

Fair and Efficient Resource Allocation for Cooperative Diversity in Ad-Hoc Wireless Networks*

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Abstract—User cooperation is a powerful solution that can significantly improve the reliability of wireless networks by using several relays to achieve diversity gains. There has been a lot of work on improving the peer-to-peer link quality of a single source-destination pair. However, how to fairly and efficiently allocate resources among multiple nodes has not received much attention yet. In this paper, we propose a novel cooperative diversity method that can achieve fair and efficient resource allocation. We shall show that fairness cannot be achieved by using fixed sets of relays in general. A *multi-state cooperation* method, where the relay set of each node can be changed, is then proposed to solve this problem. In this proposed approach, the energy is allocated among the nodes via a finite step iterative algorithm. In each step, the relay sets of nodes are changed so that each step will generate a *cooperation state*, which characterizes the cooperation relationship among the nodes. Based on the energy allocation result, the duration of each state is then optimized so as to minimize the outage probability. We shall show that the proposed method can not only guarantee fairness, but also provide significant diversity gain over conventional cooperation schemes.

Keywords— *User cooperation, Cross-layer design, Resource allocation, Fairness, Lifetime, Ad-hoc networks.*

I. INTRODUCTION

MIMO (Multiple-Input Multiple-Output) systems, where multiple antennas can be used at both the transmit and receive ends, have recently been receiving significant attention because they hold the promise of achieving huge capacity increases and diversity gains over the harsh wireless communications link [1]. Unfortunately, the use of MIMO technology may not be practical in many wireless networks and applications. For instance, nodes in a sensor network are usually small, inexpensive, and have typically severe energy constraints. Cooperative diversity or communications is one potential solution that can overcome this limitation. The fundamental idea behind cooperative diversity is based on the observation that the signals transmitted by a source node to its destination node can be also received by other nodes in a wireless environment. These nodes can then act as relays or partners to process and re-transmit the signals they receive in a distributed fashion, thereby, creating a virtual antenna array or MIMO system through the use of the relays' antennas without complicated signal design or adding more antennas at the nodes or terminals [2-6].

Sendonaris *et al* firstly proposed the idea of cooperative diversity for CDMA cellular networks [3-4]. Laneman *et al* studied various cooperative diversity schemes such as fixed relaying, selection relaying, and incremental relaying [5-6]. The work in [7] compared several cooperation protocols and presented a space-time code design criteria for amplify-and-forward relay channels. The above previous work mainly aimed at enhancing the performance in the physical layer. However, cooperative communication is inherently a network problem, as pointed out in [3], [5-6]. It would be therefore fruitful to take into account additional higher layer network issues. There have been some efforts towards this such as combining node cooperation with ARQ in the link layer [7], routing in the network layer [8], or resource allocation in the MAC layer [9-10]. From a cross-layer perspective, fairness is especially important in cooperative networks since some nodes may have more chances to be relays, or consumes more power in cooperative transmissions so that their energy may be used up very fast. In this scenario, not only the heavily-used nodes will suffer from a short lifetime, but also the other nodes will not be able to achieve the expected cooperative gain due to the lack of available relays. More seriously, these self-interested users or heavily-used terminals may refuse to cooperate in order to save their energy. Most recently, we applied a market-based approach for increasing the fairness and efficiency of ad-hoc wireless networks using cooperative beamforming [11]. In particular, a practical protocol was presented in [11] to significantly increase the lifetime and throughput of energy-constrained cooperative networks.

In this paper, our main objective is to develop an effective way to optimize the overall performance of cooperative networks across multiple layers simultaneously. Specifically, we consider energy-constrained ad-hoc wireless networks where selection relaying is adopted. Our objective is to guarantee that the lifetime of each node can be equal to a target lifetime and that the energy used in transmitting and/or relaying each node's signal is equal to its total energy. Moreover, each node can efficiently use the available energy to optimize its performance such as outage probability. To achieve this, we propose a novel cooperative diversity by using multi-state cooperation approach, where the relay set of each node is not fixed. In particular, each state corresponds to a set of nodes which run out of energy and will not cooperate anymore. We shall show that at least one node will run out of its energy in each state. Thus, the total number of states will not be greater than the number of nodes. Based on the energy

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allocation results, we allocate the node lifetime among the multiple states to determine how long a particular set of relays can serve a node. The proposed multi-state cooperation in which the heavily-used nodes are not always forced to serve as relays is a natural extension of the conventional cooperation protocols with fixed sets of relays. As an example, we shall apply the proposed method into selection relaying. Numerical results show that conventional selection relaying with fixed relay set will result in a significant decrease in the lifetime of heavily-used nodes. In contrast, the proposed framework can not only guarantee fairness, but also provide significant diversity gain over the conventional selection relaying.

