Selective Relaying in Cooperative OFDM Systems: Two-Hop Random Network

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***Abstract*— In this paper, we investigate two selective relaying schemes in cooperative OFDM systems. Selective OFDMA relaying, where the relay selection is performed in a per-subcarrier manner, and selective OFDM relaying, where one best relay among the L potential relays is selected to relay the entire OFDM block, are compared in a two-hop random network. The outage performance of Equal Bit Allocation (EBA), where each subchannel has the same number of bits, and Bit Loading (BL), where bits are adaptively allocated to each subchannel, are analyzed and compared for these two approaches. The outage analysis clearly shows that a significant performance gain can be achieved by selective OFDMA relaying, whether EBA or BL is employed, compared with selective OFDM relaying. The performance gain remains the same for different relay locations. With EBA, the performance gain increases with an increase in L and N, the number of independent subchannels. For BL, the performance gain also increases with an increase in R, the average number of bits per subchannel, in addition to L and N. Centralized and decentralized implementation issues are also considered. For EBA, selective OFDMA relaying scheme is preferred because of its superior performance and simple decentralized implementation. For BL, selective OFDMA relaying scheme is a good choice for centralized systems and selective OFDM relaying is more suitable for decentralized systems at the expense of a loss in performance.**

I. Introduction

With the increasing needs for high speed wireless applications, future networks, no matter infrastructure-based or ad hoc, will be required to provide reliable high data rate services in dynamic environments. The use of multiple antennas, which can improve the power and spectral efficiency greatly in single-link wireless communications, may be impractical in many instances due to limitations on the size and power of communications devices. Cooperative transmission, which utilizes the broadcast nature of the wireless medium and the numerous nodes in a network, is an efficient way to realize the benefits of multi-antenna transmission with only one antenna at each node (for example, see [1]-[4]).

Orthogonal Frequency Division Multiplexing (OFDM), which can enable the high bit rates demanded by current and emerging applications [5], is the underlying physical-layer technology for IEEE802.11 (WiFi) [6], as well as for digital audio [7] and video broadcasting [8]. In addition, OFDMA, a multiple access technique in which the subchannels of an OFDM symbol are shared by multiple users, has significant advantages that have made it the natural choice for commercial broadband wireless networks, such as IEEE802.16 (WiMAX) [9], as well as for the long-term evolution of third-generation cellular systems (specifically for the downlink) [10]. Currently, relay and cooperative networks with OFDM(A) transceivers have been proposed for applications in several emerging systems [11]-[12].

Although there has been a significant effort on the study of cooperative systems, there has been very little work on the use of OFDM in these networks. In most of the work in this area, OFDM is simply the underlying transmission technology. How to use OFDM to facilitate relaying in a multihop networks is still an open issue. In [13], we proposed selective OFDMA relaying, where relay selection is performed on a per-subchannel basis. An OFDM transceiver with and without coding in an idealized linear multi-hop network is considered. We have shown that a significant performance gain can be achieved by selective OFDMA relaying, compared with selective OFDM relaying, when only one "best" relay is selected to re-transmit the entire OFDM block.

In this paper, the end-to-end outage performance of selective OFDMA relaying is evaluated and compared to that of selective OFDM relaying in a more practical environment. A two-hop *random* network including the effects of path loss is addressed. Two transmission schemes, Equal Bit Allocation (EBA), where each subchannel has the same number of bits to transmit, and Bit Loading (BL), where bits are adaptively allocated to each subchannel, are addressed. It is proved that, although selective OFDMA relaying and selective OFDM relaying achieve the same diversity gain, the power gain is different. For EBA, the performance improvement of selective OFDMA relaying over selective OFDM relaying increases with an increase in L, the number of relay nodes, and N, the number of independent subchannels. For BL, an increase in R, the average number of transmission bits per subchannel, will also increase the performance improvement. We also show that the locations of the relay nodes do not affect the relative performance of these two approaches. Simulation results validate the analysis. Practical issues, such as centralized and decentralized implementations and complexity issues, are also addressed.

