

# High-Rate Interleave-Division-Multiplexing Space-Time Codes †

Li Ping, K. Y. Wu and W. K. Leung

Department of Electronic Engineering, City University of Hong Kong  
 eeliping@cityu.edu.hk, kewu@ee.cityu.edu.hk, wkleung@ee.cityu.edu.hk

**Abstract** - We propose a family of simple multi-layer space-time codes based on random interleaving and power allocation techniques. We show that the optimal space-time coding problem can be effectively resolved by superimposing several ideal low-rate codes for an AWGN channel. Simulation results demonstrate that the proposed high-rate space-time codes can achieve performance close to the theoretical limit.

## I. INTRODUCTIONS

Recently, significant progresses have been made on space-time (ST) coding [1]-[3]. However, analysis and design of space-time codes to approach the theoretical limit [4] remain difficult issues for high-rate applications.

This paper is concerned with a multi-layer ST coding scheme [5] based on random interleaving and power allocation strategies. The basic principle is very simple: several coded and interleaved signal streams are transmitted simultaneously across all antennas using carefully calculated power levels. There is no sophisticated structural design involved. The main contributions of the paper are listed below.

- For fast performance assessment, we introduce a signal-to-noise-ratio (SNR) evolution technique for fixed channels and a bounding technique for quasi-static fading channels. These techniques are simple, efficient and relatively accurate.
- A power allocation technique is developed which greatly increases the achievable rates of the proposed scheme.
- It is shown that the proposed scheme can potentially achieve the theoretical limit provided that an ideal low-rate forward error correction (FEC) code is used for each layer. This establishes a connection between ST coding and conventional additive white Gaussian noise (AWGN) channel coding problems. (*Note:* Although here a low-rate code is assumed for each layer, there is no limitation on the overall rate considering all layers. The focus of this paper is actually on high-rate systems.)
- Simulation results show that the proposed high-rate scheme can achieve performance close to the theoretical limit.

## II. SYSTEM MODEL

### A. Transmitter Principle

Consider a system with  $N$  transmit antennas and one receive antenna (an  $N \times 1$  system) in a quasi-static Rayleigh fading channel. The proposed multi-layer interleave-division-multiplexing ST (ML-IDM-ST) coding scheme is shown in

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Fig.1. The inputs are  $K$  equal-length sequences  $\{d_k, k = 1, \dots, K\}$  fed into  $K$  binary FEC encoders, generating  $c_k = \{c_{k,i}\}$  ( $c_{k,i} \in \{+1, -1\}$ ). Signals from the same encoder are referred to as a layer.

With binary-phase-shift-keying (BPSK) modulation, each  $c_k$  is independently interleaved  $N$  times, producing  $\{x_k^{(n)}, n = 1, \dots, N\}$ , where  $x_k^{(n)} = \{x_{k,j}^{(n)}, j = 1, \dots, J\}$  with  $J$  the frame length. With quadrature-phase-shift-keying (QPSK) modulation, each interleaved version of  $c_k$  is divided into two equal-length sequences to form the real and imaginary parts of  $x_k^{(n)}$ . The signals in  $\{x_k^{(n)}, n = 1, \dots, N\}$  are scaled by a common amplitude factor  $\sqrt{p_k}$  (see Fig.1) before distributed to the  $N$  transmit antennas. For the  $n$ th transmit antenna, the transmitted signal is  $\sum_{k=1}^K \sqrt{p_k} x_k^{(n)}$ . The signal received at time  $j$  is

$$y_j = \sum_{n=1}^N \alpha^{(n)} \sum_{k=1}^K \sqrt{p_k} x_{k,j}^{(n)} + n_j \quad (1)$$

where  $\alpha^{(n)}$  is the fading coefficient for the  $n$ th transmit antenna,  $\{n_j\}$  are samples of an AWGN process with zero-mean and variance  $\sigma^2 = N_0/2$  per dimension. Assume the same FEC code  $C$  with rate  $R_C$  for all layers. The overall rate  $R = KR_C$  for BPSK and  $R = 2KR_C$  for QPSK.

A key property of the above scheme is the use of different power factors  $\{p_k, k = 1, \dots, K\}$  for  $K$  layers, as will be elaborated later.