Throughout this paper, the following notations will be used. The inequality $\mathbf{x} \leq \mathbf{y}$ implies that $x_i \leq y_i$ for any i . $\lambda(\mathbf{X})$ denotes the eigenvalue of matrix \mathbf{X} . For a set \mathcal{X} , the operator $|\mathcal{X}|$ denotes the amount of elements in the set. For an event ω , the indicator function shall be denoted by $\mathcal{I}\{\omega\}$, where $\mathcal{I}\{\omega\}=1$ if ω is true. Otherwise, $\mathcal{I}\{\omega\}=0$.

The remainder of the paper is organized as follows. Section II presents the system model. In Section III, a multi-state cooperation methodology with its energy allocation scheme is presented. Section IV investigates the efficient time allocation over multiple states. Finally, numerical results and concluding remarks are presented in Sections V and VI, respectively.

II. SYSTEM MODEL

Consider a wireless network which consists of N source/relay nodes. The source node set is denoted by $\mathcal{S}=\{1, \dots, N\}$. Each source node i transmits to its destination node $d(i)$, which may not belong to \mathcal{S} . Let h_{ij} denote the channel power gain between node i and j . The small-scale fading is assumed to be Rayleigh so that the instantaneous channel gain h_{ij} is a random variable with an exponential distribution with mean value \bar{h}_{ij} . Due to the path-loss, we assume that $\bar{h}_{ij} = D_{ij}^{-\alpha}$, where D_{ij} denotes the distance between node i and node j and $\alpha \in [2, 4]$ is the path-loss factor.¹ The noise power at the receivers is denoted by σ^2 . All nodes are assumed to be energy-constrained and the total energy of each node is denoted by E^{total} .

With user cooperation, each source node may employ some nodes to serve as relays. Each cooperative transmission will be assumed to occur over two timeslots, where the source transmits to its relays in the first timeslot, with a fixed power P^s , and relays re-transmit the signal to the destination in the second timeslot. Since for a particular s-d pair, some nodes may be far away from both the source and the destination, only neighbors are selected in order to increase power efficiency and avoid error propagation. An average channel gain threshold \bar{g}_i is assigned to each node i . Then, node i can only choose the nodes j satisfying $\bar{h}_{ij} \geq \bar{g}_i$ to serve as its relays. The set of potential relays for node i is denoted by \mathcal{R}_i . In this paper, we assume that the source node itself can act as a relay in the second timeslot. That is, $i \in \mathcal{R}_i$. Assume that all relay nodes are chosen from the source node set \mathcal{S} . Hence, $\mathcal{R}_i \subseteq \mathcal{S}$, for any $i \in \mathcal{S}$. In the MAC layer, each source node with its relays can employ an orthogonal channel to avoid

¹ Here, we do not normalize the channel gain since it can be scaled by the transmission power.

multi-user interference. Without loss of generality, FDMA is assumed throughout this paper. Furthermore, each node is assumed to have a saturated queue with always packet availability.

We denote the energy that node j consumed in transmitting/relaying signals of node i to be $E_j^{(i)}$. In wireless networks, all the nodes are expected to have the same lifetime T^* . As a result, each node should consume all of its energy, E^{total} , simultaneously at the end of the target node lifetime interval $[0, T^*]$. In this case, the energy to be allocated over the even timeslots is given by $E^{max} = E^{total} - P^s T^*/2$. No nodes are allowed to have residual energy after T^* . Otherwise, its lifetime can be longer than T^* . On the other hand, the energy utilized in transmitting and/or relaying each node's information should be equal in order to guarantee fairness. In non-cooperative networks where nodes transmit directly without employing relays, this can be easily satisfied since each node's energy consumed is utilized to transmit its own information. In cooperative networks, this is not naturally achieved and hence an appropriate resource allocation is desired.

III. ENERGY ALLOCATION FOR MULTI-STATE COOPERATION

In this section, we address the energy allocation problem for multi-state cooperation. In order to satisfy the fairness and energy constraints, we provide a geometrical approach to allocate the energy state-by-state via a finite-step iteration algorithm. In each state, the energy allocation is obtained by solving a linear equation.

A. Energy Allocation and Consumption

Let us denote the energy consumption vector $\mathbf{e}^C = [e_1^C, \dots, e_N^C]$, where the i th element $e_i^C = \sum_{j=1}^N E_i^{(j)}$ is the total energy consumed by node i . Note that the energy allocated to a node is the total energy utilized in transmitting and relaying this node's information. We denote the energy allocation vector $\mathbf{e}^A = [e_1^A, \dots, e_N^A]$, where the i th element $e_i^A = \sum_{j=1}^N E_j^{(i)}$. Since the node is energy-constrained and all nodes should consume all of their energy within their target lifetime, the energy constraint is written as

$$\mathbf{e}^C = E^{max} \mathbf{1} \quad (1)$$

Due to the fairness requirements, each node should be allocated the same energy to transmit and relay its signal. Thus, we should have $e_i^A = \sum_{n=1}^N E_n^{max} / N = E^{max}$. In a vector form, the fairness constraint is given by

$$\mathbf{e}^A = E^{max} \mathbf{1} \quad (2)$$

In an non-cooperative network, the two constraints (1)-(2) are naturally satisfied since $E_i^{(i)}=0$ for $i \neq j$. As cooperative diversity is adopted, \mathbf{e}^A and \mathbf{e}^C will be shown to be linearly related. In order to derive such a relationship, a *cooperation matrix* is defined as follows.

