Analysis and Design of IDMA Systems Based on SNR Evolution and Power Allocation

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***Abstract*-We show that the performance of an interleave-division multiple-access (IDMA) system can be assessed by tracking the signal-to-noise-ratio (SNR) evolution of the iterative chip-by-chip (CBC) detection process. Based on this we develop a power allocation technique for performance optimization. Very high throughput (say 8 bits/chip) is observed for IDMA without sophisticated channel coding.**

***Index Terms*-CDMA, iterative detection, linear programming, multiuser detection, power allocation.**

I. INTRODUCTION

Recently, it has been shown that multiuser detection [1]-[5] together with unequal power control [6]-[9] can enhance the performance of code-division multiple-access (CDMA) systems. Different approaches have been investigated. For ideal (or nearly ideal) forward error control (FEC) codes, successive stripping [6][9] (or "onion peeling" [7]) can be used. For more practical codes, power allocation for iterative multiuser detection has been discussed [8].

The complexity of CDMA multiuser detection has always been a serious concern. This problem can be seen from different angles. First, the computational cost of most multiuser detection algorithms increases rapidly with the number of users (e.g., quadratically for the well-known minimum mean square error (MMSE) approach in [3]). Second, the analysis of CDMA multiuser detection is complicated due to the correlation modeling among signature sequences in random waveform systems. Large random matrix theory has been employed to tackle the problem [4][9][10], but the approach is mathematically demanding. The large system assumption also makes it less intuitive to small size systems. (However, good matching has been observed from simulation [4].)

Interleave-division multiple-access (IDMA) [11][12] is a special case of random waveform CDMA, and the accompanying chip-by-chip (CBC) estimation algorithm [11] is essentially a low-cost iterative soft-cancellation technique [3]. It has been shown [11] that IDMA with equal power control can achieve performance close to the theoretical limits at low-to-medium throughput (e.g., with a sum-rate up to 1 bit/chip in a quadrature additive white Gaussian noise (AWGN) channel).

In this paper, we first outline a signal-to-noise-ratio (SNR) evolution technique for fast performance evaluation [5][12]-[14] of the IDMA scheme. A power allocation technique is developed for further optimization. The system performance can be greatly enhanced in this way. Simulation results for IDMA based on a trivial repetition code show high throughput (8 bits/chip). This is equivalent to spreading in CDMA and so will be referred to as un-coded IDMA.

Compared with the CDMA approach [1]-[9], the IDMA approach has several noticeable features. First, the CBC algorithm is conceptually simple and can be easily analyzed. It does not involve the 'large system' assumption. The design procedure can be accomplished by exhaustive search for small systems and by linear programming (LP) for large systems. Second, the results can be easily verified since the cost of the CBC algorithm is very low. (Nevertheless, the optimality of the CBC algorithm will be shown under the assumption of ideal FEC coding and decoding.) Third, the proposed method is applicable to un-coded IDMA. Note it is difficult to apply a turbo-type multiuser detection scheme (that relies on an upper-layer soft-in-soft-out decoder) to un-coded CDMA. Finally, the principle can be extended to more general applications, such as multi-ary coded modulation [15] and multi-level space-time coding [16].

II. SYSTEM MODELING AND ANALYSIS

A. System Model and CBC Algorithm

Consider an IDMA system with K users [11][12]. At the transmitter side (see Fig. 1(a)), the information bits for user-k are first encoded by an encoder (ENC) based on a FEC code and then interleaved and transmitted over a Gaussian multiple access channel (MAC). The received signal can be written as

$$r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j), \qquad j = 1, 2, \ldots J \tag{1}$$

where $x_k(j) \in \{+1, -1\}$ is the jth chip transmitted by user-k, h_k the coefficient for user-k representing the combined effect of power control and channel loss, and $\{n(j)\}$ are samples of an AWGN process with zero-mean and variance $\sigma^2 = N_0/2$. For simplicity, we only consider real $\{h_k\}$ but the results can be easily extended to quadrature channels [17].