The paper is organized as follows. In Section II, we provide the system model and introduce selective OFDMA relaying. Section III presents the outage analysis for selective OFDMA relaying and selective OFDM relaying with EBA. The outage

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analysis with BL is given in Section IV. We address centralized and decentralized implementation issues in Section V. Finally, Section VI summarizes and concludes the paper.

II. System model and selective relaying

We consider a single source(S)-destination(D) cooperative system with L relay nodes, as shown in Fig. 1. The relays are randomly located between the source and the destination. An OFDM transceiver with N subchannels is available at each node. We assume perfect time and frequency synchronization among nodes and the inclusion of a cyclic prefix that is long enough to accommodate the channel delay spread.

A two-stage transmission protocol is adopted. In the first stage, the source transmits and the relay nodes listen - the links in this stage are called the source-relay (SR) links. In the second stage, the relays retransmit the message to the destination - the links in this stage are called the relay-destination (RD) links. Here, we adopt a *selective* decode-and-forward relaying strategy. In particular, each source subchannel can only be relayed by one relay node. The selected relay node will fully decode the received information, re-encode it, and then forward it to the destination. In the RD links, a specific subchannel can only be used by a single node. With these assumptions, interference is avoided.

We assume that the total required data rate is R_{total} bits per OFDM symbol (block). On average, each subchannel will transmit $R = R_{total}/N$ bits. Further, denote the channel response of subchannel n from the source node to relay node i and from relay node i to the destination node as $H_{sr_i}(n)$ and $H_{r_id}(n)$, respectively. In general, these include path loss, shadowing, and Rayleigh fading. For convenience, let $G_{sr_i}(n)$ and $G_{r_id}(n)$ denote the channel power gains, $\|H_{sr_i}(n)\|^2$ and $\|H_{r_id}(n)\|^2$, respectively. Also, let $G_{sr_id}(n)$ refer to $\min\{G_{sr_i}(n), G_{r_id}(n)\}$.

In previous work, OFDM is simply adopted as a physical layer technique to overcome the frequency-selective fading in the network. As shown in Fig. 1(a), one relay is selected to forward the entire OFDM block so that all the subchannels traverse the same path. In particular, the relay with the largest $\min_{n=1,\ldots,N} G_{sr_id}(n)$ is selected for EBA. For BL, the relay with the largest sum rate is selected. This type of relaying is referred to as *selective OFDM relaying*. Compared to the other relaying strategies, selective relaying requires the least amount of signaling and can be performed in a distributed way [14]. In [13], we propose a new relaying scheme, which is called *selective OFDMA relaying*. In this case, the relay selection is performed in a per-subcarrier manner. Different relays might be selected to retransmit on different subchannels. As shown in Fig. 1(b), subchannels 1 and 2 are retransmitted by relay 3, while subchannel 3 is retransmitted by relay 2. At the destination, all the subchannels are collected. In this paper, we compare the performance of these two relaying strategies with two transmission schemes: EBA and BL.

The following assumptions are made:

- A1) The destination node only uses the signal transmitted in the second stage.

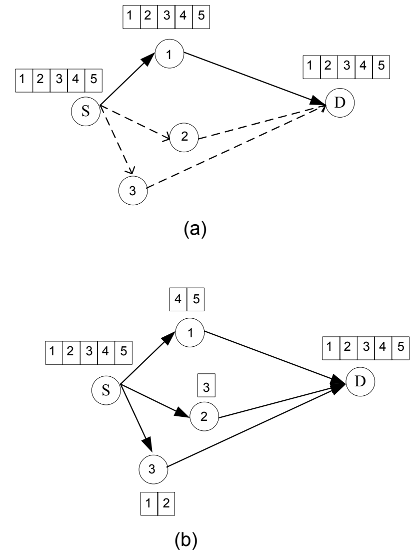

Fig. 1. (a) Selective OFDM relaying in a two-hop random network. (b) Selective OFDMA relaying in a two-hop random network.

- A2) $G_{sr_i}(n)$ and $G_{r_id}(n)$ are independent exponential random variables with mean $1/\lambda_{sr_i}$ and $1/\lambda_{r_id}$, respectively. Also, let $\lambda_i = \lambda_{sr_i} + \lambda_{r_id}$.
- A3) A high-SNR condition is assumed in deriving the diversity gain and power gain.
- A4) Each subchannel has equal transmitting power.