Definition 1 (Cooperation Matrix): The cooperation matrix is defined to be a matrix $\mathbf{A}=[a_{ij}]_{N \times N}$, where the element a_{ij} denotes the energy ratio that node i contributes to node j . That is,

$$a_{ij} = E_i^{(j)} / \sum_{n=1}^N E_n^{(j)} \quad (3)$$

Note that the cooperation matrix \mathbf{A} is determined by the cooperation scheme and the relay sets \mathcal{R}_i , for $i=1, \dots, N$. Since

$\sum_{i=1}^N a_{ij} = 1$, \mathbf{A}^T is a stochastic matrix.² The cooperation matrix of selection relaying is given by the following theorem.

Theorem 1: For selection relaying, the cooperation matrix is given by³

$$a_{ij} = \begin{cases} \frac{\Pr\{h_{ji} \geq \xi_j\}}{\sum_{n \in \mathcal{R}_j} \Pr\{h_{jn} \geq \xi_j\}} & i \in \mathcal{R}_j \\ 0 & i \notin \mathcal{R}_j, \end{cases} \quad (4)$$

where the decoding threshold of the source-relay (s-r) channel gain is given by $\xi_j = (2^{2r_j} - 1)\sigma^2 / P^s$ and r_j is the target rate of source node j .

Proof: For $i \notin \mathcal{R}_j$, node i does not serve as a relay for node j . As a result, $E_i^{(j)} = 0$ and $a_{ij} = 0$. Next, we consider node $i \in \mathcal{R}_j$. From [4], we know that node i re-transmit node j 's message, only when the s-r channel gain $h_{ji} \geq \xi_j$. Let t_j denote the time that node j uses the relay set $i \in \mathcal{R}_j$. In this case, the time that node i serves as relay for node j is $\Pr\{h_{ji} \geq \xi_j\} t_j / 2$. Since all relays that can decode transmit with the same power $P^{r(j)}$, it follows that $E_i^{(j)} = P^{r(j)} t_j \Pr\{h_{ji} \geq \xi_j\} / 2$. By substituting $E_i^{(j)}$ into (3), (4) can be obtained accordingly. ■

According to (3), the energy that node i consumes for transmitting/relaying the signal of node j can be presented as $E_i^{(j)} = a_{ij} e_j^A$. Hence, the total energy consumption of node i is obtained as $e_i^C = \sum_{j=1}^N a_{ij} e_j^A$. As a result, the energy allocation and consumption can be related by

$$\mathbf{A} \mathbf{e}^A = \mathbf{e}^C. \quad (5)$$

By substituting (1) and (2) into (5), we have

$$\mathbf{A} \mathbf{E}^{\max} \mathbf{1} = \mathbf{E}^{\max} \mathbf{1}. \quad (6)$$

According to Definition 1 and (6), the cooperation matrix \mathbf{A} should be a doubly-stochastic matrix in order to satisfy both (1) and (2). Unfortunately, \mathbf{A} cannot satisfy (6) in general because it cannot be doubly-stochastic for networks with randomly-located nodes.

B. Multi-state Cooperation: To Cooperate or Not to Cooperate

Since the energy and fairness constraints (1)-(2) cannot be satisfied simultaneously as relay sets are fixed, each node should be allowed to use a different relay set in a different cooperation state in order to satisfy (1)-(2). This motivates us to develop a cooperation method consisting of multiple relay sets, which we shall refer to as *multi-state cooperation*. In each state k , the relay set $\mathcal{R}_i(k)$ for any node i is fixed. Therefore, the cooperation state is given by the set of all relay sets. For a multi-state cooperation, the energy allocation $\mathbf{e}^A(k)$, $k=1, \dots, K$, should satisfy the energy and fairness constraints given by

$$\begin{cases} \sum_{k=1}^K \mathbf{e}^C(k) = \sum_{k=1}^K \mathbf{A}(k) \mathbf{e}^A(k) = \mathbf{E}^{\max} \mathbf{1} \\ \sum_{k=1}^K \mathbf{e}^A(k) = \mathbf{E}^{\max} \mathbf{1} \end{cases} \quad (7)$$

where K is the total number of states and $\mathbf{A}(k)$ is the cooperation matrix associated with state k . In order to obtain

$\mathbf{e}^A(k)$ that satisfies (7), a state-by-state energy allocation methodology is presented. In each state, the energy allocation should satisfy the constraint

$$\mathbf{e}^A(k) = \mathbf{e}^C(k), \quad (8)$$

which can always be satisfied by the method described next in Section III-C. However, $e_i^C(k)$ may not be equal to $e_j^C(k)$ for any $j \neq i$ in general. Therefore, some nodes will consume all of their energy in state k while others may still have residual energy to be allocated and consume in the following states. In each state, only the nodes that have residual energy can transmit and serve as relay for others. Iteratively, we can allocate the residual or remaining energy state-by-state until all of the energy is allocated in the final state.