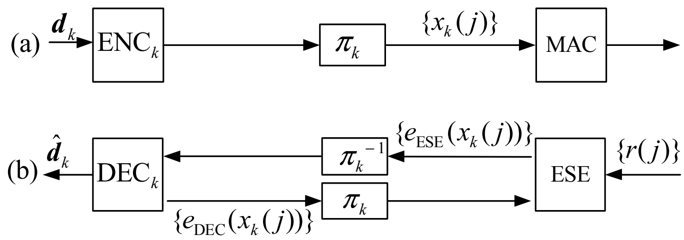

Fig. 1. (a) The transmitter for user-k. (b) A part of the receiver related to user-k.

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The receiver consists of an elementary signal estimator (ESE) and K *a posteriori* probability (APP) decoders (DECs), operating iteratively. (Only one DEC for user-k is shown in Fig. 1(b).) Let

$e_{\rm DEC}(x_k(j)) \equiv \log(\Pr(x_k(j) = +1))\text{-}\log(\Pr(x_k(j) = -1)),\ \forall j$

be the extrinsic *a posteriori* log-likelihood ratios (LLRs) generated by the DEC for user-k. For each k, we rewrite (1) as

$$r(j) = h_k x_k(j) + \zeta_k(j) \quad (2a)$$

where

$$\zeta_k(j) = \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j)\,. \quad (2b)$$

Denote by E(·) and Var(·) the mean and variance functions, respectively. We list the CBC detection algorithm as follows (with initialization $e_{\rm DEC}(x_k(j)) = 0,\ \forall k, j$) [11][12].

The CBC Algorithm:

$$\mathrm{E}(x_k(j)) = \tanh(e_{\rm DEC}(x_k(j))/2)\,, \quad \forall k, j, \quad (3a)$$

$$\mathrm{Var}(x_k(j)) = 1 - (\mathrm{E}(x_k(j)))^2\,, \quad \forall k, j, \quad (3b)$$

$$\mathrm{E}(\zeta_k(j)) = \sum_{k' \neq k} h_{k'} \mathrm{E}(x_{k'}(j))\,, \quad \forall k, j, \quad (3c)$$

$$\mathrm{Var}(\zeta_k(j)) = \sum_{k' \neq k} |h_{k'}|^2\ \mathrm{Var}(x_{k'}(j)) + \sigma^2\,, \quad \forall k, j, \quad (3d)$$

$$e_{\rm ESE}(x_k(j)) = \frac{2h_k}{\mathrm{Var}(\zeta_k(j))}(r(j) - \mathrm{E}(\zeta_k(j)))\,, \ \forall k, j. \quad (3e)$$

After (3e), the APP decoding in the DECs is performed to generate the LLRs $\{e_{\rm DEC}(x_k(j)),\ \forall k, j\}$. Then go back to (3a) for the next iteration.

B. Performance Evaluation

We now outline a performance evaluation technique [12] for the CBC algorithm. We first examine the average SNR for $\{e_{\rm DEC}(x_k(j)),\ \forall j\}$ for a fixed k. The issue is complicated as the expression of $\mathrm{Var}(\zeta_k(j))$ in (3e) is a function of j. We avoid this difficulty by replacing the detection rule in (3e) using

$$\begin{aligned} e_{\rm ESE}(x_k(j)) &= \frac{2h_k}{V_{\zeta_k}}(r(j) - \mathrm{E}(\zeta_k(j))) \\ &= \frac{2h_k}{V_{\zeta_k}}\left(h_k x_k(j) + \zeta_k(j) - \mathrm{E}(\zeta_k(j))\right) \end{aligned} \quad (4)$$

where

$$V_{\zeta_k} \equiv \sum_{k' \neq k} |h_k|^2 V_{x_{k'}} + \sigma^2 \quad (5a)$$

$$V_{x_k} \equiv \frac{1}{J} \times \sum_{j=1}^{J} \mathrm{Var}(x_k(j))\,. \quad (5b)$$

