance, we can reshape the relay set according to the residual power information. In particular, the nodes with low residual power will be excluded from the relay set even if they are located inside the relay region. In that case, $b(t)$ will be less than 1.

Suppose that there are $\mathcal{N}(t)$ nodes that are able to use cooperation at time slot t. $\mathcal{N}(t)$ can be written as

$$\mathcal{N}(t) = a(t)\mathcal{K}(t), \qquad (18)$$

where $0 \le a(t) \le 1$ describes the proportion of nodes competing for the channel. Note that $a(t)=1$, $t=1,\dots, T_{\max}$, for the Full Cooperative Protocol and $a(t)<1$ for the proposed FAP.

With opportunistic transmission, the throughput at time slot t is given by

$$c(t) = \max_{i=1,\dots,\mathcal{N}(t)} \frac{1}{2} \log_2 (1 + \rho\lambda_i(t)), \qquad (19)$$

where $\lambda_i(t)$ has a chi-squared distribution with dimension $2(\mathcal{M}(t)+1)$. According to [15], the aggregate throughput can finally be written as

$$C = \frac{1}{2} \int_0^{T_{\max}} \int_0^{\infty} \mathcal{N}(t) \log_2 (1 + \rho x) f(x) [F(x)]^{\mathcal{N}(t)-1} \, dx dt, \qquad (20)$$

where $f(.)$ and $F(.)$ denote the pdf and cdf functions, which are given by

$$f(x) = \frac{x^{\mathcal{M}(t)} e^{-x/2}}{2^{(\mathcal{M}(t)+1)} \Gamma(\mathcal{M}(t)+1)}, \qquad (21)$$

$$F(x) = \int_0^x f(\tau) d\tau. \qquad (22)$$

The aggregate throughput of an energy-constrained co-operative ad-hoc network with opportunistic transmission is presented in the following theorem.

Theorem 3: The aggregate throughput of an energy-constrained cooperative ad-hoc network with opportunistic transmission is given by

$$C = \mu_1 \int_0^{T_{\max}} \log_2 a(t)dt + \int_0^{T_{\max}} \log_2 b(t)dt + \xi T_{\max}$$
$$+ (\log_2 K - 1) T_{\max} + v T_{\max}, \qquad (23)$$

where $\mu_1 << 1$ and $v = \log_2 \left(\frac{d_0}{\beta^{1/\alpha}R_0}\right) + \mu_1 \log_2 \varsigma_1 + \log_2 \varsigma_2$, ς_1, ς_2 are scaling coefficients.

---

[3]Here it is assumed that the number of relay nodes is large enough that the throughput with cooperation is always higher than that without cooperation.

Proof: See Appendix III.

Theorem 3 provides the fundamental tradeoff between throughput and fairness. Clearly with an increase in the fairness indicator ξ, substantial throughput gains can be achieved through the third term on the right side of (23), $\psi_1 = \xi T_{\max}$. However, improved fairness is obtained at a cost expressed in the first two terms in (23), which are always non-positive. Let

$$\psi_2 = \mu_1 \int_0^{T_{\max}} \log_2 a(t) dt + \int_0^{T_{\max}} \log_2 b(t) dt. \quad (24)$$

By comparing ψ_1 and ψ_2, we can see whether fairness improves or impairs the aggregate throughput. For instance, with FAP, we have $b(t)=1$ and $1 \geq a(t) \geq 1/2$. Considering that $\mu_1 << 1$, it follows that

$$\psi_1 + \psi_2 \geq (\xi - \mu_1) T_{\max} > 0. \quad (25)$$

Recall that for the Full Cooperative Protocol, $\psi_1 + \psi_2 = 0$ ($a(t) = b(t) = 1$ and $\xi \approx 0$). Clearly a throughput gain is achieved by FAP resulting from the improvement in fairness. Simulation results presented in Section V will verify this and will show that FAP can provide a much higher aggregate throughput than the Full Cooperative Protocol.

Another important observation from Theorem 3 is that the aggregate throughput is more sensitive to the change of $b(t)$ than that of $a(t)$ because $\mu_1 << 1$. Recall that $b(t)$ and $a(t)$ are determined by the relaying strategy and the scheduling strategy, respectively. This indicates that although the fairness performance can be improved by either reshaping the relay set or revising the competition rule, they affect the throughput in different ways. The aggregate throughput is more sensitive to the decrease in cooperative diversity gain than the multiuser diversity gain. This will be further demonstrated based on the two examples presented in the following section.