A1 allows us to concentrate on the comparisons of these two relaying schemes, and the results developed here can be easily extended to the case where the source-destination link is considered. Equal transmit power in each subchannel is a good choice for EBA because channel gains are not available at the transmitter. For BL, at high SNR, varying the amount of transmit power as a function of the channel state yields minimal gains [15]. Hence, A4 is a reasonable assumption.

III. Outage analysis for EBA

In this section, we will evaluate the end-to-end outage performance of selective OFDM relaying and selective OFDMA relaying. We first consider EBA. That is, each subchannel has the same number of bits, R. An outage occurs when at least one subchannel cannot successfully support the end-to-end transmission of the R bits.

Since relay selection is performed independently for each subchannel and channel gains are independent (A2), the end-to-end outage of selective OFDMA relaying is given by

$$P_{out,EBA}^{OFDMA} = 1 - \prod_{n=1}^{N} \left(1 - P_{out,EBA}^{OFDMA}(n)\right) \approx \sum_{n=1}^{N} P_{out,EBA}^{OFDMA}(n) \tag{1}$$

where $P_{out,EBA}^{OFDMA}(n)$ is the outage probability of subchannel n, and is given as

$$P_{out,EBA}^{OFDMA}(n) = \Pr\left[\frac{1}{2}\log(1 + \max_{i=1,\ldots,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right] \tag{2}$$

where the logarithms are base-2 unless otherwise noted, and where Γ is the SNR gap determined by coding techniques and

γ is the SNR in each subchannel without fading. With perfect coding, $\Gamma = 1$, and $\Gamma = 8.8$ dB without any coding [5].

Theorem 1. The end-to-end outage of selective OFDMA relaying scheme with EBA transmission is given by

$$P_{out,EBA}^{OFDMA} \approx \prod_{i=1}^{L} \lambda_i N \left(2^{2R}-1\right)^L \left(\frac{\Gamma}{\gamma}\right)^L \quad (3)$$

Proof: Rewrite (2) as

$$\begin{aligned} P_{out,EBA}^{OFDMA}(n) &= \Pr\left[\max_{i=1,\ldots,L} G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] \\ &= \prod_{i=1}^{L} \Pr\left[G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] \end{aligned} \quad (4)$$

The last step comes from A2; we also know that $G_{sr_id}(n)$ is an exponential random variable with parameter $\lambda_i = \lambda_{sr_i} + \lambda_{r_id}$ [16]. Then

$$\begin{aligned} \Pr\left[G_{sr_id}(n) < \frac{(2^{2R}-1)\Gamma}{\gamma}\right] &= 1 - \exp\left(-\lambda_i \frac{(2^{2R}-1)\Gamma}{\gamma}\right) \\ &\approx \lambda_i \frac{(2^{2R}-1)\Gamma}{\gamma} \end{aligned} \quad (5)$$

The last step is a high-SNR approximation. Substituting (4) and (5) into (1), we obtain (3). ■

From Theorem 1, we can see that selective OFDMA relaying can achieve L-fold diversity gain, which is the maximum for EBA. Also, λ_i varies with different relay locations, that is, the outage probability varies with relay locations.

Proposition 1: For the same N, L, and R, the minimum outage probability is achieved when the L relay nodes are located in the middle of the path from the source to the destination.

Proof: See Appendix I.

In this case, L relay nodes form a relay cluster. We assume that the distance between the source (destination) and the relay nodes is much larger than the distance between any two relay nodes. Also, the distance between any two relay nodes is sufficiently large so that the channel gains of different relay nodes are independent. Therefore, $G_{sr_i}(n)$ and $G_{r_id}(n)$ are i.i.d. random variables.