Here V_{ζ_k} (unlike $\mathrm{Var}(\zeta_k(j))$ in (3e)) is not a function of j, which makes the following analysis easier. The average SNR (averaged over j) for $e_{\rm ESE}(x_k(j))$ in (4) is given by [12]

$$snr_k = \frac{\mathrm{E}(|h_k x_k(j)|^2)}{V_{\zeta_k}} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 V_{x_{k'}} + \sigma^2}\,. \quad (6)$$

Recall that $\mathrm{Var}(x_k(j))$ in (3b) is calculated based on $e_{\rm DEC}(x_k(j))$, so V_{x_k} in (5b) is a function of snr_k. We express this function as

$$V_{x_k} \equiv f(snr_k)\,. \quad (7)$$

We also define the bit-error-rate (BER) performance for the DEC_k as a function of snr_k as

$$\mathrm{BER} \equiv g(snr_k). \quad (8)$$

Both $f(\cdot)$ and $g(\cdot)$ can be obtained by using the Monte-Carlo method. Sufficiently large J should be used for this purpose to ensure that the inputs to the DECs are approximately un-correlated. As an example, Fig. 2 shows $f(\cdot)$ and $g(\cdot)$ for a length-16 repetition code with $J = 8192$.

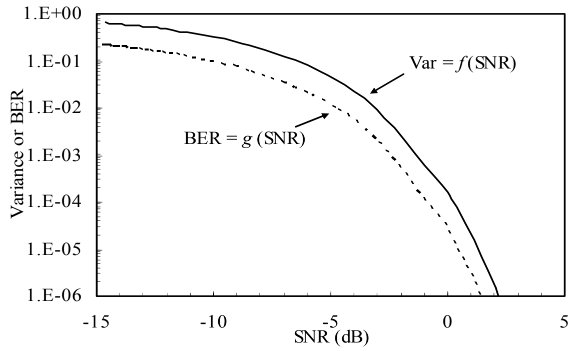

Fig. 2. The $f(\cdot)$ and $g(\cdot)$ functions for a length-16 repetition code. SNR = E_c/N_0 where Ec is energy per coded bit.

Combining (6) and (7), we have

$$snr_{k\_\mathrm{new}} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 f(snr_{k'\_\mathrm{old}}) + \sigma^2}\,, \ \forall k \quad (9)$$

where snr_{k_new} and snr_{k_old}, respectively, are snr_k values after and before one iteration of the CBC algorithm. Initially, we start with $f(snr_{k\_\mathrm{old}}) = 1$, for all k, implying no feedback from the DECs. Repeating (9), we can track the SNR evolution for the iterative process in the CBC algorithm. During the final iteration, we can estimate the BER performance by substituting the final values $\{snr_{k\_\mathrm{final}}\}$ into (8).

C. An Example: Un-coded IDMA

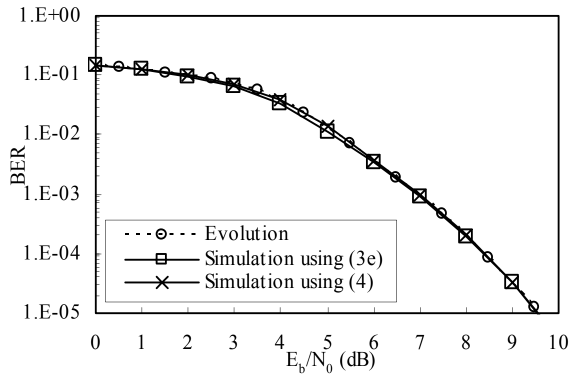

Fig. 3. Comparison of performance obtained by evolution and simulations based on a length-16 repetition code with QPSK modulation. $K = 16$, $J = 8192$ and $h_k = 1$ for all k.

Fig. 3 compares the simulated and predicted (using SNR evolution) IDMA performance for a system with $K = 16$ users based on a length-16 repetition code and $J = 8192$ (the corresponding information length is 512). Repetition coding in

IDMA is really equivalent to spreading in CDMA and hence the name "un-coded IDMA".

We can see from Fig. 3 that the evolution result and the simulated results based on both (3e) and (4) are very close. This indicates that the SNR evolution technique provides a conceptually simple, fast and reasonably accurate way for performance assessment.