C. FAP-R and FAP-S

We take two examples to illustrate how the aggregate throughput varies under different strategies for improving fairness. Here the residual power information of each node is further exploited. In particular, we propose to use *price* to reshape the relay set. Each node k charges a price, $\mathcal{P}_k$, which is a monotonically decreasing function of its residual power $\tilde{P}_k$, $k=1, \ldots, K$. For each source node, instead of using the entire relay set $\mathbf{R}_k$ (the nodes located inside the relay region $\mathcal{R}_k$), only the ones with an affordable price are selected. Let $\mathbf{R}_k'$ represent node k's new relay set. Clearly, $\mathbf{R}_k' \subseteq \mathbf{R}_k = \{j : d_{kj} \leq R_k\}$. The relay selection problem can be then formulated as

$$\max_{\mathbf{R}_k'} c_{\mathbf{R}_k'} = \frac{1}{2} \log_2 \left(1 + \rho \left(|h_{k,D(k)}|^2 + \sum_{j \in \mathbf{R}_k'} |h_{j,D(k)}|^2 \right) \right) \quad (26)$$

subject to $\sum_{j \in \mathbf{R}_k'} \mathcal{P}_j \leq \mathrm{P}_k$

where $\mathcal{P}_j$ is the price of the j-th node in relay set $\mathbf{R}_k'$ and P_k is the total price that node k can afford. There are numerous options for determining P_k and the price function. In this paper, we simply let $\mathcal{P}_j = -\tilde{P}_j$ and require that for any j-th node in relay set $\mathbf{R}_k'$, $\mathcal{P}_j \leq \bar{\mathcal{P}}/l$. Here, $\bar{\mathcal{P}}$ is the mean price of the whole network. Note that this is a stronger condition than the one given by (26). Parameter l should be carefully adjusted to achieve good fairness performance. With a small l, the variance of the nodes' residual power will decrease. However, the transmission power of each node will increase due to a smaller number of relays. A large l, on the other hand, can help to increase the size of the relay set while incurring a larger variance of the nodes' residual power. This clearly indicates uneven power consumption. Numerical results presented in Section V will show that an appropriate choice of this parameter l can significantly improve fairness. Note that in this paper, an exhaustive search is performed for the optimal value of l. An analytical approach, which is more desirable, is out of the scope of this paper and might be investigated in future work.

The *FAir cooperative Protocol with Reshaped relay set* (which is referred to as *FAP-R*) is described below.

Algorithm 3 FAP-R

1: For each source node k, select the relay set $\mathbf{R}_k' = \{j : \mathcal{P}_j \leq \bar{\mathcal{P}}/l, d_{kj} \leq R_k\}$.

2: Compare W_k and the total required power of the relays Ψ_k:
   If $W_k \geq \Psi_k$, compute the transmission rate request according to (2).
   Else, compute the transmission rate request according to (5).

3: Compare the rate requests of all the pairs and select the optimal one $(k^*, D(k^*))$.

4: Update the power reward of node k^* and its relays.

5: Update the price of node k^* and its relays, and broadcast the mean price of the whole network.

Compared to FAP, additional price information for each node needs to be exchanged here so as to further improve the fairness performance. This information update, however, is still distributive since each node can update its price information without knowing that of other nodes. Nevertheless, the broadcasting of the mean price of the whole network may incur some additional overhead.

Similarly, we can require that only the nodes with a lower price than $\bar{\mathcal{P}}/l$ can compete for the time slot. In this case, the resulting *FAir cooperative Protocol with adjusted Scheduling* (which is referred to as *FAP-S*) can be described as follows.

It is clear that both FAP-R and FAP-S are variants of the proposed FAP. To further improve the fairness performance of FAP, the residual power information is exploited to reshape the relay set or to revise the scheduling strategy in the cases of FAP-R and FAP-S, respectively. The simulation results presented in Section V will show that FAP-S leads to a higher aggregate throughput than FAP-R.

Algorithm 4 FAP-S

1: For each source node k, compare its price $\mathcal{P}_k$ and $\bar{\mathcal{P}}/l$:
   If $\mathcal{P}_k > \bar{\mathcal{P}}/l$, set the transmission rate request to zero.
   Else, compare W_k and the total required power of the relays Ψ_k:
   If $W_k \geq \Psi_k$, compute the transmission rate request according to (2).
   Else, compute the transmission rate request according to (5).
2: Compare the rate requests of all the pairs and select the optimal one $(k^*, D(k^*))$.
3: Update the power reward of node k^* and its relays.
4: Update the price of node k^* and its relays, and broadcast the mean price of the whole network.

V. SIMULATION RESULTS

Assume that K source nodes are uniformly distributed in a circular area with unit radius $R_0 = 1$. All the nodes always have packets to transmit in each time slot, and, for each transmission, the average received SNR ρ is 0 dB. Assume a unit noise power and a total power constraint $P_t = 150$ for each node.[4] Let the distance between any source node and its destination node be fixed as d_0. The required average error probability ratio of the s-d channel to the s-r channel, β, is 100 and the path loss exponent α is 4. The initial power reward of each node is given by W_0. With a small W_0, no nodes can afford cooperation and the throughput will be the same as that of Direct Transmission. On the other hand, a large W_0 will lead to full cooperation and the fairness cannot be guaranteed. In our simulations, W_0 is set to be $\rho \cdot d_0^\alpha / 2$.