For selective OFDM relaying, one relay is selected to retransmit the entire OFDM symbol. With A2, the end-to-end outage of selective OFDM relaying is given by

$$P_{out,EBA}^{OFDM} = \prod_{i=1}^{L} P_{out,EBA,r_i}^{OFDM} \quad (6)$$

where P_{out,EBA,r_i}^{OFDM} is the end-to-end outage probability through relay i, and is given as

$$\begin{aligned} &P_{out,EBA,r_i}^{OFDM} \\ &= 1 - \prod_{n=1}^{N}\left(1 - \Pr\left[\frac{1}{2}\log(1 + G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right]\right) \end{aligned} \quad (7)$$

Theorem 2. The end-to-end outage of selective OFDM relaying scheme with EBA transmission is given by

$$P_{out,EBA}^{OFDM} \approx \prod_{i=1}^{L} \lambda_i N^L \left(2^{2R}-1\right)^L \left(\frac{\Gamma}{\gamma}\right)^L \quad (8)$$

Proof: Rewrite (7) as

$$P_{out,EBA,r_i}^{OFDM} = \Pr\left[\frac{1}{2}\log(1 + \min_{n=1,..,N} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < R\right] \quad (9)$$

From A2, we know that $\min_{n=1,..,N} G_{sr_id}(n)$ is an exponential random variable with parameter $N\lambda_i$. Hence

$$P_{out,EBA,r_i}^{OFDM} \approx N\lambda_i \frac{(2^{2R}-1)\Gamma}{\gamma} \quad (10)$$

Eq. (8) follows easily by substituting (10) into (6). ■

As for selective OFDMA relaying, the same L-fold diversity gain can be achieved[‡]. The power gain, however, is different. Comparing (3) and (8), we see that the performance improvement of selective OFDMA relaying over selective OFDM relaying is

$$\Delta\gamma_{EBA} = 10\frac{L-1}{L}\log_{10} N \quad (11)$$

From (11), we can see that selective OFDMA relaying always has better performance than selective OFDM relaying. The performance gap remains the same for different relay locations and different data rate R; it increases with an increase in the number of relays and the number of subchannels.

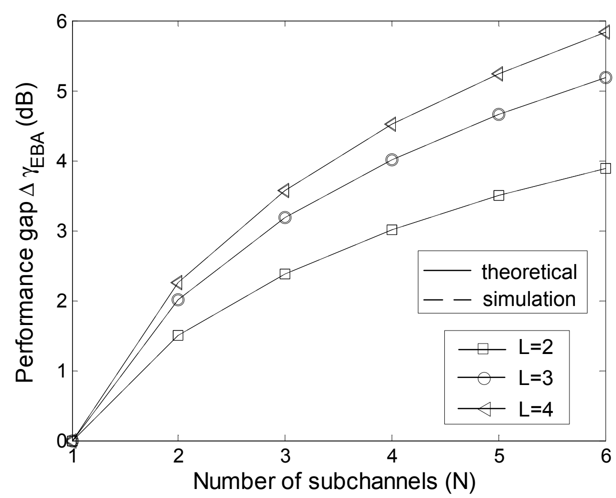

Fig. 2. Performance improvement of selective OFDMA relaying over selective OFDM relaying with $L = 2$, 3, and 4 relays.

Fig. 2 illustrates the performance improvement of selective OFDMA relaying over selective OFDM relaying with EBA. In the simulation, we assume that the L relay nodes are located in the middle of the source-to-destination path. We also assume that each relay node has the same distance to the source and the destination. The channels between the source and each relay and the channels between each relay and the destination are independent. A perfect match can be observed between the theoretical and simulation results. As expected, the performance gap increases with an increase in L and N. An almost 6-dB performance improvement can be achieved by selective OFDMA relaying when there are $L = 4$ relays and $N = 6$ subchannels.

[‡]In [13], we showed that no diversity gain can be obtained for selective OFDM relaying if the relay node with the highest combined SNR is chosen. Here, the relay with the largest minimum SNR among the N subchannels is chosen, and L-fold diversity gain can be achieved.

IV. Outage analysis for BL

So far, we have shown that selective OFDMA relaying and selective OFDM relaying both achieve L-fold diversity gain with EBA; however, only space diversity is exploited. To take advantage of frequency diversity, BL is adopted. In this section, we analyze and compare the outage of OFDMA relaying and OFDM relaying with BL.