III. Power Allocation with Ideal Coding

We now examine a power allocation technique for systems with ideal FEC coding. We assume that h_k includes both the power control factor and the channel loss. Thus the distribution of $\{|h_k|^2\}$ can be adjusted through power control. As we will see, only one iteration is required with ideal FEC coding and decoding, which results in an ideal successive stripping process [6][9][18].

For illustration, we consider a two-user system with $|h_1|^2 \le |h_2|^2$. Recall from (3) the initialization $e_{DEC}(x_k(j)) = 0$ (and so $\mathrm{Var}(x_k(j)) = 1$), $\forall k, j$. This means that $V_{x_k} = 1$ for both $k = 1$ and 2. We first detect the signal for user-2. From (6), we have $snr_2 = |h_2|^2/(|h_1|^2+\sigma^2)$ and the capacity for user-2 by treating the signal from user-1 as additive noise is

$$\frac{1}{2}\log_2\left(1+\frac{|h_2|^2}{|h_1|^2+\sigma^2}\right). \tag{10}$$

Now suppose that we properly select $|h_2|^2$ to ensure error-free decoding for DEC-2, i.e., $V_{x_2} = 0$ (after the decoding of user-2). Substituting this into (6) for user-1, we have $snr_1=|h_1|^2/\sigma^2$ and the capacity for user-1 is (conditional on the successful decoding of user-2)

$$\frac{1}{2}\log_2\left(1+\frac{|h_1|^2}{\sigma^2}\right). \tag{11}$$

If h_1 is also properly chosen, we can also achieve error-free detection for user-1.The capacity of the overall system can be expressed as

$$\frac{1}{2}\log_2\left(1+\frac{|h_1|^2+|h_2|^2}{\sigma^2}\right)=\frac{1}{2}\log_2\left(1+\frac{|h_2|^2}{|h_1|^2+\sigma^2}\right)+\frac{1}{2}\log_2\left(1+\frac{|h_1|^2}{\sigma^2}\right). \tag{12}$$

From (10)-(12), we can see that if both users achieve their individual capacities, the capacity of the overall system is also achievable [18]. The principle can be generalized to more users.

Note that the capacities in (10)-(12) are for un-constrained signaling waveforms. For binary codes, the capacities are different. However, if we assume that very low-rate codes are used (for systems with a large number of users), the difference diminishes asymptotically.

The above discussion also shows that with ideal coding, the capacity of an IDMA system is only limited by the received power.

IV. Power Allocation and Linear Programming

We now proceed to consider power allocation for practical FEC codes. When K is small, we can carry out an exhaustive search of all possible power levels based on (9). This quickly becomes impractical for a large K. We now outline a linear programming (LP) technique for a large K.

To facilitate the discussion, we quantize the received power P into M+1 discrete values: $\{P(m),\ m = 0, 1, \ldots, M\}$ with $P(m\text{-}1) < P(m)$, $m = 1, \ldots M$. This means that $P(m) = |h_k|^2$ for some k. Again we assume that h_k includes both the power control factor and the channel loss.

We partition the K users into M+1 groups according to their power levels. Let $\lambda(m)$ denote the number of users assigned with power level $P(m)$ and let $y(m)$ denote the total power of these $\lambda(m)$ users. As such,

$$\sum_m \lambda(m) = K \tag{12a}$$

$$y(m) = \lambda(m)P(m) \tag{12b}$$

and the total received power is

$$P_{\text{total}} = \sum_m \lambda(m)P(m) = \sum_m y(m).$$

Denote by $snr(m)$ the SNR for the users in the mth group with power $P(m)$. Define

$$I = \sum_m y(m) f(snr(m)) + \sigma^2 \tag{13}$$

which is the total power (including noise) in the ESE output. When K is large, (6) can be approximated as

$$snr(m)_{\text{new}} = \frac{P(m)}{I - P(m) f(snr(m)_{\text{old}})} \approx \frac{P(m)}{I}. \tag{14}$$

