Lecture 2. Fading Channel

• Characteristics of Fading Channels
• Modeling of Fading Channels
• Discrete-time Input/Output Model
Radio Propagation in Free Space

- **Speed**: \( c = 299,792,458 \text{ m/s} \)
- **Isotropic**
  - Received power at a particular location decays with distance: \( P \sim r^{-2} \)

\[ s(t) = \cos 2\pi ft \]
\[ y(t) = \frac{\beta \cos 2\pi f(t - r / c)}{r} \]
Radio Propagation in Terrestrial Environment

Transmission Power: $P_t$

Received Power: $P_r$

Distance between transmitter and receiver: $r$

Fluctuation around the local mean: small-scale fading

Fluctuation of the local mean: large-scale fading
Large-scale Fading

- **Large-scale fading** -- *Log-normal shadowing*
  - Attributed to the random variation of propagation environment.
  - Empirically modeled as a *log-normal* random variable with mean $\mu$ and variance $\sigma^2$.
  - Mean $\mu$ is determined by the path loss: $\mu \sim r^\alpha$, $\alpha > 2$.

$\log(P_r / P_t)$

$\log(r)$

- $\alpha$ is called path-loss factor, which depends on the propagation environment.
- The higher the density of obstacles, the larger $\alpha$. 
Small-scale Fading

- Small-scale fading
  - Attributed to 1) multiple arriving paths; and 2) movements of the transmitter and/or receiver.
A Simple Two-path Model (1)

- Reflecting wall, fixed antenna

✓ Transmitted signal: \( s(t) = \cos 2\pi ft \)

✓ Received signal: 
\[
y(t) = \frac{\beta \cos 2\pi f(t - r/c)}{r} - \frac{\beta \cos 2\pi f(t - (2d - r)/c)}{2d - r}
\]

The phase difference between the two waves:

\[
\Delta \phi = \left( \frac{2\pi f(2d - r)}{c} + \pi \right) - \left( \frac{2\pi fr}{c} \right)
\]

\[
= \frac{4\pi f}{c}(d - r) + \pi
\]

What happens if \( \Delta \phi \) changes from \( \pi \) to \( 2\pi \)?

- \( \Delta \phi = 2k\pi \): Constructively add
- \( \Delta \phi = (2k+1)\pi \): Destructively add
Coherence Bandwidth vs. Delay Spread

- An appreciable change of received signal will be observed if the frequency of transmitted signal $f$ changes by:
  $$\frac{c}{4(d-r)}$$

- The difference between the propagation delays along the two signal paths is:
  $$\frac{2d-r}{c} - \frac{r}{c} = \frac{2(d-r)}{c}$$

- $T_d \sim \frac{1}{W_c}$
A Simple Two-path Model (2)

- Reflecting wall, moving antenna

✓ Transmitted signal: \( s(t) = \cos 2\pi ft \)

✓ Received signal:

\[
y(t) = \frac{\beta \cos 2\pi f(t - r(t)/c)}{r(t)} - \frac{\beta \cos 2\pi f(t - (2d - r(t))/c)}{2d - r(t)}
\]

Doppler Shift: \( D_1 = -\frac{fv}{c} \)

\[
= \frac{\beta \cos 2\pi f((1 - v/c)t - r_0/c)}{r_0 + vt} - \frac{\beta \cos 2\pi f((1 + v/c)t + (r_0 - 2d)/c)}{2d - r_0 - vt}
\]

Doppler Shift: \( D_2 = \frac{fv}{c} \)

\[
\approx \frac{2\beta \sin 2\pi f(vt/c + (r_0 - d)/c)\sin 2\pi f(t - d/c)}{r_0 + vt}
\]
Coherence Time vs. Doppler Spread

- An appreciable change of received signal will be observed if time $t$ changes by:
  \[
  \frac{c}{4fv} \leq T_c
  \]

- The difference between the Doppler shifts of the two paths is:
  \[
  D_s = D_2 - D_1 = 2fv/c
  \]

- $T_c \sim \frac{1}{D_s}$

Coherence Time $T_c$

Doppler Spread $D_s$
Modeling of Fading Channels

A fading channel can be modeled as a Linear Time-Varying system.

\[ y(t) = \int_{-\infty}^{\infty} h(\tau, t)s(t - \tau)d\tau \]

- Suppose that \( h(\tau, t) \) is a deterministic function of time \( t \) and delay \( \tau \).

Time-variant transfer function: \( H(f, t) = \mathcal{F}_t(h(\tau, t)) \)

Delay Doppler function: \( S(\tau, \nu) = \mathcal{F}_t(h(\tau, t)) \)

Doppler-variant transfer function: \( D(f, \nu) = \mathcal{F}_t(S(\tau, \nu)) = \mathcal{F}_t(H(f, t)) \)

\( \mathcal{F}(\cdot) \) denotes Fourier transform.
Modeling of Fading Channels

Transmitted signal $s(t)$

Channel

Received signal $y(t)$

- A fading channel can be modeled as a **Linear Time-Varying** system.