The fairness performance of Direct Transmission, Full Cooperative Protocol and the proposed FAP are presented in Table I, with the total number of nodes K ranging from 20 to 250. As we have demonstrated in Section IV. A, the fairness indicators of the Direct Transmission and the Full Cooperative Protocol, ξ_d and ξ_f, will quickly decrease as the number of nodes K increases. In contrast, the proposed FAP can achieve a much better fairness performance. A closer observation of this table shows that when $K=250$ nodes, the fairness indicator of FAP, ξ_a, is 0.48, and is much higher than ξ_d and ξ_f which are close to zero.

From (15) we know that ξ_a of the proposed FAP is lower bounded by M/K, which implies that for a given total number of nodes K, better fairness performance can be achieved by increasing the number of relays M. As previously indicated in Section IV. A, there are two ways to enlarge the relay region: (1) increase the distance between the source node and the destination node d_0; or (2) decrease the required average error probability ratio of the s-d channel to the s-r channel β. Here, we consider the first case. Table II shows the values of ξ_f and ξ_a for the Full Cooperative Protocol and the proposed FAP for different values of d_0 when the number of nodes K is 250. It is clear that with an increase in d_0, a substantial improvement in ξ_a can be observed. For instance, when $d_0 = 6$ (where nearly all the nodes act as a relay for each other), ξ_a is found

[4]We omit the unit here since we only care about the signal-to-noise ratio.

TABLE II
FAIRNESS PERFORMANCE OF THE FULL COOPERATIVE PROTOCOL AND
FAP UNDER DIFFERENT VALUES OF d_0 $(K = 250)$

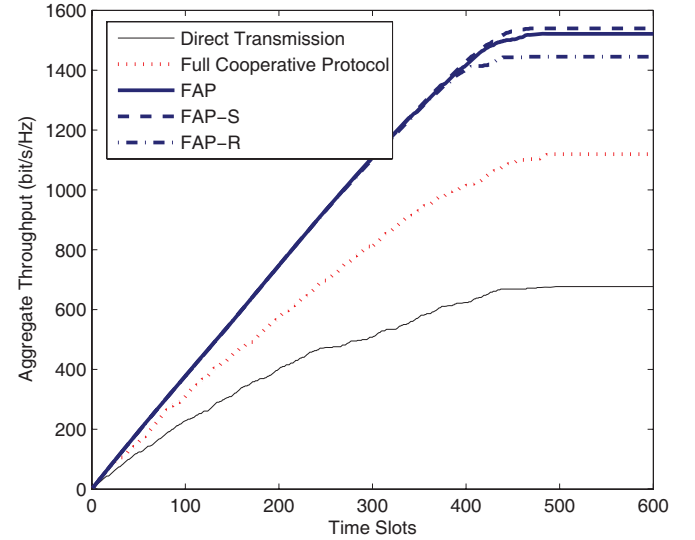|	$d_0 = 2$	$d_0 = 2.5$	$d_0 = 3$	$d_0 = 4$	$d_0 = 6$
Full Cooperative Protocol ξ_f	0.03	0.04	0.03	0.06	0.27
FAP ξ_a	0.32	0.45	0.48	0.66	0.87

Fig. 3. Throughput comparison of Direct Transmission, Full Cooperative Protocol, FAP, FAP-S with $l = 5$ and FAP-R with $l = 8$. $K = 250$ and $d_0 = 3$.

to be 0.87, which is quite close to 1. On the other hand, ξ_f of the Full Cooperative Protocol remains small regardless of the increase in d_0. It is noted that when d_0 changes from 4 to 6, ξ_f sharply increases from 0.05 to 0.2. This benefit actually comes from the decreasing variance of M_k instead of the increasing number of relays. In this case, nearly all the nodes have the same number of relays, i.e., $M_k \approx K$, $k=1,\ldots, K$. As a result, each node has an approximately equal probability to access the channel, and the fairness performance can be then greatly improved. Nevertheless, ξ_f is still much lower than ξ_a.