Using (13) and (14) we have the update rule

$$I_{\text{new}} = \sum_m y(m) f\left(\frac{P(m)}{I_{\text{old}}}\right) + \sigma^2. \tag{15}$$

where I_{new} and I_{old} denote, respectively, the values of I after and before the ESE operation. Eqn. (15) characterizes the evolution of the total interference variance at each iteration. If the iterative detection converges, I_{new} should be lower than I_{old}. Equivalently, we can write the convergence condition as

$$\sum_m y(m) f\left(\frac{P(m)}{I}\right) + \sigma^2 \le (1-\delta)I,\ I_{\min} \le I \le I_{\max} \tag{16}$$

where $0 < \delta < 1$ is a decay factor that controls the convergence speed. $I_{\max}$ and $I_{\min}$ specify the total interference at the beginning and end of the iterative detection. As (16) is linear with respect to $\{y(m)\}$, we solve the problem by LP as follows.

<u>**The LP Process:**</u> Find $\{y(m)\}$ to minimize the cost function:

$$P_{\text{total}} = \sum_m y(m) \tag{17a}$$

subject to

$$\sum_m y(m) f\left(\frac{P(m)}{I}\right) + \sigma^2 \le (1-\delta)I,\ I_{\min} \le I < I_{\max}, \tag{17b}$$

$$y(m) \ge 0, \quad m = 0, 1, \ldots, M. \tag{17c}$$

<u>**Note (i):**</u> In practice, both P and I should be quantized. We need to determine the searching ranges, which are measured by $P_{\min}$, $P_{\max}$, $I_{\min}$ and $I_{\max}$. Let us quantize P and I as

$$P(m) = \alpha^m P_{\min}, \quad m = 0, 1, \ldots, M, \tag{18a}$$

$$I(n) = \beta^n I_{\min}, \quad n = 0, 1, \ldots, N, \tag{18b}$$

with $P(0) = P_{\min} > 0$, $P(M) = P_{\max}$, $I(0) = I_{\min} > 0$, $I(N) = I_{\max}$, $\alpha > 1$ and $\beta > 1$. Then (17b) becomes

$$\sum_m y(m) f\left(\alpha^m \beta^{-n} \gamma\right) + \sigma^2 \leq (1-\delta)\beta^n I_{\min} . \quad (19)$$

Denote by

$$\gamma = P_{\min}/I_{\min} \quad (20)$$

the minimum SNR at the end of the iterative decoding process. Here γ can be determined by the desired BER, i.e., $\gamma = g^{-1}(\text{BER}_{\text{specified}})$ with $g(\cdot)$ given in (8). Since

$$K = \sum_m \lambda(m) = \sum_m \frac{y(m)}{P(m)} = \sum_m \frac{y(m)\alpha^{-m}}{\gamma I_{\min}} , \quad (21)$$

we have

$$\sum_m y(m)\alpha^{-m} - K\gamma I_{\min} = 0 . \quad (22)$$

We can replace (17b) by (19). We can also include (22) as part of the LP constraints. In this way, $I_{\min}$ becomes an optimization variable and $P_{\min}$ can be determined from $I_{\min}$ by (20).

Note (ii): We still need to determine M and N in (18). In general, we just use a sufficient large M, since the LP will automatically determine the maximum power level used. We start with a relatively small N. If after the LP,

$$I_{\max} = \sum_m y(m) \times 1 + \sigma^2 > I(N) \quad (23)$$

then the quantization range in (18b) is not sufficient and we increase N and repeat the LP until $I_{\max} \leq I(N)$.

Note (iii): In the above, δ controls the convergence speed. A larger δ generally leads to faster convergence, i.e., fewer iterations, but larger P_{total}. See Section V.B.

Note (iv): Once $\{y(m)\}$ are known, the numbers of users in each group, $\{\lambda(m)\}$, can be obtained from (12b).

Note (v): Linear programming has also been applied to solve the power allocation problem for CDMA based on a large system assumption [8].

V. Numerical Results

In this section we present numerical results to illustrate the proposed power allocation scheme for both un-coded and coded IDMA systems.

A. Un-coded IDMA

We adopt a simple system model to illustrate the proposed method for power allocation. Each user's information data are encoded by a rate-1/16 (R=1/16) repetition code. The resulting signals are interleaved by randomly generated interleavers, and transmitted over an AWGN channel with QPSK modulation.