Let $\mathbf{e}^{\max}(k)$ denote the residual energy vector, where the i th element $e_i^{\max}(k)$ is the node i 's residual energy in state k . In the initial state 1, $\mathbf{e}^{\max}(1) = \mathbf{E}^{\max} \mathbf{1}$. After energy allocation in state k , the residual energy $\mathbf{e}^{\max}(k+1)$ of the next state is given by

$$\mathbf{e}^{\max}(k+1) = \mathbf{e}^{\max}(k) - \mathbf{e}^A(k). \quad (9)$$

Constrained by $e_i^{\max}(k)$, the relay set of node i in state k , $\mathcal{R}_i(k)$, is given by

$$\mathcal{R}_i(k) = \begin{cases} \{j : \bar{h}_{ij} \geq \bar{g}_i, e_j^{\max}(k) > 0, j \in \mathcal{S}\} & e_i^{\max}(k) > 0 \\ \{i\} & e_i^{\max}(k) = 0 \end{cases} \quad (10)$$

In Section III-C, we will present an energy allocation method, where at least one node will run out of energy in each state. Therefore, all of the energy can be allocated within $K \leq N$ states so that the residual energy vector in the final state K satisfies

$$\mathbf{e}^{\max}(K+1) = \mathbf{0}. \quad (11)$$

According to (8), (9), and (11), (7) is satisfied in a K -state cooperation.

C. Energy Allocation in One State

In this part, the analytical result for energy allocation in one state is presented. For a particular state k , the cooperation matrix $\mathbf{A}(k)$ is determined by the relay sets given by (10). Thus, $\mathbf{e}^A(k)$ should be a nonnegative solution to the following equation

$$\mathbf{A}(k) \mathbf{e}^A(k) = \mathbf{e}^A(k), \quad (12)$$

and satisfy the residual energy constraint in state k , which is given by

$$\mathbf{e}^A(k) \leq \mathbf{e}^{\max}(k). \quad (13)$$

Since $\mathbf{A}^T(k)$ is a stochastic matrix, there must be an eigenvalue $\lambda(\mathbf{A}(k)) = \lambda(\mathbf{A}^T(k)) = 1$ [12]. Thus, Eqn. (12) must have nontrivial solutions. The solution space is the eigenspace of $\mathbf{A}(k)$ with respect to $\lambda(\mathbf{A}(k)) = 1$. In order to obtain $\mathbf{e}^A(k)$, a *network decomposition* methodology is first introduced in order to find an orthogonal basis set of the solution space. For convenience, we can ignore the index k in this part since we are considering energy allocation of a particular state.

In each state, the cooperative network can be decomposed into disjoint sub-networks, where the nodes in different sub-networks do not cooperate with each other.⁴ Intuitively, the energy allocations in each sub-network are independent.

⁴ In this first state, a network may consist of only one sub-network if its topology is not clustered.

² A stochastic matrix is a nonnegative matrix in which each row sum is equal to 1. A doubly-stochastic matrix is a nonnegative matrix in which each row sum as well as each column sum is equal to 1 [12].

³ Note that $\Pr\{h_{jj} \geq \xi_j\} = 1$. Hence, a node can always retransmit its own message in the second timeslot. For Rayleigh fading, specifically, we have $\Pr\{h_{ji} \geq \xi_j\} = \exp(-\xi_j / \bar{h}_{ji})$.

Mathematically, for a given cooperation matrix \mathbf{A} , the whole network \mathcal{S} can be decomposed into M disjoint sub-networks, $\mathcal{S}^{(m)} = \{n_1^{(m)}, \dots, n_{|\mathcal{S}^{(m)}|}^{(m)}\}$ for $m=1, \dots, M$, which satisfy the following

- (1) $\mathcal{S} = \bigcup_{m=1}^M \mathcal{S}^{(m)}$ with $\mathcal{S}^{(m)} \cap \mathcal{S}^{(n)} = \emptyset, \forall m \neq n$.
- (2) $a_{ij} = a_{ji} = 0$ for $\forall i \in \mathcal{S}^{(m)}$ and $j \in \mathcal{S}^{(n)}, m \neq n$.
- (3) Each sub-network \mathcal{S}_m cannot be decomposed into multiple disjoint subsets satisfying properties (1) and (2).

For a given \mathbf{A} , the network decomposition can be implemented by the following graph-theoretic algorithm.