With BL, an outage occurs when the sum rate of all subchannels is less than NR bits. The end-to-end outage of selective OFDMA relaying with BL is

$$P_{out,BL}^{OFDMA} = \Pr\left[\sum_{n=1}^{N}\frac{1}{2}\log(1+\max_{i=1,\dots,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \tag{12}$$

Theorem 3. The end-to-end outage of selective OFDMA relaying scheme with BL transmission is given by

$$P_{out,BL}^{OFDMA} \approx \prod_{i=1}^{L}\lambda_i^N 2^{2NRL}\left(L\ln 2\right)^{N-1} \times \left(\prod_{j=1}^{N-1}\frac{1}{j}(2NR)^{N-1}\right)\left(\frac{\Gamma}{\gamma}\right)^{-NL} \tag{13}$$

Proof: See Appendix II.

From Theorem 3, we can see that selective OFDMA relaying can achieve (NL)-fold diversity gain with BL. This means that all diversity gains are exploited by selective OFDMA relaying with BL. Also, as for EBA, the outage probability varies for different relay locations; for the same N, L and R, the outage probability achieves the minimum when L relay nodes are located in the middle of the path from the source to the destination.

For selective OFDM relying, one *best* relay is selected for retransmission. With A2, the end-to-end outage of selective OFDM relaying with BL is

$$P_{out,BL}^{OFDM} = \prod_{i=1}^{L}\Pr\left[\sum_{n=1}^{N}\frac{1}{2}\log(1+G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \tag{14}$$

Theorem 4. The end-to-end outage of selective OFDM relaying scheme with BL transmission is given by

$$P_{out,BL}^{OFDM} \approx \prod_{i=1}^{L}\lambda_i^N 2^{2NRL}\left(\ln 2\right)^{L(N-1)} \times \left(\prod_{j=1}^{N-1}\frac{1}{j}(2NR)^{N-1}\right)^{L}\left(\frac{\Gamma}{\gamma}\right)^{-NL} \tag{15}$$

Proof: See Appendix III.

Comparing (13) and (15), we see that selective OFDM relaying achieves the same NL-fold diversity gain as selective OFDMA relaying. The power gain, however, is different. The performance improvement of selective OFDMA relaying over selective OFDM relaying is

$$\Delta\gamma_{BL} = \frac{10(N-1)}{N}\log_{10}\left((CR)^{(L-1)/L}L^{-1/L}\right) \tag{16}$$

where $C = 2N(\ln 2)\left(\prod_{j=1}^{N-1}\frac{1}{j}\right)^{1/(N-1)}$

From (16), we see that selective OFDMA relaying always has better performance than selective OFDM relaying. The performance gap remains the same for different relay locations; it increases with an increase in L, N and R.

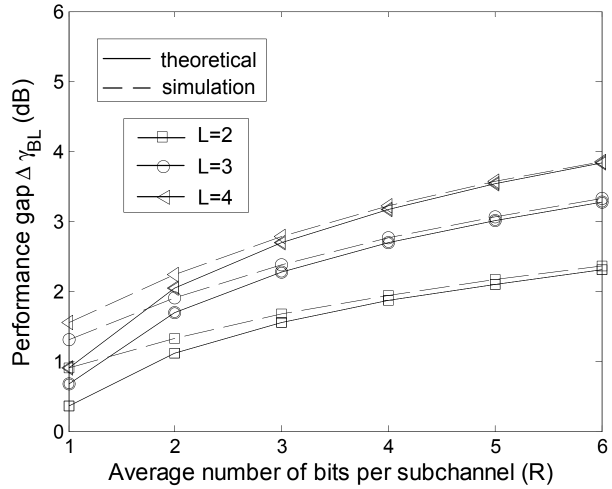

Fig. 3. Performance improvement of selective OFDMA relaying over selective OFDM relaying with $L = 2$, 3, and 4 relays ($N = 2$).

Fig. 3 illustrates the performance improvement of selective OFDMA relaying over selective OFDM relaying with BL. The same simulation environment as in Section III is adopted. We see that the analysis is quite accurate, especially for high data rate R. As expected, the performance gain increases with an increase in L and R.

V. Centralized and decentralized implementations

In the previous section, we showed that selective OFDMA relaying always outperforms selective OFDM relaying, no matter if EBA or BL is used. In this section, we will address issues related to centralized and decentralized implementations of these techniques.