We perform LP using the $f(\cdot)$ and $g(\cdot)$ functions in Fig. 2. The optimization target is to minimize the average E_b/N_0 to achieve BER = 10^{-4}. Table I lists the optimization results for K = 16 and 64 with the sum-rates KR = 2 and 8 bits/chip respectively (taking QPSK modulation into account). We can see that equal power is the best solution for KR = 2 bits/chip but unequal power allocation is necessary to ensure good performance for KR = 8 bits/chip.

TABLE I
RELATIVE POWER LEVELS FOR UN-CDOED IDMA SYSTEMS OBTAINED BY LP

	(power level (dB)) × (user number)
K=16	0×16
K=64	0×24, 7.976×12, 9.116×2, 13.673×8, 14.813×4, 19.37×6 20.51×8

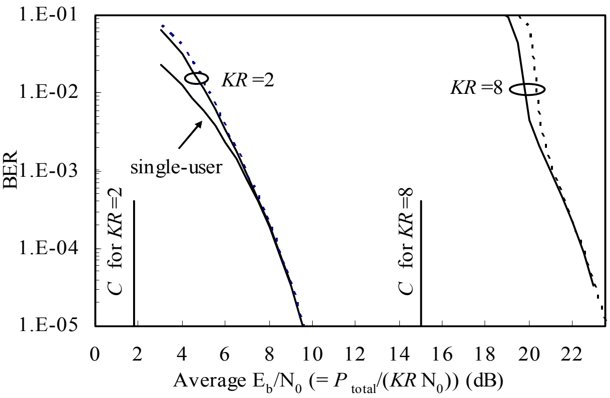

Fig. 4. The lowest-power-users' BER performance of un-coded IDMA systems obtained by simulation (solid lines) and SNR evolution (dashed lines) with sum-rates 1/8 (single user), 2 and 8 bits/chip. The corresponding capacities (abbreviated to C) are plotted for reference. Iteration number = 30. J = 8192.

Fig. 4 shows the BER performance of the users with the lowest power level obtained by simulation (solid lines). The performance of other users is always better. The performance curves estimated using the SNR evolution outlined in Section II with 30 iterations (dashed lines) are included for comparison. The single-user performance is also plotted for reference. Notice that the gap between the capacity and the 64-user IDMA performance is only about 8 dB measured at BER =10^{-4}. This is quite close to the gap between the capacity and the single-user performance with un-coded QPSK modulation.

Systems with even higher throughputs can also be designed. It appears that the throughput is only limited by the SNR, not by the number of users (as in traditional CDMA systems [5]).

B. Coded IDMA

We employ the same transmission model as in Section V.A except changing the repetition code into a concatenation of a rate-1/2 convolutional code with generator polynomials $(23, 35)_8$ followed by a rate-1/8 repetition code. Functions $f(\cdot)$ and $g(\cdot)$ obtained by Monte-Carlo methods are shown in Fig. 5.

The performance trend for coded IDMA is very similar to that of the un-coded IDMA in Fig. 4 except that the gap to the capacity is narrowed. We will focus on a system with K = 32 users and sum-rate KR = 4 bits/chip. More results can be found in [12].

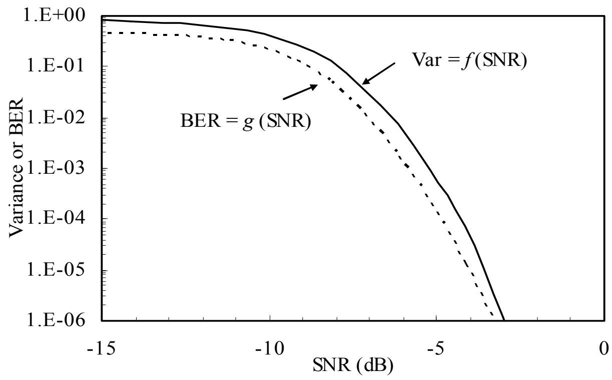

Fig. 5. The variance and BER as functions of SNR for a rate-1/2 convolutional code followed by a rate-1/8 repetition code. J = 32768.