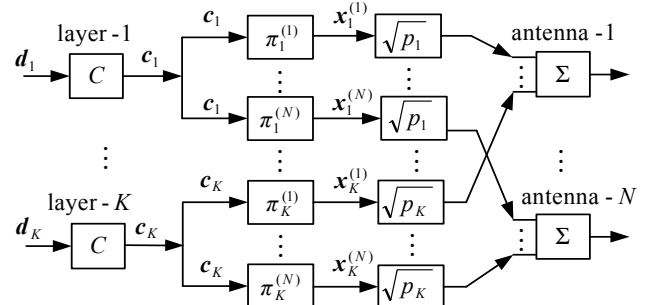


Figure 1. The transmitter structure of a  $N$ -antenna,  $K$ -layer ML-IDM-ST code, where  $\pi_k^{(n)}$  is the interleaver for layer- $k$  on the  $n$ th transmit antenna.

### B. Decoding Principle

We employ a sub-optimal iterative decoder [5], which consists of an elementary signal estimator (ESE) and  $K$  a posteriori probability (APP) decoders (DECs), operating iteratively [5]. Fig. 2 illustrates a part of the receiver structure in which only the DEC for layer- $k$  (denoted by DEC- $k$ ) is shown. The DECs for other layers are connected to the ESE in the same way as DEC- $k$ .

We rewrite (1) as

$$y_j = \alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} \quad (2)$$

where

$$\zeta_{k,j}^{(n)} = \sum_{(n',k') \neq (n,k)} \alpha^{(n')} \sqrt{p_{k'}} x_{k',j}^{(n')} + n_j. \quad (3)$$

Denote by  $E(\cdot)$  and  $\text{Var}(\cdot)$  the mean and variance functions, respectively. The main operations involved in decoding are listed below for BPSK modulation and real  $\alpha \equiv \{\alpha^{(n)}\}$  [5]. (Refer to Fig. 2 for the notations involved.) A generalization to QPSK modulation and complex  $\alpha$  can be found in [5].

**(a) Initialization:** Set  $\tilde{L}(x_{k,j}^{(n)}) = 0 \quad \forall k, n, j.$

**(b) Main iteration:**  
 $E(x_{k,j}^{(n)}) = \tanh(\tilde{L}(x_{k,j}^{(n)})/2) \quad \forall k, n, j. \quad (4a)$

$$\text{Var}(x_{k,j}^{(n)}) = 1 - (E(x_{k,j}^{(n)}))^2 \quad \forall k, n, j. \quad (4b)$$

$$E(\zeta_{k,j}^{(n)}) = \sum_{(n',k') \neq (n,k)} \alpha^{(n')} \sqrt{p_{k'}} E(x_{k',j}^{(n')}) \quad \forall k, n, j. \quad (5a)$$

$$\text{Var}(\zeta_{k,j}^{(n)}) = \sum_{(n',k') \neq (n,k)} |\alpha^{(n')}|^2 p_{k'} \text{Var}(x_{k',j}^{(n')}) + \sigma^2 \quad \forall k, n, j. \quad (5b)$$

$$\text{Ext}(x_{k,j}^{(n)}) = \frac{2\alpha^{(n)} \sqrt{p_k}}{\text{Var}(\zeta_{k,j}^{(n)})} \cdot (y_j - E(\zeta_{k,j}^{(n)})) \quad \forall k, n, j. \quad (6)$$

$$\tilde{L}(c_{k,i}) = \sum_{(n,j) \in S(c_{k,i})} \text{Ext}(x_{k,j}^{(n)}) \quad \forall k, i. \quad (7)$$

In (7),  $S(c_{k,i})$  is the index set of  $N$  replicas in  $\{x_{k,j}^{(n)}, \forall n, j\}$  related to  $c_{k,i}$  (for all  $(n, j)$  combinations). Each DEC- $k$  carries out an APP decoding at this stage using  $\{\tilde{L}(c_{k,i}), \forall i\}$  as the input to produce *a posteriori* log-likelihood ratios (LLRs)  $\{L(c_{k,i}), \forall i\}$  for  $c_k$ , which are used to update  $\{\tilde{L}(x_{k,j}^{(n)}), \forall n, j\}$  as (assuming  $(n, j) \in S(c_{k,i})$ )

$$\tilde{L}(x_{k,j}^{(n)}) = L(c_{k,i}) - \text{Ext}(x_{k,j}^{(n)}) \quad \forall k, n, j. \quad (8)$$

Then go back to (4) for the next iteration. After the final iteration, each DEC- $k$  makes a hard decision  $\hat{d}_k$  for  $d_k$ . The cost per coded bit per layer involved in (4)~(8) is independent of  $K$  and grows only linearly with  $N$  [5].