Network Decomposition Algorithm

- Step 0:** Initialize, $M=1, \mathcal{S}^{(1)} = \emptyset$;
- Step 1:** If $\bigcup_{m=1}^M \mathcal{S}^{(m)} = \mathcal{S}$, go to End;
- Step 2:** Choose one node s such that $s \notin \bigcup_{m=1}^M \mathcal{S}^{(m)}$. Let $\mathcal{T} = \{s\}$ and $\mathcal{S}^{(M+1)} = \{s\}$;
- Step 3:** If $\mathcal{T} = \emptyset$, go to Step 6;
- Step 4:** $\mathcal{T} = \{n : a_{np} + a_{pn} > 0, p \in \mathcal{S}^{(M+1)}, n \notin \mathcal{S}^{(M+1)}\}$;
- Step 5:** $\mathcal{S}^{(M+1)} = \mathcal{S}^{(M+1)} \cup \mathcal{T}$, go to Step 3;
- Step 6:** $M=M+1$, go to Step 1.
- End**

It is noted that each sub-network $\mathcal{S}^{(m)}$ will have its own cooperation matrix $\mathbf{A}^{(m)}$ as described in the next theorem.

Theorem 2: The optimum energy allocation vector that satisfies (12)-(13) is obtained by

$$\mathbf{e}^A = \sum_{m=1}^M \left(\min_{i \in \mathcal{S}^{(m)}} \frac{e_i^{\max}}{b_i^{(m)}} \right) \mathbf{b}^{(m)}, \quad (14)$$

where the orthogonal basis $\mathbf{b}^{(m)} = [b_1^{(m)}, \dots, b_N^{(m)}]^T$, for $m=1, \dots, M$, are non-negative. They are given by

$$\mathbf{b}^{(m)} = \mathbf{P} \times \begin{bmatrix} \mathbf{0}_{(\sum_{r=1}^{m-1} |\mathcal{S}^{(r)}|) \times 1} \\ \left[\begin{array}{cccc} 1 & 1 & \dots & 1 \end{array} \right]^{-1} \left[\begin{array}{c} 1 \\ \mathbf{0}_{(|\mathcal{S}^{(m)}|-1) \times 1} \end{array} \right] \\ \mathbf{0}_{(\sum_{r=m+1}^M |\mathcal{S}^{(r)}|) \times 1} \end{bmatrix}, \quad (15)$$

where \mathbf{P} is the permutation matrix with elements $p_{uv} = \mathcal{I}\left\{ \left(u = n_i^{(m)}, v = i + \sum_{r=1}^{m-1} |\mathcal{S}^{(r)}| \right) \right\}$, and $\mathcal{A}^{(m)}$ is a $(|\mathcal{S}^{(m)}|-1)$ -by- $|\mathcal{S}^{(m)}|$ matrix obtained by deleting an arbitrary one row of $(\mathbf{A}^{(m)} - \mathbf{I})$. Here, the cooperation matrix of $\mathcal{S}^{(m)}$, $\mathbf{A}^{(m)}$, is the m th diagonal block in the diagonal block matrix

$$\mathbf{P}^T \mathbf{A} \mathbf{P} = \begin{bmatrix} \mathbf{A}^{(1)} & & \\ & \ddots & \\ & & \mathbf{A}^{(M)} \end{bmatrix}. \quad (16)$$

Due to space limitation, we omit the proof of this Theorem.

D. Energy Allocation in Multiple Cooperation States

Having obtained the energy allocation in one state, our proposed multi-state energy allocation algorithm can be presented as follows.

Multi-state Energy Allocation Algorithm

- Step 0:** Initialize $k=1$, and set $\mathbf{e}^{\max}(1) = E^{\max} \mathbf{1}$;
- Step 1:** Generate $\mathcal{R}_i(k)$, for $i=1, \dots, N$ by (9) and $\mathbf{A}(k)$ by (3);
- Step 2:** Decompose the network into $\mathcal{S}^{(m)}, m=1, \dots, M$ using the network decomposition algorithm;
- Step 3:** Obtain the energy allocation vector $\mathbf{e}^A(k)$ by (14)-(16);
- Step 4:** Calculate the residual energy vector $\mathbf{e}^{\max}(k+1)$ by (9);
- Step 5:** $k=k+1$;
- Step 6:** If $\mathbf{e}^{\max}(k) > 0$, go to Step 1.
- End**

It must be noted here that Theorem 2 shows that the energy allocation is always feasible since the solution is nonnegative. One can easily see that Eqn. (12)'s solution space characterized by $\mathbf{b}^{(m)}, m=1, \dots, M$, contains infinite amount of energy allocation vectors satisfying (13). The optimum energy allocation presented in Theorem 2 chooses the solution vector with the maximum 1-norm. The optimality of choosing such an energy allocation vector can be explained as follows. From (10), it can be seen that the number of each node's relays decreases as the state index k increases. Notice that the energy efficiency is higher as more relays are used. Clearly with an increasing state index k the energy efficiency will go down. Therefore, as much as possible energy should be allocated in each state during the iteration. This is why we choose the energy allocation vector with the maximum 1-norm.