Recall our earlier statement that δ can be used to control the trade-off between convergence speed and transmission power. Consider two different δ values (see Table II): Case-I with δ = 0.2 and Case-II with δ = 0.02. In Fig. 6, we plot the SNR evolution process for users with different power levels for Case-I with δ = 0.2, measured at an average channel E_b/N_0 = 11.4 dB. We can also estimate the BER performance during the iterative process by combining the results in Figs. 5 and 6. It is seen that seven iterations are sufficient to achieve most gain.

Next we compare the performance of Case-I and Case-II shown in Fig. 7. We can make the following observations.

- With δ= 0.2, the convergence is relatively fast. For a small iteration number (It = 5), δ= 0.2 results in better performance.
- With δ= 0.02, the convergence is slower but the performance with a large iteration number (It = 30) is better.

Clearly, we can see that δ can be used to control the trade-off between the iteration number and performance.

TABLE II
POWER OPTIMIZATION RESULTS WITH DIFFERENT CONVERGENCE SPEEDS

	δ	(power level (dB)) × (user number)
Case-I	0.2	0×12, 3.167×3, 3.959×2, 5.543×3, 6.334×2, 7.918×4, 9.502×1, 10.293×3, 11.085×2
Case-II	0.02	0×15, 3.167×2, 3.959×6, 7.126×7, 7.918×2

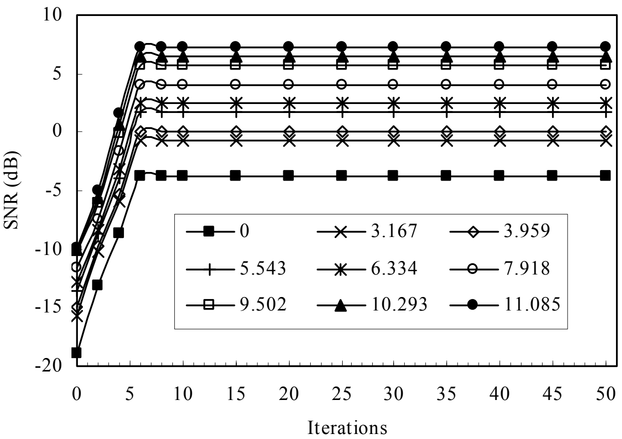

Fig. 6. The SNRs at the output of the ESE after each iteration for Case-I with δ = 0.2. K = 32 users. KR = 4 bits/chip. Average channel E_b/N_0 = 11.4 dB.

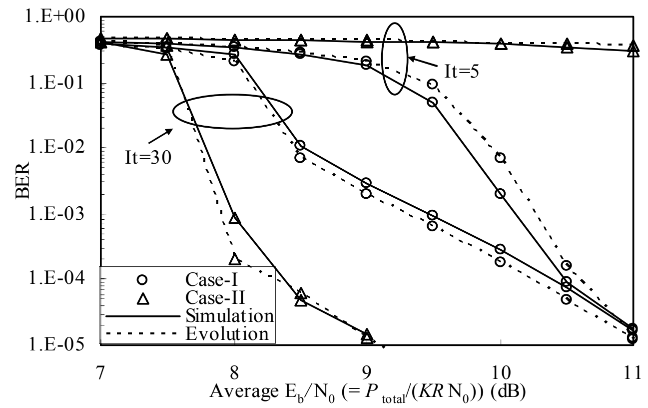

Fig. 7. The lowest-power-users' BER performance of a coded IDMA system obtained by simulation (solid lines) and SNR evolution (dashed lines) with K = 32 users, sum-rate KR = 4 bits/chip and J = 32768.

VI. CONCLUSIONS

The SNR evolution provides a fast and relatively accurate method to assess the performance of the iterative CBC detection algorithm for IDMA systems. Using this technique as a searching tool, we have developed a linear programming technique to optimize the performance of IDMA systems. Both analytical and simulation results show that IDMA throughput is power limited, and is not interference limited as in conventional CDMA with signal-user detection and equal power control.

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