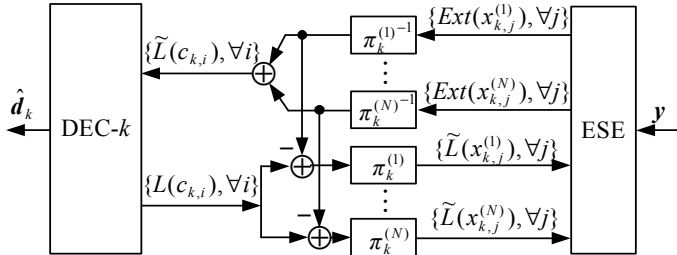


Figure 2. A part of the receiver structure of the ML-IDM-ST code related to layer- $k$ , where  $\mathbf{y} = \{y_j\}$ . The DECs for other layers are connected to the ESE in the same way as DEC- $k$ .

### III. PERFORMANCE ANALYSIS

We now derive a simple performance assessment technique for ML-IDM-ST codes. The method is semi-analytical since some of the functions involved (related to the FEC codes) are pre-calculated by simulation (similar to [6][7]). It provides a fast technique to predict the performance of ML-IDM-ST codes without time consuming simulations.

#### A. Performance Analysis with Fixed Fading Coefficients

We first consider the performance analysis of ML-IDM-ST codes with real and fixed  $\alpha$ . Notice that  $\text{Var}(\zeta_{k,j}^{(n)})$  in (6) is a function of  $j$  and this makes the analysis of (6) a difficult task. To avoid this, we introduce an approximation of (6). Define

$$V_{c_k} \equiv \frac{1}{L} \times \sum_{i=1}^L (1 - \tanh^2(\beta L(c_{k,i})/2)). \quad (9)$$

where  $L$  is the codeword length of  $C$ , and  $\beta$  is a constant (see below). Approximating  $\text{Var}(x_{k,j}^{(n)})$  by  $V_{c_k}$  in (5b), we get

$$\text{Var}(\zeta_{k,j}^{(n)}) \approx V_{\zeta_k^{(n)}} \equiv \sum_{(n',k') \neq (n,k)} |\alpha^{(n')}|^2 p_{k'} V_{c_k} + \sigma^2. \quad (10)$$

Substituting (10) into (6), we have

$$\text{Ext}(x_{k,j}^{(n)}) = \frac{2\alpha^{(n)} \sqrt{p_k}}{V_{\zeta_k^{(n)}}} \cdot (\alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - E(\zeta_{k,j}^{(n)})). \quad (11)$$

**Notes:** (i) The coefficient  $2\alpha^{(n)} \sqrt{p_k} / V_{\zeta_k^{(n)}}$  in (11) is not a function of  $j$ , which makes it easier for the discussions in (13) and (16) below. (ii) Using (11) leads to certain performance loss compared with using (6). (iii) In (9),  $\beta$  is chosen to minimize the loss mentioned in (ii). As a rule of thumb, we use  $\beta = (N-1)/N$ .

In (11),  $\alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)}$  and  $\zeta_{k,j}^{(n)} - E(\zeta_{k,j}^{(n)})$  represent signal and distortion components, respectively. Since  $x_{k,j}^{(n)} = \pm 1$ , signal power  $E(|\alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)}|^2) = |\alpha^{(n)}|^2 p_k$ . We approximate the average noise power after soft cancellation (for fixed  $k$  and  $n$ ) by  $V_{\zeta_k^{(n)}}$ ,

$$E(|\zeta_{k,j}^{(n)} - E(\zeta_{k,j}^{(n)})|^2) \approx V_{\zeta_k^{(n)}} \quad (12)$$

The average signal-to-noise ratio (SNR) of  $\{\text{Ext}(x_{k,j}^{(n)}), \forall j\}$ , denoted by  $\text{snr}_k^{(n)}$ , is

$$\text{snr}_k^{(n)} = \frac{E(|\alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)}|^2)}{V_{\zeta_k^{(n)}}} = \frac{|\alpha^{(n)}|^2 p_k}{\sum_{(n',k') \neq (n,k)} |\alpha^{(n')}|^2 p_{k'} V_{c_k} + \sigma^2} \quad (13)$$

Now substituting (11) into (7), we have

$$\tilde{L}(c_{k,i}) = \sum_{(n,j) \in S(c_{k,i})} \frac{2\alpha^{(n)} \sqrt{p_k}}{V_{\zeta_k^{(n)}}} \cdot (\alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - E(\zeta_{k,j}^{(n)})) \quad (14)$$

Besides a scaling factor of 2, (14) can be regarded as a maximum ratio combining (MRC) of  $N$  independent distorted signals:

$$\left\{ \alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - E(\zeta_{k,j}^{(n)}) \mid (n, j) \in S(c_{k,i}) \right\}.$$