Fig. 3 further shows how the improvement on fairness leads to a throughput gain. During the first 50 time slots, the Full Cooperative Protocol can achieve nearly the same throughput as FAP. However, since some popular nodes run out of energy rapidly, the number of available relays keeps decreasing. That is why the rate of change on throughput declines with time. The aggregate throughput with the Full Cooperative Protocol will remain constant after about 500 time slots, which implies that all the nodes have run out of energy. In contrast, FAP can better schedule the node transmissions so that the number of relays remains nearly constant for a rather long period of time (say, nearly 250 slots). The throughput gain over the Full Cooperative Protocol keeps increasing with time, with an ultimate gain of about 30%. Thus we can see that both the Full Cooperative Protocol and FAP can achieve a significant throughput gain over Direct Transmission due to the cooperative diversity gain.

In FAP, we use power reward to balance the nodes' transmission so that no node can keep accessing the channel. However, some "popular" nodes (which have a large relay set

TABLE I
FAIRNESS COMPARISON OF DIRECT TRANSMISSION, THE FULL COOPERATIVE PROTOCOL AND THE PROPOSED FAP ($d_0 = 3$)

	$K = 20$	$K = 50$	$K = 100$	$K = 150$	$K = 200$	$K = 250$
Direct Transmission ξ_d	0.2	0.11	0.08	0.06	0.05	0.03
Full Cooperative Protocol ξ_f	0.16	0.14	0.07	0.04	0.04	0.04
FAP ξ_a	0.31	0.34	0.35	0.41	0.48	0.48

and hence they would have more chances to act as relays) still suffer from a shorter lifetime compared to others. A power reward can help to prevent these nodes from continuing to occupy the time slot. Nevertheless, compared to the other nodes, it is still easier for them to earn enough power reward (by acting as relays for other nodes) for transmission. As a result, they have more chances to transmit, which leads to a faster power consumption. In the FAP-R and FAP-S protocols proposed in Section IV. C, the residual power information of the nodes is further exploited to improve fairness which is limited by the lifetime of the popular nodes. Table III shows the fairness and throughput gains achieved by FAP-R and FAP-S over FAP. It can be clearly seen that both FAP-R and FAP-S can bring better fairness performance than FAP. A closer observation of this table also shows that a higher gain in fairness can be achieved by FAP-R. This is because every node always has a better chance to transmit as a relay than as a source node. By using the power reward, on average, a node with M relays needs to act as a relay M times before it can transmit as a source again. Therefore, it will be more effective to modify the relay selection strategy than the scheduling strategy. From Table III, it can be seen that when *l*=8, a 37% gain in fairness can be achieved by FAP-R.

The improvement in fairness, however, does not necessarily turn into a throughput gain. As Table III shows, the throughput of FAP-R is always lower than that of FAP, which implies that the throughput gain provided by the third term in (23) is less than the loss incurred by the second term. On the other hand, a slight throughput gain can be achieved by FAP-S. As indicated in Theorem 3, the aggregate throughput is more sensitive to the decrease in the cooperative diversity gain than in the multiuser diversity gain. Therefore, FAP-S can obtain a much higher throughput than the FAP-R, although a better fairness performance can be achieved by FAP-R.

Fig. 4 shows how the number of nodes changes with time when Direct Transmission, Full Cooperative Protocol, FAP, FAP-R (*l*=8) and FAP-S (*l*=5) are adopted. Clearly both the Direct Transmission and the Full Cooperative Protocol suffer from a small ξ. Some nodes run out of energy very rapidly, and the number of nodes decreases almost linearly with time. In contrast, with FAP and FAP-S, no nodes run out of energy in the first 250 time slots, indicating a significant improvement in fairness. FAP-R can achieve the best fairness performance. This improvement, however, is obtained at the cost of sacrificing throughput. As Fig. 3 shows, FAP-R has a lower aggregate throughput than both FAP and FAP-S. Nevertheless, substantial throughput gains can be observed over the Direct Transmission and the Full Cooperative Protocol. FAP and FAP-S achieve the highest aggregate throughput.

Figs. 5 and 6 further presents the throughput performance of each single node. Here the x-axis is the throughput gain
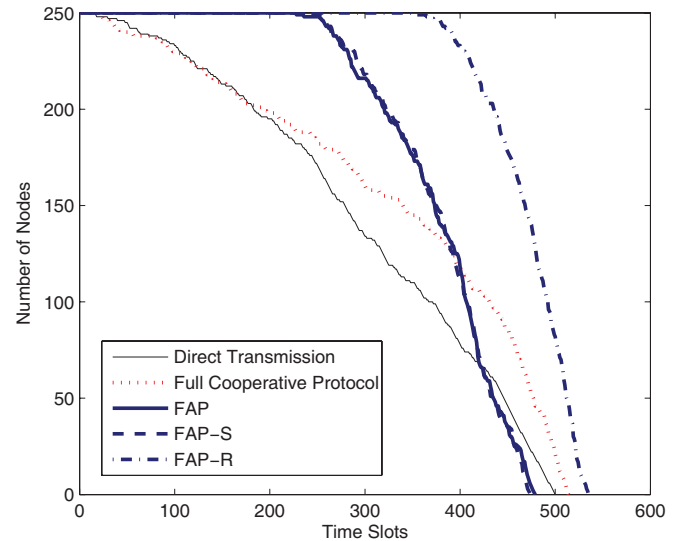