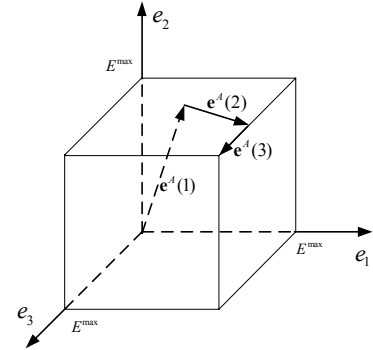


Fig. 1: Geometrical Interpretation of Multi-State Energy Allocation

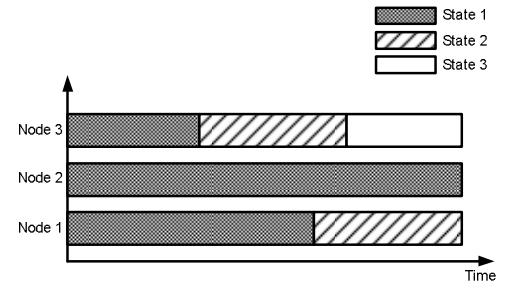


Fig. 2: Multi-State Cooperation in the Time Domain

From (14) it can be also seen that with the proposed energy allocation, at least one node in each $\mathcal{S}^{(m)}$ satisfies $e_i^A(k) = e_i^{\max}(k)$. This implies that at least one node runs out of energy in each state. Therefore, (11) can be satisfied by a K -state ($K \leq N$) energy allocation. From a geometrical perspective, the

multi-state energy allocation can be characterized by a K -part curve in \mathbf{R}^N starting from the origin point $\mathbf{0}$ to $E^{\max}\mathbf{1}$. The k th part of the curve, $\mathbf{e}^A(k)$, belongs to the eigenspace of $\mathbf{A}(k)$ with respect to $\lambda(\mathbf{A}(k))=1$, and the cumulative energy allocation vector $\sum_{l=1}^k \mathbf{e}^A(l)$ is bounded by the super-cube characterized by $E^{\max}\mathbf{1}$. For instance, Fig. 1 shows an energy allocation in a three node network. In this case, node 2 is allocated all of its energy in state 1, while nodes 1 and 3 still have residual energy. Next, node 1 is allocated all of its residual energy in state 2. Finally, node 3 runs out of its energy in the last state. Given the above energy allocation outcome, node 2 will always transmit with its state 1's relay set $\mathcal{R}_2(1)=\{1, 2, 3\}$ throughout its entire lifetime. Node 1, however, will use its state 1's relay set, $\mathcal{R}_1(1)=\{1, 2, 3\}$ and state 2's relay set, $\mathcal{R}_1(2)=\{1, 3\}$, in a time sharing manner. Node 3 will have three different relay sets, namely, $\{1, 2, 3\}$, $\{1, 3\}$, and $\{3\}$, which are used in disjoint time durations. Fig. 2 shows how multi-state cooperation evolves in the time domain, where one can see that all nodes will have the same lifetime. How to determine the optimal state duration will be addressed in the next Section.

IV. OPTIMAL STATE DURATION

In this section, we investigate how long the set of nodes $\mathcal{R}_i(k)$ can serve as the relay set of node i . Given a target lifetime, we shall allocate the whole lifetime over the multiple states of a node. The optimal state duration, which determines the amount of time each state would exist or how long $\mathcal{R}_i(k)$ can serve as the relay set of node i , will maximize the utility such as the outage probability. Assume that $\mathcal{R}_i(k)$ serves as a relay set of node i for $t_i(k)$ seconds. Here, $t_i(k)$ denotes the duration of state k . Subject to the target lifetime T^* , the lifetime constraint for each node i is presented by

$$\sum_{k=1}^K t_i(k) = T^*. \quad (17)$$

The node i 's utility in state k is denoted by $u_i(k)$, which can be represented as a function of the average power in state k . That is, $u_i(k)=U_{i,k}(e_i^A(k)/t_i(k))$, where the utility function $U_{i,k}(x)$ is an increasing function of x satisfying $U_{i,k}(0)=0$. For selection relaying, the probability that outage does not occur, $1-P_{out}$, is adopted to measure the utility of each node. The following theorem gives the utility function for selection relaying.