Following the discussion in [8] on MRC, and recalling that the  $N$  replicas of each  $c_{k,i}$  are transmitted from  $N$  different antennas, the average SNR for  $\{\tilde{L}(c_{k,i}), \forall i\}$  is given by

$$snr_k = \sum_{n=1}^N snr_k^{(n)} = \sum_{n=1}^N \frac{|\alpha^{(n)}|^2 p_k}{\sum_{(n',k') \neq (n,k)} |\alpha^{(n')}|^2 p_{k'} V_{c_{k'}} + \sigma^2}. \quad (15)$$

Assume that  $\{\tilde{L}(c_{k,i}), \forall i\}$  can be approximately treated as LLRs of  $\{c_{k,i}, \forall i\}$  generated from the observations of a real AWGN channel with SNR equal to  $snr_k$ . This implies that the distortion components among  $\{\tilde{L}(c_{k,i}), \forall i\}$  in (14) are approximately un-correlated when the frame length  $J \rightarrow \infty$ . Recall that  $V_{c_k}$  in (9) is calculated based on the output  $\{L(c_{k,i}), \forall i\}$  of DEC- $k$ , so it is a function of  $snr_k$ ,

$$V_{c_k} = f(sn�_k). \quad (16)$$

In general, there is no closed-form expression for  $f(\cdot)$ , but it can be easily obtained by the Monte Carlo method for  $C$  in an AWGN channel with specified SNRs. Similarly we can define the frame-error-rate (FER) performance for DEC- $k$ , denoted by  $FER_k$ , as a function of  $snr_k$ ,

$$FER_k = g(sn�_k) \quad (17)$$

which can also be obtained by simulation. From (15) and (16),

$$snr_{k\_new} = \sum_{n=1}^N \frac{|\alpha^{(n)}|^2 p_k}{\sum_{(n',k') \neq (n,k)} |\alpha^{(n')}|^2 p_{k'} f(sn�_{k\_old}) + \sigma^2} \quad (18)$$

where  $snr_{k\_new}$  and  $snr_{k\_old}$  are  $snr_k$  values after and before one iteration. At the start, we initialize  $f(sn�_{k\_old}) = 1 \forall k$ , implying no feedback from DECs. Repeating (18), we can track the SNR evolution for the iterative process with any given  $\alpha$ . After the final iteration, we can estimate the FER performance of layer- $k$  using (17):

$$FER_k = g(sn�_{k\_final}), \quad k=1, 2, \dots \quad (19)$$

The above discussion can be generalized to QPSK modulation and complex  $\alpha$ , and will arrive at the same result as (18). The details are omitted due to the space limitation. For reference, please refer to [9], in which a similar derivation is presented for a related interleave-division multiple-access scheme.

### B. Performance Analysis in Quasi-Static Rayleigh Fading Channels

We now consider a quasi-static Rayleigh fading channel where  $\alpha$  remain unchanged during one frame and vary independently from frame to frame. Considering all possible  $\alpha$ , the average FER of layer- $k$  in an ML-IDM-ST code over a quasi-static Rayleigh fading channel can be estimated as

$$FER_{k\_fading} = \int g(sn�_{k\_final}) p(\alpha) d\alpha, \quad (20)$$

where  $p(\alpha) = p(\alpha^{(1)}) \cdots p(\alpha^{(N)})$  is the joint probability density function (pdf) of  $\{\alpha^{(1)}, \dots, \alpha^{(N)}\}$ . (**Note:**  $snr_{k\_final}$  is a function of  $\alpha$ .) The computation of (20) is relatively difficult since it involves an  $N$ -fold multiple integral. To avoid this difficulty, we introduce a bounding technique. Denote  $\lambda \equiv \sum_n |\alpha^{(n)}|^2 = \alpha \alpha^H$ . From (15),  $snr_k$  can be bounded as (see Appendix)

$$\frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} V_{c_{k'}} - \lambda p_k V_{c_k} / N + \sigma^2} \leq snr_k \leq \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} V_{c_{k'}} - \lambda p_k V_{c_k} + \sigma^2} \quad (21)$$

where the first equality holds when  $|\alpha^{(1)}|^2 = \dots = |\alpha^{(N)}|^2 = \lambda/N$  and the second equality holds when  $|\alpha^{(n)}|^2 = \lambda$  and  $|\alpha^{(n')}|^2 = 0, \forall n' \neq n$ . (**Notes:** (i) The lower bound corresponds to the uniform-fading situation when all  $\{\alpha^{(n)}\}$  have an equal amplitude. (ii) The upper bound corresponds to the cross-antenna-interference-free situation that the arrival signal is from a single transmit antenna.)