With selective relaying, the best relay(s) should be selected based on the channel gains of the SR and RD links. If a central controller is available (such as a base station in a cellular network or an access point in a mesh network), it can collect all the channel information and then assign the transmission. In this scenario, selective OFDMA relaying is clearly preferred for its superior performance.

For ad hoc networks, decentralized relay selection strategies are preferred. Each relay node can obtain the channel gains of its own SR and RD links by listening to the RTS (Request-To-Send) signal and CTS (Clear-To-Send) signal. A similar decentralized relay selection algorithm as that in [14] can be adopted here. In this algorithm, each relay sets a timer based on a parameter. The larger the parameter is, the shorter the timer should be. In this way, the timer of the relay with the largest parameter will expire first. That relay then sends a flag signal. All other relays, while waiting for their timer to reduce to zero, are in listening mode. As soon as they hear the flag signal, they back off. This method requires that all the relays in a cluster hear each other.

For EBA, both selective OFDMA relaying and selective OFDM relaying can be implemented with the above decentralized algorithm. From (9), we know that, with selective OFDM relaying, each relay can set the timer according to the smallest $G_{sr_id}(n)$ among N subchannels. The one with the largest $\min_{n=1,...,N} G_{sr_id}(n)$ is selected. In the case of selective OFDMA relaying, however, relay selection needs to be performed in a per-subchannel manner, i.e., the best relay is selected for each subchannel. From (2), we know that relay i sets a timer for subchannel n according to $G_{sr_id}(n)$. In this way, each subchannel has a timer at each relay, which would significantly increase the selection delay: N-fold compared to selective OFDM relaying. Therefore, with a decentralized EBA, selective OFDMA relaying achieves better performance but at the expense of a larger selection delay.

For BL, selective OFDMA relaying needs the channel information of all SR and RD links to perform relay selection and bit loading. Hence, a significant amount of communications overhead is required for relays to exchange their channel information. Selective OFDM relaying, however, only needs the channel information of its own SR and RD links. Each relay calculates the maximum rate based on its own channel gains, and then sets the timer according to its own rate. The relay with the largest rate is selected to relay all the subchannels. We see that, for BL, selective OFDMA relaying achieves better performance at the expense of more communications overhead.

Another important practical issue is synchronization (such as timing and frequency offset estimation) among nodes, especially for selective OFDMA relaying. In selective OFDMA relaying, multiple relay nodes transmit simultaneously to the destination node. How to deal with multiple time and frequency offsets is a challenging problem. For OFDM relaying, only one relay node transmits to the destination. Traditional synchronization techniques can be adopted to estimate and compensate the offset.

In summary, for EBA, selective OFDMA relaying is always a good choice whether centralized control is available or not. For BL, selective OFDMA relaying is preferred for the case with centralized control; otherwise, selective OFDM relaying is more suitable.

VI. Conclusions

In this paper, we analyzed and compared the outage performance of selective OFDMA relaying and selective OFDM relaying in a two-hop random network. We showed that the same diversity gain can be achieved by each relaying scheme; the power gains, however, are different. Simulation results validated our analysis and showed that superior performance can always be achieved by selective OFDMA relaying, and the performance improvement remains the same with different relay locations. This approach is preferred for centralized systems because of its good performance. For decentralized systems, selective OFDMA relaying works well with EBA; if BL is employed, selective OFDM relaying is a good choice because of its simpler implementation.

Future work will include the consideration of a more practical environment; in particular, synchronization for the selective OFDMA relaying is an important issue and requires more investigation.