Fig. 4. Lifetime comparison of Direct Transmission, Full Cooperative Protocol, FAP, FAP-S with $l = 5$ and FAP-R with $l = 8$. $K = 250$ and $d_0 = 3$.

per node achieved by the Full Cooperative Protocol and the proposed FAP compared to Direct Transmission. It can be clearly seen that with full cooperation, almost half of the nodes suffer from throughput loss due to relaying. Some of them spend all the energy helping others so that their own throughputs are close to zero. In contrast, with FAP, over 80% of the nodes can benefit from cooperation. Most of them enjoy a throughput gain varying from 20% to 50%. By using power reward, the proposed FAP can not only enhance the minimum lifetime, but also decrease the throughput gap among all the nodes. It improves the fairness performance in both the effort and the outcome aspects. This is consistent with [16], where it was shown that an equal throughput gain per node can be approached by performing *energy fairness*, i.e., the energy a node contributes to others is equal to what the others contribute to this node. The proposed FAP can be regarded as a distributed implementation of the optimal resource allocation presented in [16].

VI. CONCLUSION

In this paper, we addressed the fairness issue in energy-constrained cooperative ad-hoc networks. We demonstrated that nodes will suffer from disparate lifetimes if full node cooperation is adopted. Some nodes may be over-utilized as relays and run out of energy quite fast, indicating a severe unfair resource allocation. To improve the fairness performance, we proposed a novel multiuser cooperative protocol, FAP, where a power reward is adopted by each node to evaluate the power contributed to and by others. Compared to the Direct Transmission and the Full Cooperative Protocol, the proposed

TABLE III
FAIRNESS GAIN AND THROUGHPUT GAIN ACHIEVED BY FAP-R AND FAP-S OVER FAP ($K = 250, d_0 = 3$)

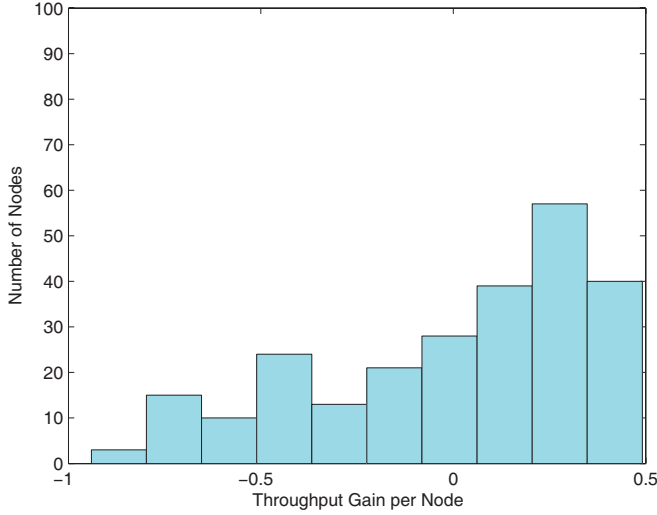|		$l=2$	$l=5$	$l=8$	$l=15$	$l=20$	$l=30$
FAP-S	$(\xi_{FAP-S} - \xi_a)/\xi_a$	0.02%	6%	2%	4%	5%	0.1%
	$(C_{FAP-S} - C_a)/C_a$	0.6%	1%	0.1%	0.4%	0.5%	0.2%
FAP-R	$(\xi_{FAP-R} - \xi_a)/\xi_a$	1%	16%	37%	25%	22%	15%
	$(C_{FAP-R} - C_a)/C_a$	−14%	−7%	−4%	−4%	−3%	−3%

Fig. 5.   Throughput gain per node achieved by Full Cooperative Protocol over Direct Transmission. $K = 250$ and $d_0 = 3$.
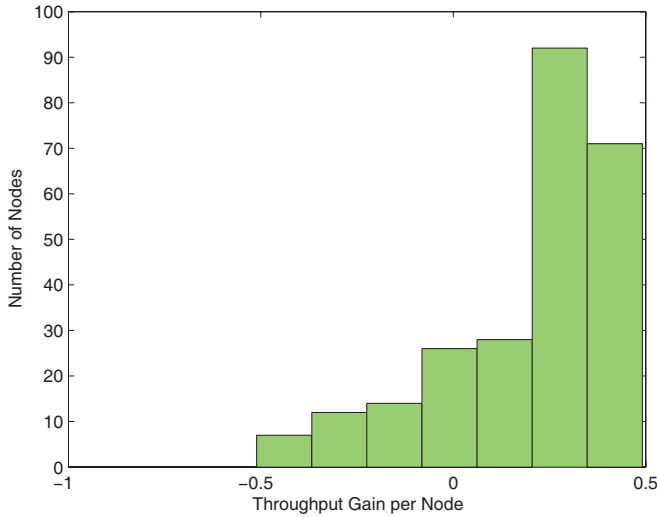

Fig. 6.   Throughput gain per node achieved by FAP over Direct Transmission. $K = 250$ and $d_0 = 3$.