It can be verified that  $g(\cdot)$  in (17) is a decreasing function, so  $FER_{k\_fading}$  in (20) can be bounded as

$$\int g(\overline{snr_{k\_final}}) p(\lambda) d\lambda \leq FER_{k\_fading} \leq \int g(\overline{snr_{k\_final}}) p(\lambda) d\lambda, \quad (22)$$

where  $p(\lambda) = \lambda^{N-1} e^{-\lambda} / (N-1)!$  is the pdf of  $\lambda$ ,  $\overline{snr_{k\_final}}$  and  $\overline{snr_{k\_final}}$  are calculated using the following two iterations respectively

$$\overline{snr_{k\_new}} = \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} f(\overline{snr_{k\_old}}) - \lambda p_k f(\overline{snr_{k\_old}}) + \sigma^2}, \quad (23a)$$

$$\overline{snr_{k\_new}} = \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} f(\overline{snr_{k\_old}}) - \lambda p_k f(\overline{snr_{k\_old}}) / N + \sigma^2}, \quad (23b)$$

with  $f(\overline{snr_{k\_old}})$  and  $f(\overline{snr_{k\_old}})$  both initialized to 1  $\forall k$ .

Compared with (20), the lower and upper bounds in (22) are much easier to generate as they involve only one-fold integrals.

Several approximations are involved in the bounding technique developed above. Nevertheless, simulation results show that the bounds (especially the upper bound) are quite close to the simulated performance.

## IV. POWER ALLOCATION WITH IDEAL CODING

We now show that the SNR evolution techniques developed above can be used to design and optimize ML-IDM-ST codes.

It can be verified that, for fixed  $\lambda$  and  $P \equiv \sum_k p_k$ , the outcome of (21) depends on the distribution of  $\{p_1, \dots, p_K\}$ . From previous work [3], proper power allocation can improve the system performance. The intuition is that, with a high probability, strong signals can be correctly detected first so their interference to weak signals can be correctly cancelled, which in turn facilitates the detection of weak signals. The overall performance can be potentially improved in this way. In this section, we consider the power allocation with ideal FEC code  $C$  and we will consider non-ideal  $C$  in the next section.

Consider the first iteration with  $V_{c_k} = 1, \forall k$ . (As we will see, only one iteration is required with ideal coding [10][11].) Decoding starts from the top layer. For each layer- $k$ , assume that the contributions from layers  $k' > k$  has been removed from the received signal. Thus during the decoding for layer- $k$ ,  $V_{c_{k'}} = 0$  for  $k' > k$ . We now show how to determine  $p_k$  so that signals of layer- $k$  can be successively decoded and its interference to lower layers can be removed. With  $V_{c_{k'}} = 0$  for

$k' > k$  and  $V_{c_{k'}} = 1$  for  $k' \leq k$ , (**Note:** In this case, the approximations in (11)-(12) are accurate.) we have (from (21))

$$\text{snr}_k \geq \frac{\lambda p_k}{\lambda \sum_{k' < k} p_{k'} + \lambda(N-1)p_k / N + \sigma^2}. \quad (24)$$

For a real AWGN channel with Gaussian inputs, the minimum SNR required for reliable communication at rate  $R_C$  is [12]

$$\rho_{\min} = 2^{2R_C} - 1. \quad (25)$$

We first assume that  $\lambda$  is known *a priori* at the transmitter. Recall that the layer- $k$  outputs of the ESE are treated approximately as the LLRs obtained from the outputs of a real AWGN channel with SNR given in (24) (see III.A). Assume that  $\{p_1, \dots, p_{k-1}\}$  have been determined. We choose  $p_k$  according to (25)<sup>1</sup> as

$$\frac{\lambda p_k}{\lambda \sum_{k' < k} p_{k'} + \lambda(N-1)p_k / N + \sigma^2} = 2^{2R_C} - 1. \quad (26)$$

If  $C$  is capacity achieving, we can successfully decode layer- $k$ . We then proceed to determine  $p_{k+1}$  for layer- $(k+1)$  and so on.