Appendix I
Proof of Proposition 1

For the same N, L, R, and γ, the outage probability achieves its minimum when λ_i, $i = 1, ..., L$, is a minimum. From A2, we know that $1/\lambda_{sr_i}$ and $1/\lambda_{r_id}$ are the means of $G_{sr_i}(n)$ and $G_{r_id}(n)$, respectively. With path loss, $1/\lambda_{sr_i}$ and $1/\lambda_{r_id}$ equal to $1/d^{\alpha}_{sr_i}$ and $1/d^{\alpha}_{r_id}$, respectively. Hence,

$$\lambda_i = \lambda_{sr_i} + \lambda_{r_id} = d^{\alpha}_{sr_i} + d^{\alpha}_{r_id} \tag{17}$$

where d_{sr_i} and d_{r_id} are the distance from the source to relay i and the distance from relay i to the destination, respectively; α is the path loss exponent. We can easily show that

$$d^{\alpha}_{sr_i} + d^{\alpha}_{sr_i} \geq 2\left(\frac{d}{2}\right)^{\alpha} \tag{18}$$

where d is the distance from the source to the destination; equality holds when relay i is located in the middle of the path from the source to the destination, that is, $d_{sr_i} = d_{r_id} = d/2$. Hence, the outage achieves its minimum when the relays are located in the middle of the path from the source to the destination.

Appendix II
Proof of Theorem 3

The proof of Theorem 3 requires the results of Theorem 1 in [17], which is rewritten here:

Theorem 1 in [17]: Let u_s and v_s be two independent random variables with the property that

$$\lim_{s\to\infty} s \cdot \Pr[u_s < t] = f(t)$$
$$\lim_{s\to\infty} s^d \cdot \Pr[v_s < t] = g(t),$$

where $f(t)$ and $g(t)$ are monotone increasing and integrable, and $f'(t)$ is integrable. Then

$$\lim_{s\to\infty} s^{d+1} \cdot \Pr[u_s + v_s < t] = \int_0^t g(t-x) f'(x) dx. \tag{19}$$

The end-to-end outage of OFDMA-relaying with BL is

$$\begin{aligned} P^{OFDMA}_{out,BL} &= \Pr\left[\sum_{n=1}^{N} \frac{1}{2}\log(1 + \max_{i=1,...,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \\ &\approx \Pr\left[\sum_{n=1}^{N} \frac{1}{2}\log(\max_{i=1,...,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < NR\right] \end{aligned} \tag{20}$$

The last step comes from the fact that $\log_2(1+x) \approx \log_2 x$, when $x \gg 1$ [15]. Let $u_n = \log(\max_{i=1,...,L} G_{sr_id}(n)\frac{\gamma}{\Gamma})$, then

$$\begin{aligned} \Pr[u_n < t] &= \Pr\left[(\max_{i=1,...,L} G_{sr_id}(n)\frac{\gamma}{\Gamma}) < 2^t\right] \\ &= \prod_{i=1}^{L} \Pr\left[G_{sr_id}(n) < \frac{2^t\Gamma}{\gamma}\right] \end{aligned} \tag{21}$$

We know that $G_{sr_id}(n)$ is an exponential random variable with parameter $\lambda_i = \lambda_{sr_i} + \lambda_{r_id}$ [16]. Then

$$\Pr[u_n < t] = \prod_{i=1}^{L} \left(1 - \exp\left(-\lambda_i \frac{2^t \Gamma}{\gamma}\right)\right) \tag{22}$$

Therefore

$$\lim_{\gamma\to\infty} \gamma^L \cdot \Pr[u_n < t] = \left(2^t \Gamma\right)^L \prod_{i=1}^{L} \lambda_i \tag{23}$$

Let $g_1(t) = f(t) = \left(2^t\Gamma\right)^L \prod_{i=1}^{L} \lambda_i$ and $f'(t) = 2^{tL} L \Gamma^L \prod_{i=1}^{L} \lambda_i \ln 2$. Applying Theorem 1 in [17], we get

$$\begin{aligned}\lim_{\gamma\to\infty} \gamma^{2L} \cdot \Pr[u_1 + u_2 < t] &= \int_0^t g_1(t-x) f'(x) dx \\ &= \prod_{i=1}^{L} \lambda_i^2 2^{tL} \Gamma^{2L} \left(L \ln 2\right) t\end{aligned} \tag{24}$$

Repeating the application of Theorem 1 in [17], that is, let $g_2(t) = \prod_{i=1}^{L} \lambda_i^2 2^{tL} \Gamma^{2L} \left(L \ln 2\right) t$ and $f'(t) = 2^{tL} L \Gamma^L \prod_{i=1}^{L} \lambda_i \ln 2$, then

$$\begin{aligned}&\lim_{\gamma\to\infty} \gamma^{3L} \cdot \Pr\left[\sum_{n=1}^{3} u_n < t\right] \\ &= \prod_{i=1}^{L} \lambda_i^3 2^{tL} \Gamma^{3L} \left(L \ln 2\right)^2 \left(\frac{1}{2} t^2\right) = g_3(t)\end{aligned} \tag{25}$$