Theorem 3: For selection relaying, the utility function of node i in state k is given by

$$U_{i,k}\left(\frac{e_i^A(k)}{t_i(k)}\right) = \sum_{\mathcal{D} \in \mathcal{R}_i(k)} \left\{ \frac{\sum_{j \in \mathcal{D}} \exp\left(-\frac{\eta_j(k)}{\bar{h}_{jd(i)}}\right) \prod_{n \neq j} \frac{\bar{h}_{jd(i)}}{\bar{h}_{jd(i)} - \bar{h}_{nd(i)}}}{\prod_{j \in \mathcal{D}} \exp\left(-\frac{\xi_j}{\bar{h}_{ij}}\right) \prod_{j \notin \mathcal{D}} \left[1 - \exp\left(-\frac{\xi_j}{\bar{h}_{ij}}\right)\right]} \right\}, \quad (18)$$

where the decoding set \mathcal{D} is any subset of the relay set $\mathcal{R}_i(k)$ and the outage threshold is given by

$$\eta_i(k) = \left[(2^{2r_i} - 1) \sigma^2 \sum_{j \in \mathcal{R}_i(k)} \exp\left(-\frac{\xi_j}{\bar{h}_{ij}}\right) \right] / \left[2e_i^A(k) / t_i(k) \right]. \quad (19)$$

Proof: By recalling the proof of Theorem 1, we know that transmission power of each relay is given by $2e_i^A(k) / (t_i(k) \sum_{j \in \mathcal{R}_i(k)} \exp(-\xi_j / \bar{h}_{ij}))$. Thus, the outage threshold (19) is

obtained. For selection relaying, the outage probability can be given by

$$P_{out,i}(k) = \sum_{\mathcal{D} \in \mathcal{R}_i(k)} \Pr\left\{ \sum_{j \in \mathcal{D}} h_{jd(i)} < \eta_i(k) \mid \mathcal{D} \right\} \Pr\{\mathcal{D}\}, \quad (20)$$

where the probability of a particular \mathcal{D} can be obtained by

$$\Pr\{\mathcal{D}\} = \prod_{j \in \mathcal{D}} \exp(-\xi_j / \bar{h}_{ij}) \prod_{j \notin \mathcal{D}} \left[1 - \exp(-\xi_j / \bar{h}_{ij}) \right]. \quad (21)$$

By using the generating function [9], the conditioned outage probability is obtained by⁵

$$\Pr\left\{ \sum_{j \in \mathcal{D}} h_{jd(i)} < \eta_i(k) \mid \mathcal{D} \right\} = \sum_{j \in \mathcal{D}} \left(1 - \exp\left(-\frac{\eta_i(k)}{\bar{h}_{jd(i)}}\right) \right) \prod_{n \neq j} \frac{\bar{h}_{jd(i)}}{\bar{h}_{jd(i)} - \bar{h}_{nd(i)}}. \quad (22)$$

By substituting (20)-(22) into $u_i(k)=1-P_{out}$, (18) can be obtained. ■

Given the above, the state duration optimization problem for node i can then be formulated as

$$\begin{aligned} & \text{Maximize} \quad \sum_{k=1}^K \frac{t_i(k)}{T^*} U_{i,k}\left(\frac{e_i^A(k)}{t_i(k)}\right) \\ & \text{Subject to} \quad \begin{cases} \sum_{k=1}^K t_i(k) = T^* \\ t_i(k) \geq 0 \end{cases} \end{aligned} \quad (23)$$

By solving the optimization problem (23), the optimal state duration can be obtained.

V. NUMERICAL RESULTS

In this section, numerical results are presented to compare the performance of multi-state cooperation and conventional cooperation with fixed relay sets. Direct transmission where nodes do not use any relay is also considered to provide a baseline reference to compare cooperation gains. For conventional cooperation, the power consumed for transmitting/relaying each node's signal is equal to the transmission power of direct transmission. This kind of cooperation shall also be referred to as full cooperation. The path-loss factor α is assumed to be 4 and the noise power is assumed to be $\sigma^2=1$.⁶ The target rate of each s-d pair is $r_i=1$ bit/s/Hz for any i . Consider a network with four s-d pairs satisfying $h_{id(i)}=1$ for $i=1, \dots, 4$, where the coordinates of the source nodes 1-4 are $(-0.5, \sqrt{3}/2)$, $(0, 0.1)$, $(0, -0.1)$, and $(-0.5, -\sqrt{3}/2)$, respectively. Likewise, the coordinates of their destination nodes $d(1)$ - $d(4)$ are $(0.5, \sqrt{3}/2)$, $(-1, 0.1)$, $(-1, -0.1)$, and $(0.5, -\sqrt{3}/2)$. Here assume the thresholds $\bar{g}_i=0.5$, $i=1, \dots, 4$. It can be seen that nodes 1 and 4 are far away from each other so that they will not use each other as a relay. In addition, such a network topology is symmetric with respect to the x -axis. Therefore, the performance of nodes 1 and 4 is the same, as well as that of nodes 2 and 3. We shall use the average Signal-to-Noise Ratio (SNR) of direct transmission given by $E^{\text{total}}/(T^* \sigma^2)$ to characterize the SNR of the network. Without loss of generality, we can normalize lifetime so that $T^*=1$. We assume that the transmission power of the source nodes in the first timeslot is $E^{\text{total}}/(2T^*)$. Then, with full cooperation, the average total power of the relays of each node should be $3E^{\text{total}}/(2T^*)$.