Starting from  $\frac{\lambda p_1}{\lambda(N-1)p_1 / N + \sigma^2} = 2^{2R_C} - 1$ , we can generate  $p_1, \dots, p_K$  recursively according to (26). The total power required can be calculated as

$$P_{\text{IDM}} = \sum_k p_k = \frac{\sigma^2}{\lambda} \left( \left( \frac{2^{2R_C} - 1 + N}{N - (2^{2R_C} - 1)(N-1)} \right)^{\frac{R}{2R_C}} - 1 \right), \quad (27)$$

where  $R = 2KR_C$  is the total rate. (Here we consider complex signaling so the rate for each layer is  $2R_C$ .)

Recall that the capacity of a (complex)  $N \times 1$  system for any given  $\alpha$  is [4]

$$\text{CAP} = \log_2(1 + \alpha \alpha^H \rho / N) = \log_2(1 + \lambda P / \sigma^2) \quad (28)$$

where

$$\rho = NP / \sigma^2 \quad (29)$$

is the average SNR per receive antenna. For any given transmission rate  $R$ , the minimum required value of  $P$  for reliable transmission, denoted by  $P_{\min}$ , can be calculated from (28) by setting  $\text{CAP} = R$  as

$$P_{\min} = (2^R - 1)\sigma^2 / \lambda. \quad (30)$$

In general, there is a gap between  $P_{\text{IDM}}$  and  $P_{\min}$  even when  $C$  is capacity achieving. However, the following equation shows that using a very low-rate code  $C$  can narrow this gap.

$$\lim_{R_C \rightarrow 0} (P_{\text{IDM}} - P_{\min}) = \frac{\sigma^2}{\lambda} \lim_{R_C \rightarrow 0} \left( \left( \frac{2^{2R_C} - 1 + N}{N - (2^{2R_C} - 1)(N-1)} \right)^{\frac{R}{2R_C}} - 2^R \right) = 0 \quad (31)$$

Hence the ML-IDM-ST code can indeed achieve capacity provided that an ideal low-rate code  $C$  is used.

Next we consider the situation when  $\lambda$  is a random variable unknown at the transmitter but the distribution of  $\lambda$  is known at

<sup>1</sup> The formula is for an FEC code with un-constraint waveform. For a binary FEC code, the minimum SNR required, denoted by  $\rho_{\min}'$ , is larger than  $\rho_{\min}$  [12]. However, the gap between  $\rho_{\min}'$  and  $\rho_{\min}$  approaches 0, when  $R_C \rightarrow 0$ .

the transmitter. In this case, we use the so-called outage capacity  $\beta_{\text{out}}$  [13] as the performance criterion, where

$$\beta_{\text{out}} \equiv \Pr(\text{CAP} < R). \quad (32)$$

Given  $\beta_{\text{out}}$  and  $R$ , we find  $\lambda_R$  using the distribution of  $\lambda$  such that

$$\beta_{\text{out}} = \Pr(\lambda < \lambda_R) \quad (33)$$

Theoretically, the minimum value of  $P$  to achieve any given  $\beta_{\text{out}}$  at rate  $R$ , again denoted by  $P_{\min}$ , can be found by solving the following equation (obtained by substituting (28) and (33) into (32))

$$R = \log_2(1 + \lambda_R P_{\min} / \sigma^2). \quad (34)$$

Now suppose that we allocate power for an ML-IDM-ST code by setting  $\lambda = \lambda_R$  in (26) and obtain  $P_{\text{IDM}}$  using (27). When  $C$  is capacity achieving, decoding fails only when  $\lambda < \lambda_R$  so the decoding failure probability (i.e., FER) is given by  $\beta_{\text{out}}$ . If (31) holds, i.e.,  $P_{\text{IDM}} = P_{\min}$  when  $R_C \rightarrow 0$ , an ML-IDM-ST code can approach the theoretical limit in quasi-static fading channels with ideal low-rate FEC coding.

## V. POWER ALLOCATION WITH PRACTICAL CODING AND ITERATIVE DECODING

With non-ideal FEC coding, we use the iterative decoding procedure introduced in Section II at the receiver. In this case, since there is no closed-form power allocation strategy (as (26) known to us, we search for a distribution of  $\{p_k\}$  leading to optimal system performance under the constraint of  $\sum_k p_k = P$ . For a small  $K$ , an exhaustive search is possible and some searching results are given in Section VI. For a large  $K$ , a linear programming technique similar to that discussed in [14] can be used. Eqn. (20) can be used for performance evaluation in the search, but the complexity involved is high. Alternatively, we can use the bounds in (22) for low-cost performance estimation but we omit details here.