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)s(t - \tau)d\tau$$

- Suppose that $h(\tau, t)$ is a WSS process with uncorrelated scatterers:
  The autocorrelation function $R_h(\tau_1, t_1, \tau_2, t_2) = E[h(\tau_1, t_1)h^*(\tau_2, t_2)] = R_h(\tau, \Delta t)$

  **Time-variant transfer function:** $R_H(\Delta f, \Delta t) = \mathcal{F}_\tau (R_h(\tau, \Delta t))$

  **Delay Doppler function:** $R_S(\tau, \nu) = \mathcal{F}_{\Delta \tau} (R_h(\tau, \Delta t))$

  **Doppler-variant transfer function:** $R_D(\Delta f, \nu) = \mathcal{F}_\tau (R_S(\tau, \nu)) = \mathcal{F}_{\Delta \tau} (R_H(\Delta f, \Delta t))$
Coherence Bandwidth vs. Delay Spread

- Let $\Delta t = 0$:

  $$|R_H(\Delta f, 0)|$$

  $\Delta f$

  Coherence Bandwidth $W_c$

  $$F^{-1}_{\Delta f}(\cdot)$$

  $F_{\tau}(\cdot)$

  $$R_h(\tau, 0)$$

  Delay Spread $T_d$

  $$T_d \sim \frac{1}{W_c}$$

- Flat fading: Signal Bandwidth $W \ll$ Coherence Bandwidth $W_c$
  Symbol Time $T \gg$ Delay Spread $T_d$

- Frequency-selective fading: Signal Bandwidth $W \gg$ Coherence Bandwidth $W_c$
  Symbol Time $T \ll$ Delay Spread $T_d$
Coherence Time vs. Doppler Spread

- Let $\tau = 0$:
  \[ R_h(0, \Delta t) \]

  \[ \Delta t \]

  Coherence Time
  \[ T_c \]

  \[ \mathcal{F}_{\Delta t}(\cdot) \rightarrow \mathcal{F}_{\nu}^{-1}(\cdot) \]

  Doppler Spread
  \[ D_s \]

  \[ R_s(0, \nu) \]

  \[ \nu \]

- Slow fading: Symbol Time $T \ll$ Coherence Time $T_c$
  Signal Bandwidth $W \gg$ Doppler Spread $D_s$

- Fast fading: Symbol Time $T \gg$ Coherence Time $T_c$
  Signal Bandwidth $W \ll$ Doppler Spread $D_s$

\[ T_c \sim \frac{1}{D_s} \]
Discrete-time Input/Output Model

- \( y(t) = \int_{-\infty}^{\infty} h(\tau, t)s(t - \tau)d\tau = \sum_{n} s[n] \sum_{i} a_{i}(t) \Psi_{T_{0}}(t - \tau_{i}(t) - nT_{0}) \)

\[ h(\tau, t) = \sum_{i} a_{i}(t) \delta(\tau - \tau_{i}(t)) \quad \text{and} \quad s(t) = \sum_{n} s[n] \Psi_{T_{0}}(t - nT_{0}) \]

- \( a_{i}(t) \): attenuation at time \( t \) of path \( i \)
- \( \tau_{i}(t) \): propagation delay at time \( t \) of path \( i \)

- \( \Psi_{T_{0}}(t) \): modulation pulse

**Sampled output at** \( t=mT_{0} \):

\[ y[m] = \sum_{n} s[n] \sum_{i} a_{i}(mT_{0}) \Psi_{T_{0}}((m-n)T_{0} - \tau_{i}(mT_{0})) \]

Let \( l=m-n \).

\[ y[m] = \sum_{l} s[m-l] \sum_{i} a_{i}(mT_{0}) \Psi_{T_{0}}(lT_{0} - \tau_{i}(mT_{0})) \]

- **Discrete-time Input/Output Model:**

\[ y[m] = \sum_{l} h_{l}[m]s[m-l] + z[m] \]
Discrete-time Input/Output Model

\[ y[m] = \sum_{l=0}^{L-1} h_l[m] s[m-l] + z[m] \]

- How many channel filter taps can be obtained (how to determine the value of \( L \))?
  
  Delay spread: \( T_d = \max_{i,j} |\tau_i - \tau_j| \)  
  
  Sampling rate: \( W \)  
  
  - Flat fading: \( L=1 \)  
  
  - Frequency-selective fading: \( L >> 1 \)

- How fast does the channel filter tap gain \( h_l[m] \) vary with time in one symbol time \( T \)?  
  
  Depends on the coherence time \( T_c \).
  
  - Slow fading: \( h_l[m] \) can be regarded as constant in \( T \)  
  
  - Fast fading: \( h_l[m] \) varies with time in \( T \)
More about $h_l[m]$

- Without line-of-sight (LOS) paths: $h_l[m]$ can be modeled as a zero-mean complex Gaussian random variable $\mathcal{CN}(0, \sigma_s^2)$.
  - The magnitude follows Rayleigh distribution.

- With one LOS path: $h_l[m]$ can be modeled as

$$h_l[m] = \sqrt{\frac{k}{k+1}} \sigma_s e^{i\vartheta} + \sqrt{\frac{1}{k+1}} \mathcal{CN}(0, \sigma_s^2)$$

  $k$: the ratio of the power in the LOS path and that in the scattered paths.
  - The magnitude follows Ricean distribution.
More about $h_i[m]$

- **Availability of Channel State Information (CSI)**
  
  **CSIR**: CSI is available at the receiver side.
  - available through channel measurement
  - required for coherent detection

  **CSIT**: CSI is available at the transmitter side.
  - available through channel measurement and feedback
  - optional

- **Ergodicity**
  
  - indispensable condition to achieve the Shannon capacity
  - may not hold in delay-limited scenarios
Summary

- Fading Channel
  - Large-scale fading
    - Path loss
    - Shadowing
  - Small-scale fading
    - Delay spread (caused by multipath)
    - Doppler spread (caused by mobility)
- Discrete-time input/output model

\[ y[m] = \sum_{l=0}^{L-1} h_l[m] s[m-l] + z[m] \]