⁵ For randomly distributed node, we can assume that $\bar{h}_{jd(i)} \neq \bar{h}_{nd(i)}$, for $j \neq n$.

⁶ Here, we drop the unit in the following text.

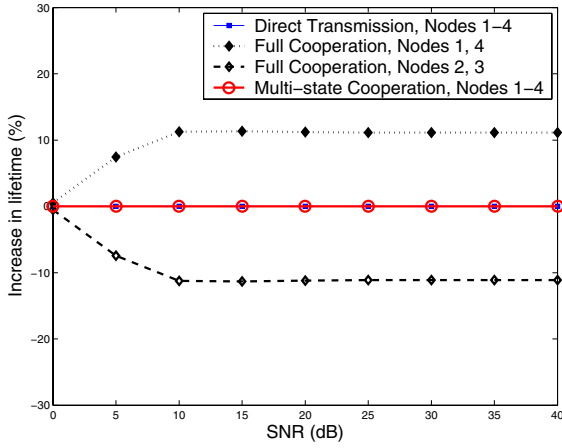


Fig. 3: Increase in node lifetime with direct transmission, full cooperation and multi-state cooperation

Fig. 3 presents the increase in lifetime compared to T^* with direct transmission, full cooperation and multi-state cooperation. With the direct transmission and the multi-state cooperation, the lifetime of each node is equal to T^* . With full cooperation, however, the lifetime of nodes 1 and 4 is increased by about 11% T^* , while the lifetime of nodes 2 and 3 is decreased by about 11% T^* in the high SNR region. It can be seen that the gap between the maximum and minimum lifetime increases with SNR. This is because the probability that a relay can decode correctly increases with SNR. By serving more nodes, nodes 2 and 3 will consume a larger amount of power in the high SNR region.

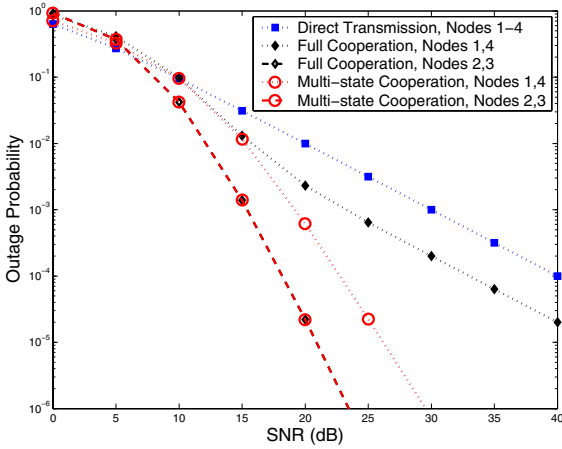


Fig. 4: Outage probability averaged over the lifetime of each node with direct transmission, full cooperation and multi-state cooperation

Fig. 4 presents the outage probability averaged over the lifetime of each node with direct transmission, full cooperation and multi-state cooperation. Since direct transmission cannot achieve diversity gain, the outage probability of all nodes approximately decays as $1/\text{SNR}$ [1]. With full cooperation, the outage probability of nodes 1 and 4 decays as $0.2/\text{SNR}$. This is because, as shown in Fig. 4, they transmit directly without relay for 20% of their lifetime, when the outage probability is approximately equal to $1/\text{SNR}$. Since our proposed framework can efficiently allocate time among the states, nodes 1 and 4 can transmit with relays much longer while transmitting directly for only a very short time with

much higher transmission power. Although the power of relays is lower compared to full cooperation, multi-state cooperation can benefit from sufficient long time when spatial diversity gain of 3 is achieved. A closer observation shows that node 2 or 3's outage probability with full cooperation is equal to that with multi-state cooperation. This is simply due to the fact that with multi-state cooperation, the average total power of node 2 or 3's relays is also equal to $3E^{\text{total}}/(2T^*)$. By calculating the outage probability averaged over all nodes, it is found that the average outage probability of full cooperation is approximately equal to $0.1/\text{SNR}$, while that of our proposed framework is $1/\text{SNR}^{2.5}$.

VI. CONCLUSIONS

In this paper, we presented a fair and efficient cooperative diversity method by using multi-state cooperation. With this proposed approach, all nodes can run out of energy simultaneously and each node is allocated an equal amount of energy so that fairness is guaranteed. We proposed a multi-state energy allocation method to jointly allocate energy and change the relay sets. Given the energy allocation results, the optimal state duration of each node is found so as to minimize each node's outage probability. It was demonstrated that an equal lifetime of all nodes can be guaranteed. This is in contrast to the unfair cooperation which will result in a significant decrease in the lifetime of heavily-used nodes. For instance, the decrease in lifetime of heavily-used nodes is 11% for selection relaying with fixed relay set. Furthermore, the proposed framework can guarantee a diversity gain greater than 2 on average compared to full cooperation, which achieves only a diversity gain of 1 on average.

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