## VI. NUMERICAL RESULTS

In our simulation, QPSK modulation is always assumed. We apply different rotations  $\{0, \pi/2K, \dots, (K-1)\pi/2K\}$  to signals from different layers to make the interference from other layers more Gaussian-like. We call each  $\mathbf{d}_k$  a frame and  $\{\mathbf{d}_k, k=1, \dots, K\}$  a super-frame. The frame error rate (FER) and super-frame error rate (SFER) are defined accordingly. Clearly,  $\text{SFER} \geq \text{FER}$ .

Consider  $2 \times 1$  and  $4 \times 1$  ML-IDM-ST codes employing a rate-1/3 turbo code with generator  $G(x) = (1+x+x^3)/(1+x^2+x^3)$  for all layers. Fig. 3 shows the simulated FER and SFER performance of these codes with  $R = 4$  bits per channel use, corresponding to  $K = 6$ . The power levels of different layers are listed in Tab.1, which are obtained using exhaustive searching. Corresponding outage capacities [13] are also included in Fig.3, which are theoretical limits of SFER. In practice, FER can be a more useful performance measurement, as in case of error it is only necessary to discard the erroneous frames, instead of a complete super-frame. It is observed that the FER curves in Fig.3 are quite close to the outage capacity curves.

Fig. 4 shows the comparison between the simulation results and the bound results in (22) for the codes in Fig. 3. As we can

see, the upper bounds are relatively tight. The lower bound is much looser than the upper bound with  $N = 4$ .

Table 1. Powers allocated to different layers, where  $P$  is the total power.

$N$	Layer					
	1	2	3	4	5	6
2	$0.45P$	$0.254P$	$0.142P$	$0.082P$	$0.045P$	$0.027P$
4	$0.44P$	$0.257P$	$0.138P$	$0.086P$	$0.046P$	$0.033P$

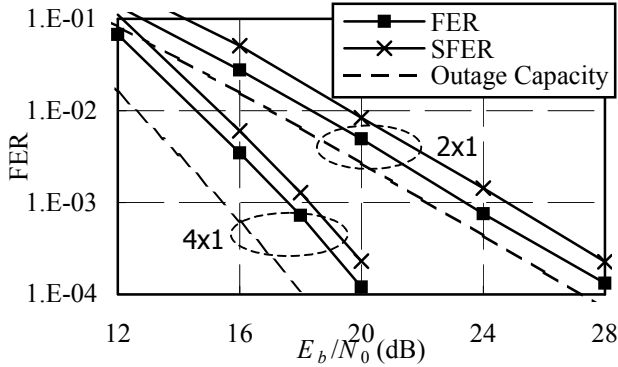


Figure 3. Simulated FER and SFER performance of 2x1 and 4x1 ML-IDM-ST codes with  $R = 4$  bits per channel use over a quasi-static Rayleigh fading channel. QPSK modulation is used. The number of information bits per layer = 4096, iteration number = 30, and  $K = 6$ .

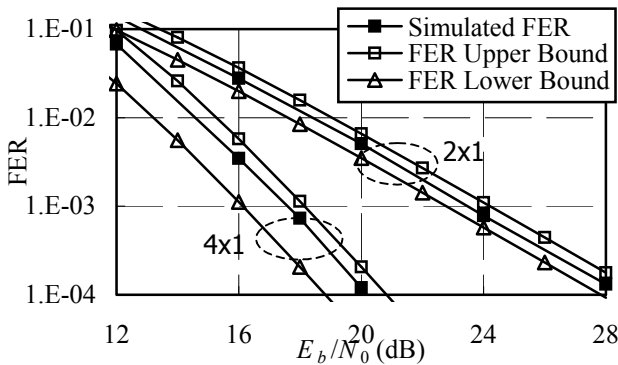


Figure 4. Comparison between the bound and simulation results for the ML-IDM-ST codes in Fig. 3.

## VII. CONCLUSIONS

We have derived SNR evolution and bounding techniques for ML-IDM-ST codes, based on which we have developed power allocation techniques for high-rate schemes. These techniques are simple and efficient. Both theoretical analysis and simulation results show that the proposed scheme can achieve performance close to the theoretical limits.

## APPENDIX. THE PROOF OF EQN. (21)

In this appendix, we only present the proof of the lower bound in (21). The proof of the upper bound in (21) can be derived using induction and is omitted here due to the space limitation.