FAP can significantly enhance both the fairness indicator and the aggregate throughput. We further analyzed the relationship between fairness and throughput, and proposed two price-aware protocols, FAP-R and FAP-S, based on the tradeoff characterization. We showed that although FAP-R can achieve a better fairness performance, its throughput is always lower than that of FAP-S, because the aggregate throughput is more sensitive to the reduced cooperative diversity than the multiuser diversity gain.

It should be noted that although opportunistic transmission is adopted in this paper, the idea of power reward can be

applied to other multiple access schemes. The performance analysis in a random-access-based energy-constrained ad-hoc network, including the fairness evaluation and the tradeoff characterization of fairness and throughput, is of potential interest. Another interesting extension of this work is to combine the routing design. For instance, the power reward information may be further taken into consideration when determining the optimal path [17-18].

APPENDIX A
PROOF OF THEOREM 1

Proof: $T^d_{\min}$ is the number of time slots that it takes for any node to run out of energy. With opportunistic transmission, the time slots are allocated to each node with equal probability. Therefore, the solution for $T^d_{\min}$ can be restated as a combinatorics problem, namely, a ball drawing problem which is described as: A bag contains K balls and at each time a single ball is drawn with replacement. How many drawings X are needed until any ball is drawn N_d times?

To compute the probability of $\Pr[X=x]$, we have to get all possible integer sequences $\{a^l_i\}$ which satisfy

$$
\begin{cases}
\sum_{i=1}^{y} a^l_i = x - N_d \\
0 \le a^l_1 \le ... \le a^l_y \le N_d - 1
\end{cases}, \tag{27}
$$

where $y = \min(K\text{-}1, x\text{-}N_d)$. Assume that there are L such sequences $\{a^l_i\}$. Here each sequence $\{a^l_i\}$ represents an event: In the first x-1 time slots, some ball is drawn N_d-1 times, and the remaining $x - N_d$ time slots are allocated to y balls, each of which is drawn a^l_i times. Rewrite (27) as

$$
\sum_{j=1}^{z_l} k^l_j b^l_j = x - N_d, \tag{28}
$$

where both $\{k^l_j\}$ and $\{b^l_j\}$ are positive integers. z_l represents how many unequal non-zero items are in $\{a^l_i\}$. $b^l_j \in \{a^l_i\}$ and $b^l_1 < b^l_2 < ... < b^l_{z_l}$. With $\{k^l_j\}$ and $\{b^l_j\}$, the probability of the event that in the first x-1 time slots some ball A is drawn N_d-1 times and the rest of them are drawn less than N_d times is given by

$$
K \binom{x-1}{N_d - 1} \left[\frac{1}{K}\right]^{x-1} \sum_{l=1}^{L} \prod_{i=1}^{z_l}
$$
$$
\left[K - 1 - \sum_{j=1}^{i-1} k^l_j\right] \binom{x - N_d - \sum_{j=1}^{i-1} b^l_j}{b^l_i} / (k^l_i!) . \tag{29}
$$

Finally, Theorem 1 can be obtained by observing that the probability of the event that the ball A is drawn at the x-th time slot is $1/K$. ∎

APPENDIX B
PROOF OF THEOREM 2

Proof: The solution for $T_{\min}^f$ is again a ball drawing problem. However, instead of drawing one ball each time, M balls need to be drawn with replacement. This is not a trivial combinatorics problem. Therefore, we resort to upper and lower bounds on the minimum lifetime.

To obtain the lower bound, note that the event that M balls are drawn with replacement in each trial is equivalent to drawing one ball without replacement and then putting the balls back into the bag every M drawings (the total number of drawings of the latter case should be divided by M so as to be consistent with the first one). Now let us consider two cases. A bag contains K balls. In Case one, each time one ball is drawn with replacement. The drawing process will be terminated once any ball is drawn $M N_d$ times. In Case two, the ball is drawn without replacement and all the balls are put back into the bag every M drawings. Obviously, Case one needs fewer drawings,[5] and this results in a lower bound for $T_{\min}^f$.

