Zigzag Codes and Concatenated Zigzag Codes

Li Ping$^1$ and Nam Phamdo$^2$

$^1$Department of Electronics Engineering, City University of Hong Kong, Hong Kong
$^2$Department of Electrical and Computer Engineering, SUNY-Stony Brook, Stony Brook, NY 11794-2350 USA
eilping@cityu.edu.hk, phamdo@sbee.sunysb.edu

ABSTRACT

This paper introduces a family of error-correcting codes called zigzag codes. A zigzag code is described by a highly structured zigzag graph. Due to the structural properties of the graph, very low-complexity soft-in, soft-out decoding rules can be implemented. We present a decoding rule, based on the Max-Log-MAP (MLM) formulation, which requires a total of only 20 addition-equivalent operations per information bit, per iteration. Simulation of a rate-1/2, four-dimensional concatenated zigzag code with interleaver length 65536 yields a bit error rate (BER) of at 0.9 dB and 1.4 × 10$^{-5}$ at 10$^{-5}$.

The MLM of the information bits can be determined as:

$$L[d(i,j)] = d(i,j) + W(F[p(i-1)], d(i,2), \ldots, d(i,J)), \ldots,$$

where $F[p(notifiable)] = +\infty$, $B[p(I)] = p(I)$, and

$$W(a_1, a_2, \ldots, a_n) \triangleq \left[ \prod_{j=1}^{n} \frac{a_j}{1+|a_j|} \right].$$

The MLM of the parity bits can be determined as:

$$L[d(i,j)] = d(i,j) + W(F[p(i-1)], d(i,1), d(i,2), \ldots, d(i,J), B[p(i)]) .$$

CONCATENATED ZIGZAG CODES

A concatenated zigzag code is described by a triplet $(I,J,K)$. Let $D_k = \pi_k(D)$ be an interleaved version of the data matrix $D$, $k = 1,2,\ldots,$ $K$. For each $D_k$, we form an $I \times 1$ parity column vector $P_k$ according to the figure. The transmit codeword consists of $[D_1, P_1, P_2, \ldots, P_K]$ and the overall code rate is $J/(J+K)$. It can be shown that the overall decoding cost for a $K$-dimensional concatenated zigzag code is $K(4+4/J)$ arithmetic equivalent operations per information bit per iteration (AEO/IB/Iter), e.g., with $K = J = 4$, the decoding cost is 20 AEO/IB/Iter. As comparison, the complexity cost of a “standard” turbo code is 192 floating point operations per information bit per iteration.

Simulation results of a rate-1/2 concatenated zigzag code with interleaver length of 400 ($([I,J,K] = (100, 4, 4))$ are presented below, and is compared with 16-state turbo code. It is seen that the performance of the turbo code is slightly better at relatively high BER. However, the zigzag code surpasses the turbo code for BER at about 10$^{-5}$. This implies that the proposed code is useful for data communication systems where very low BER is required.