In this way, we can easily prove the following equation by induction,

$$g_N(t) = \prod_{i=1}^{L} \lambda_i^N 2^{tL} \Gamma^{NL} \left(L \ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j} t^{N-1}\right). \tag{26}$$

Thus, sufficiently high SNR,

$$\begin{aligned}&\Pr\left[\sum_{n=1}^{N} u_n < t\right] \\ &= \prod_{i=1}^{L} \lambda_i^N 2^{tL} \left(L \ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j} t^{N-1}\right) \left(\frac{\Gamma}{\gamma}\right)^{NL}\end{aligned} \tag{27}$$

With $t = 2NR$ and substituting (27) into (20), (13) is obtained.

Appendix III
Proof of Theorem 4

The end-to-end outage of OFDM-relaying with BL is

$$P_{out,BL}^{OFDM} = \prod_{i=1}^{L} \Pr\left[\sum_{n=1}^{N} \frac{1}{2} \log(1 + G_{sr_id}(n) \frac{\gamma}{\Gamma}) < NR\right] \tag{28}$$

Letting $L = 1$ in Theorem 3, we get

$$\begin{aligned}&\Pr\left[\sum_{n=1}^{N} \frac{1}{2} \log(1 + G_{sr_id}(n) \frac{\gamma}{\Gamma}) < NR\right] \\ &\approx \lambda_i^N 2^{2NR} \left(\ln 2\right)^{N-1} \left(\prod_{j=1}^{N-1} \frac{1}{j} (2NR)^{N-1}\right) \left(\frac{\Gamma}{\gamma}\right)^{N}\end{aligned} \tag{29}$$

Substituting (29) into (28), gives (15).

References

[1] F. H. P. Fitzek and M. D. Katz, *Cooperation in Wireless Networks: Principles and Applications*, Springer, 2006.
[2] A. Scaglione, D. Goeckel, and J. N. Laneman, “Cooperative communications in mobile ad hoc networks: rethinking the link abstraction,” *IEEE Sig. Proc. Mag.*, vol. 23, no. 9, pp. 18-29, Sep. 2006.
[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” *IEEE Trans. on Inform. Th.*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
[4] J. N. Laneman and G. W. Wornell, “Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks,” *IEEE Trans. on Inform. Th.*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.
[5] Y. Li and G. L. Stuber, *Orthogonal Frequency Division Multiplexing for Wireless Communications*, Springer, 2006.
[6] IEEE802.11, *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, IEEE August 1999.
[7] ETSI, *Digital Audio Broadcasting (DAB)*, 2nd ed., May 1997, ETS 300 401.
[8] ETSI, *Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television*, Aug. 1997, EN 300 744 V1.1.2.
[9] IEEE802.16-2004, *IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface*, IEEE, 2004.
[10] H. Ekstrom, A. Furuskar, J. Karlsson, M. Meyer, S. Parkvall, J. Torsner, and M. Wahlqvist, “Technical solutions for the 3G long-term evolution,” *IEEE Commun. Mag.*, vol. 44, no. 3, pp. 38-45, Mar. 2006.
[11] [Online]. Available: http://www.ieee802.org/16/relay/
[12] [Online]. Available: http://grouper.ieee.org/groups/802/11/
[13] L. Dai, B. Gui, and L. J. Cimini, Jr., “Selective relaying in OFDM multihop cooperative networks,” in *Proc. of IEEE WCNC 2007*, Hong Kong, Mar. 2007.
[14] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.
[15] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge University Press, 2005.
[16] S. M. Ross. *Introduction to Probability Models*, Academic Press, 9th edition, 2006.
[17] J. N. Laneman, “Limiting analysis of outage probabilities for diversity schemes in fading channels,” in *Proc. of IEEE Globecom*, vol. 3, pp. 1242-1246, Dec. 2003.

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