The upper bound can be obtained if we neglect the power transmitted by the relays. Then, the problem becomes: A bag contains K balls. Each time one ball is drawn with replacement. How many drawings are needed until any ball is drawn $2N_d$ times? Here the maximum allowable number of drawings increases to $2N_d$ since each time the source node only transmits with half power. ∎

APPENDIX C
PROOF OF THEOREM 3

Proof: Let $s = \int_0^\infty n \log_2 (1+x) f(x) [F(x)]^{n-1} dx$, where x has a chi-squared distribution with dimension $2(m+1)$. In our model, n and m correspond to the number of nodes in the network and the number of available relay nodes, respectively. $f(x)$ can be approximated by

$$\tilde{f}(x) = \begin{cases} f(x) & x_1 \leq x \leq x_2 \\ 0 & otherwise \end{cases}, \qquad (30)$$

where $f(x_1)=f(x_2)=\theta$, and θ is a small number. We have $F(x_1)\approx 0$ and $F(x_2)\approx 1$. s can then be written as

$$s = \int_{x_1}^{x_2} n \log_2 (x) f(x) [F(x)]^{n-1} dx$$
$$= \int_0^1 \log_2 (F^{-1}(z^{1/n})) dz, \qquad (31)$$

where $F^{-1}(.)$ is the inverse function of $F(x)$, $x_1 \leq x \leq x_2$. We now introduce a lemma.

Lemma 1: For large n and m, s can be written as $s = \mu_1 \log_2 n + \log_2 m + C_0$, with $\mu_1 <<1$ and C_0 is a constant.

Proof: We first find out how s varies with m under a given n.

Let $\epsilon^{1/n} = 1 - \delta$, where δ is a small number. Clearly, $\epsilon \to 0$ when $n \to \infty$. (31) can be approximated by

$$s = \int_0^\varepsilon \log_2(F^{-1}(z^{1/n})) dz + \int_\varepsilon^1 \log_2(F^{-1}(1-\delta)) dz. \quad (32)$$

[5]An intuitive explanation is that in Case one, it is possible that one ball is continuously drawn $2N_d$ times, which will never happen in Case two.

By applying the mean-value theorem, (32) can be written as

$$s = \epsilon \log_2(x_0) + (1-\epsilon) \log_2(F^{-1}(1-\delta)), \qquad (33)$$

where $x_1 \leq x_0 \leq x_2$. With a large n, $\epsilon \to 0$ and we have $s \approx \log_2(F^{-1}(1-\delta))$. As $F^{-1}(1-\delta)=\Theta(m)$, s should have the form $s(m)= \log_2 m + C_0$.

(33) also implies that with a large n, s does not change as n increase. Now we fix m and find out how s varies with n. From (31) we have

$$\frac{ds}{dn} = \lim_{\Delta n \to 0} \frac{s(n + \Delta n) - s(n)}{\Delta n}$$
$$= \lim_{\Delta n \to 0} \frac{1}{\Delta n} \int_0^1 \log_2 \frac{F^{-1}(z^{\frac{1}{n+\Delta n}})}{F^{-1}(z^{\frac{1}{n}})} dz. \qquad (34)$$

Similar to (32-33), with a large n, (34) can be approximated by

$$\frac{ds}{dn} \approx \lim_{\Delta n \to 0} \frac{1}{\Delta n} \log_2 \frac{F^{-1}((1-\delta)^{\frac{n}{n+\Delta n}})}{F^{-1}(1-\delta)}. \qquad (35)$$

Using a Taylor Series expansion, we have

$$F^{-1}(1-\delta) = x_2 - \delta/\theta + o(\delta) \, and \, (1-\delta)^{\frac{n}{n+\Delta n}}$$
$$= 1 - \frac{n}{n+\Delta n} \delta + o(\delta). \qquad (36)$$

By substituting (36) into (35) and ignoring the high-order terms of δ, it can be obtained that

$$\log_2 \frac{F^{-1}((1-\delta)^{\frac{n}{n+\Delta n}})}{F^{-1}(1-\delta)} \approx \log_2 \frac{x_2 - \frac{n}{n+\Delta n}\delta/\theta}{x_2 - \delta/\theta}$$
$$= \log_2 \left(1 + \frac{\Delta n}{n+\Delta n} \cdot \frac{\delta}{\theta x_2 - \delta}\right). \qquad (37)$$

Let τ be an arbitrary number which satisfies $\tau <<1/\delta$. With a large m, an appropriate θ can always be found so that $\theta x_2 = 1/\tau >> \delta$. Therefore, it follows that

$$\frac{ds}{dn} \approx \lim_{\Delta n \to 0} \frac{1}{\Delta n} \log_2(1 + \frac{\Delta n}{n+\Delta n} \tau \delta) = \frac{\mu_1}{n}, \qquad (38)$$

where $\mu_1=\tau\delta <<1$. Therefore, $s(n)$ should have the form $s(n)= \mu_1 \log_2 n + C_0$, with $\mu_1 <<1$. ∎

Lemma 1 is verified by the numerical results shown in Figs. 7 and 8. It can be clearly seen from Fig. 7 that s logarithmically increases with n. When m increases, significant improvement in s can be observed. In contrast, Fig. 8 indicates that, although s again logarithmically increases with m, increasing n can only provide negligible benefits.

