

An Efficient Symbol-by-Symbol MAP Decoding Algorithm for the Golay Code

Li Ping and Kwan L. Yeung, e-mail: eeliping@cityu.edu.hk

Department of Electronics Engineering, City University of Hong Kong, Hong Kong

Abstract

An efficient symbol-by-symbol soft-in/soft-out MAP decoding algorithm for the Golay code is presented based on the coset partitioning principle. Its application in the iterative decoding of concatenated Golay codes is examined. Its advantage in constructing short frame length concatenated codes is demonstrated.

I. Introduction

The MAP (maximum *a posteriori*) symbol-by-symbol soft-in/soft-out decoding for the convolutional codes plays an crucial rule in the iterative decoding of turbo codes. Here we will show that efficient MAP decoding can also be devised for the Golay code.

II. The hexacode, the SPC code and the Golay code

The following [1] are two mappings from the elements of GF[4], denoted by $\{S^{(m)}\}$, to 4×1 vectors over GF[2].

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ \text{even} & \frac{1}{S^{(0)}} & \frac{-1}{S^{(1)}} & \frac{-1}{S^{(2)}} & \frac{1}{S^{(3)}} & \text{odd} & \frac{-1}{S^{(0)}} & \frac{1}{S^{(1)}} & \frac{1}{S^{(2)}} & \frac{-1}{S^{(3)}} \end{array}$$

Applying the even interpretations to the codewords in the hexacode [1] results in $64 \ 4 \times 6$ arrays, referred to as H^e . Let P^e be the length-6 even SPC (Single Parity Check) codes. Let $h = \{h[i,j]\} = \{h_1, h_2, h_3, h_4, h_5, h_6\} \in H^e$. Then half of the Golay code C with elements in $\{1, -1\}$, denoted by C^e , can be constructed as [1]

$$C^e = \{p * h \mid p \in P^e, h \in H^e\} \quad (1a)$$

$$p * h \triangleq [p_1 h_1, p_2 h_2, p_3 h_3, p_4 h_4, p_5 h_5, p_6 h_6] \quad (1b)$$

The other half of C can be constructed similarly using the odd interpretations and odd SPC code.

III. The MAP decoding algorithm of C

Let the received signals be $x = c + n$, where n is an AWGN array with variance σ^2 . Denote $v = \{v[i,j]\} = \{2x[i,j]/\sigma^2\}$. The MAP decoder for C produces,

$$L[i, j] = \log \left(\sum_{\substack{c \in C \\ c[i,j]=+1}} e^{<c,v>/2} / \sum_{\substack{c \in C \\ c[i,j]=-1}} e^{<c,v>/2} \right) \quad (2)$$

From (1), every h determines a subset of 32 codewords, referred to as an even h -coset. The 128 h -cosets (including even and odd) form a coset partition of C. Within an even h -coset η generated by h , the summations in (2) can be partially evaluated as,

$$\sum_{\substack{c \in \eta \\ c[i,j]=\pm 1}} e^{<c,v>/2} = \sum_{\substack{p \in P^e \\ p_j = c[i,j]/h[i,j]}} e^{<p,w>/2} \quad (3)$$

where $w \triangleq [<h_1, v_1>, \dots, <h_6, v_6>]$.

Eqn. (3) is essentially a MAP decoding for P^e . Due to the simple structure of P^e , (3) can be calculated as,

$$\sum_{\substack{p \in P^e \\ p_j = \pm 1}} e^{<p,w>/2} = \frac{a_j^\pm}{2} \left(\frac{\prod_{k=1}^6 (a_k^+ + a_k^-)}{a_j^+ + a_j^-} \pm \frac{\prod_{k=1}^6 (a_k^+ - a_k^-)}{a_j^+ - a_j^-} \right) \quad (4)$$

where $a_j^\pm = e^{\pm w_j/2}$. The results of (4) can be combined

to produce the summations in (2). Efficient combining techniques can also be developed using the coset partitioning principle [2]. The computational cost of the resultant algorithm is about 103 additions and 75 multiplications per information bit. This is roughly comparable to that of a MAP decoding for a 16-state convolutional code.

IV. Application

The performances of concatenated Golay codes and turbo codes are compared in Fig.1 for short frame length typically required by mobile applications. The encoding and decoding processes for the concatenated Golay codes is similar to that of [3], incorporating the algorithm described above. The decoding complexity of the two approaches are similar but the memory usage of the Golay code based scheme is considerably lower.

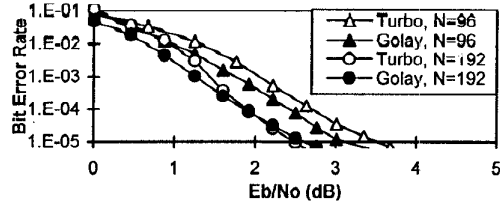


Fig.3 Performance comparisons of the concatenated Golay codes and Turbo codes (generator polynomials: 21 and 25). Rate=1/3. Iteration number=10. N is the number of information bits in a frame.

References

- [1] A. Vardy and Y. Ber'ry, "More efficient soft decoding of the Golay codes", *IEEE Trans Inform. Theory*, vol. IT-37, pp.667-672, May 1991.
- [2] Li Ping, S. Chan and K.L. Yeung "Max-Log-MAP filtering algorithm for decoding product F_{24} code", Proc. IEEE 1997 International Conference on Communications, pp. 1366-1370, June 1997.
- [3] Li Ping, S. Chan and K.L. Yeung, "Iterative decoding of multidimensional concatenated single parity check codes", Proc. IEEE 1998 International Conference on Communications, Atlanta, USA, June 1998.