Substituting  $\sum_n |\alpha^{(n)}|^2 = \lambda$  into (15), we have

$$snr_k = \frac{\sum_{n=1}^N \frac{|\alpha^{(n)}|^2 p_k}{\sum_{n',k'} |\alpha^{(n')}|^2 p_{k'} V_{c_{k'}} - |\alpha^{(n)}|^2 p_k V_{c_k} + \sigma^2}}{\sum_{n=1}^N \frac{A}{A - |\alpha^{(n)}|^2 p_k} - N}$$

$$= \frac{1}{V_{c_k}} \left( \sum_{n=1}^N \frac{A}{A - |\alpha^{(n)}|^2 p_k} - N \right) \quad (A1)$$

where  $A \equiv (\lambda \sum_{k'} p_{k'} V_{c_{k'}} + \sigma^2) / V_{c_k}$ . Multiplying both sides of (A1) by  $V_{c_k} \sum_{n=1}^N (A - |\alpha^{(n)}|^2 p_k)$ , we have (using Schwartz inequality)

$$\begin{aligned} & snr_k V_{c_k} \sum_{n=1}^N (A - |\alpha^{(n)}|^2 p_k) \\ &= \sum_{n=1}^N \frac{A}{A - |\alpha^{(n)}|^2 p_k} \sum_{n=1}^N (A - |\alpha^{(n)}|^2 p_k) - N \sum_{n=1}^N (A - |\alpha^{(n)}|^2 p_k) \\ &\geq \left( \sum_{n=1}^N \sqrt{\frac{A}{A - |\alpha^{(n)}|^2 p_k}} \sqrt{A - |\alpha^{(n)}|^2 p_k} \right)^2 - N(NA - \lambda p_k) \\ &= N^2 A - N^2 A + N \lambda p_k = N \lambda p_k \end{aligned} \quad (A2)$$

where the equality holds when  $|\alpha^{(1)}|^2 = \dots = |\alpha^{(N)}|^2 = \lambda / N$ . Eqn. (A2) leads to

$$snr_k \geq \frac{N \lambda p_k}{V_{c_k} \sum_{n=1}^N (A - |\alpha^{(n)}|^2 p_k)} = \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} V_{c_{k'}} - \lambda p_k V_{c_k} / N + \sigma^2}$$

## REFERENCES

- [1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance analysis and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.
- [2] H.-J. Su and E. Geraniotis, "Space-time turbo codes with full antenna diversity," *IEEE Trans. Commun.*, vol. 49, pp. 47-57, Jan. 2001.
- [3] L. H. C. Jason, M. Tao, and R. S. Cheng, "Optimal power allocation scheme on generalized layered space-time coding systems," in *Proc. IEEE ICC 01*, Helsinki, Finland, June 2001, pp. 1706-1710.
- [4] G. J. Foschini and M. Gans, "On the limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, Mar. 1998.
- [5] K. Y. Wu, W. K. Leung, and Li Ping, "A simple approach to near-optimal multiple transmit antenna space-time codes," in *Proc. IEEE ICC'03*, Alaska, USA, May 2003, pp. 2603-2607. ([www.ee.cityu.edu.hk/~liping](http://www.ee.cityu.edu.hk/~liping)).
- [6] F. N. Brannstrom, T. M. Aulin, and L. K. Rasmussen, "Iterative decoders for trellis code multiple-access," *IEEE Trans. on Commun.*, vol. 50, pp. 1478-1485, Sept. 2002.
- [7] S. Brück, U. Sorger, S. Gligorevic, and N. Stolte, "Interleaving for outer convolutional codes in DS-CDMA Systems," *IEEE Trans. Commun.*, vol. 48, pp. 1100-1107, July 2000.
- [8] T. S. Rappaport, *Wireless Communications Principle and Practice*. Prentice-Hall, 1996.
- [9] Li Ping, L. Liu, K. Y. Wu, and W. K. Leung, "Interleave-division multiple-access," submitted for publication. Available at: [www.ee.cityu.edu.hk/~liping](http://www.ee.cityu.edu.hk/~liping).
- [10] N. Chayat and S. Shamai, "Convergence properties of iterative soft onion peeling," in *Proc. ITW'99*, Kruger National Park, South Africa, June 1999, pp. 9-11.
- [11] G. Caire, S. Guemghar, A. Roumy, and S. Verdu, "Maximizing the spectral efficiency of coded CDMA under successive decoding," *IEEE Inform. Theory*, vol. 50, pp. 152-164, Jan. 2004.
- [12] J. G. Proakis, *Digital Communications*, McGraw-Hill, 1995.
- [13] D. Cui and A. M. Haimovich, "Performance of parallel concatenated space-time codes," *IEEE Commun. Lett.*, vol. 5, pp. 236-238, June 2001.
- [14] Li Ping and Lihai Liu, "Analysis and design for IDMA systems based on SNR evolution and power allocation," in *Proc. IEEE VTC'04 Fall*.