Let $\Omega(t) = \int_0^\infty \frac{1}{2} \mathcal{N}(t) \log_2 (1 + \rho x) f(x) [F(x)]^{\mathcal{N}(t)-1} dx$. According to Lemma 1, $\Omega(t)$ can be rewritten as

$$\Omega(t) = \mu_1 \log_2 \varsigma_1 \mathcal{N}(t) + \log_2 \varsigma_2 \mathcal{M}(t), \qquad (39)$$

where ς_1, ς_2 are scaling coefficients and $\mu_1 <<1$.
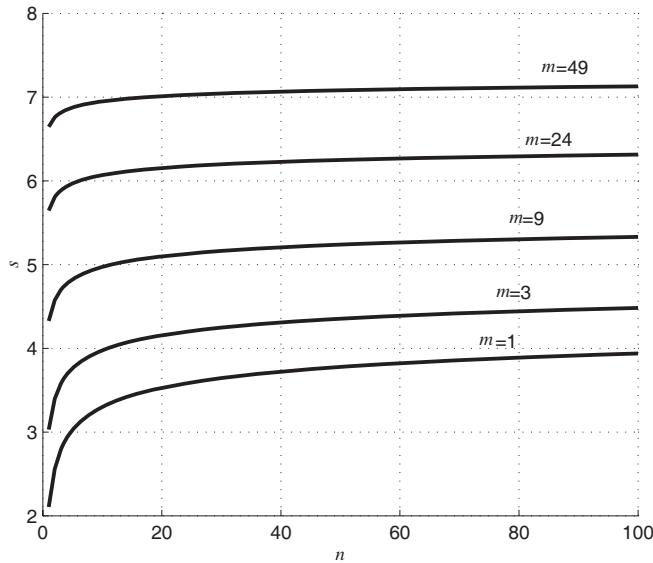By combining (17-18) and (20), we obtain

Fig. 7. Variation of s as a function of n for different values of m. s increases logarithmically with n and a significant improvement in s can be observed as m increases.
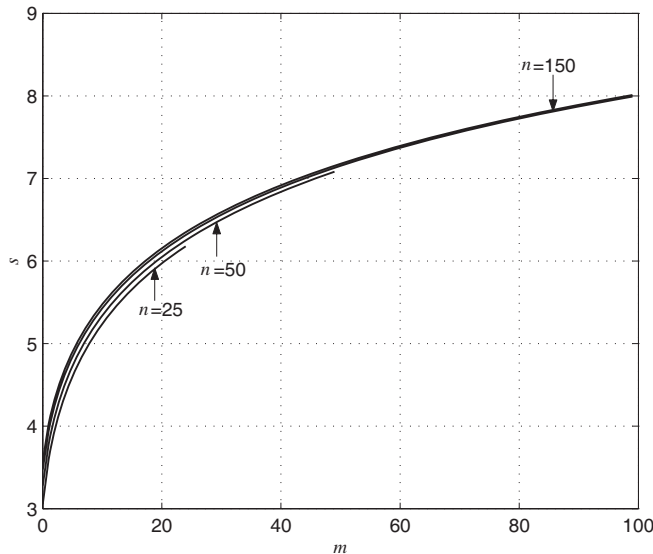

Fig. 8. Variation of s as a function of m for different values of n. Again, s increases logarithmically with m. However, an increase of n can only provide negligible benefits to s.

$$C = \int_0^{T_{\max}} \Omega(t)$$
$$= \mu_1 \int_0^{T_{\max}} \log_2 a(t)dt + \int_0^{T_{\max}} \log_2 b(t)dt$$
$$+ (\mu_1 + 1) \int_0^{T_{\max}} \log_2 \mathcal{K}(t)dt + \upsilon T_{\max}, \tag{40}$$

where $\upsilon = \log_2 \left(\frac{d_0}{\beta^{1/\alpha} R_0} \right) + \mu_1 \log_2 \varsigma_1 + \log_2 \varsigma_2$.
Finally, (23) can be obtained by substituting (16) into (40). ∎

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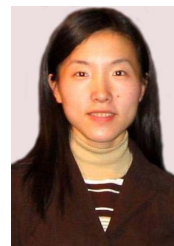
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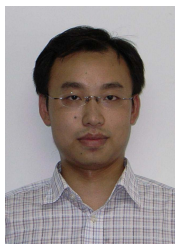
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