

On Interleave-Division Multiple-Access¹

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Abstract – This paper provides a comprehensive study of IDMA systems. We first outline the basic IDMA principles in single-path and multi-path environments. We then describe a simple semi-analytical technique to assess the performance of IDMA systems, based on which we develop a power allocation scheme for performance optimization. We also discuss the use of low-rate codes to further enhance the power efficiency of IDMA systems. Simulation results demonstrate the advantages of the IDMA scheme in terms of both bandwidth and power efficiencies. For example, with simple convolutional/repetition codes an overall throughput of 8 bits/chip is achieved in single antenna systems. With turbo-Hadamard codes, performance at 1.4 dB away from the theoretical limit is demonstrated in a Gaussian MAC.

I. INTRODUCTION

Multisuser detection (MUD) has been widely studied for code-division multiple-access (CDMA) systems and significant progress has been made recently [1]-[6]. However, complexity has always been a stringent concern for MUD. Much research effort has been devoted to this issue in pursuit of simpler solutions without compromising performance.

Interleave-division multiple-access (IDMA) is a recently proposed scheme that employs chip-level interleavers for user separation [7][8]. IDMA inherits many advantages from CDMA, e.g., diversity against fading and mitigation of the worst-case other-cell user interference. Furthermore, it allows a very simple chip-by-chip (CBC) iterative MUD strategy [7] with complexity (per user) independent of the user number.

This paper provides a comprehensive study of IDMA systems. The IDMA detection principles in single-path and multi-path environments are outlined. A simple semi-analytical technique is developed for performance assessment. The technique is simple and efficient. It can be used as a searching tool in a performance optimization procedure based on power allocation. It is shown that very high spectral efficiency (e.g., 8 bits/chip) can be achieved with the power allocation strategy. Finally, the use of low-rate codes is discussed to further enhance the power efficiency of IDMA systems.

II. IDMA TRANSMITTER AND RECEIVER PRINCIPLES

A. Transmitter Structure

The upper part of Fig. 1 shows the transmitter structure of an IDMA system with K simultaneous users. Interleaving is the only mechanism for user separation here [7][8].

The input data sequence \mathbf{d}_k of user- k is encoded based on a low-rate forward error correction (FEC) code C . The coded sequence is then interleaved by a chip-level interleaver π_k , producing $\mathbf{x}_k \equiv [x_k(1), \dots, x_k(j), \dots, x_k(J)]^T$, where J is the frame length. We will call the elements in \mathbf{x}_k “chips”.

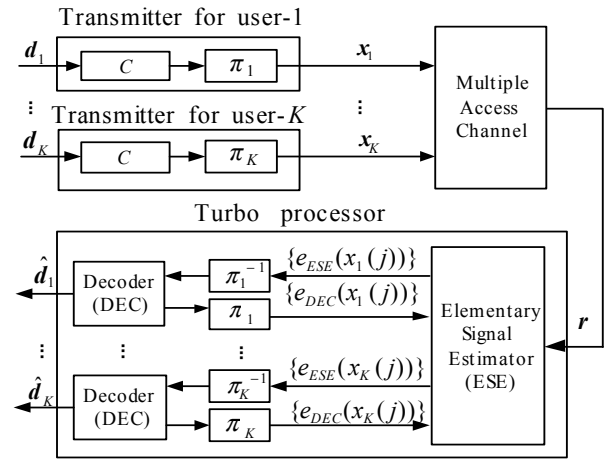


Figure 1. Transmitter and (iterative) receiver structures of an IDMA scheme with K simultaneous users.

The key principle of IDMA is that the interleavers $\{\pi_k\}$ should be different for different users. We assume that the interleavers are generated independently and randomly. These interleavers disperse the coded sequences so that the adjacent chips are approximately uncorrelated, which facilitates the simple chip-by-chip detection scheme discussed below.

B. Receiver Structure

We adopt a sub-optimal receiver structure, as illustrated in Fig. 1, which consists of an elementary signal estimator (ESE) and K single-user *a posteriori* probability (APP) decoders (DECs). The ESE and DECs operate in a turbo-type iterative manner [7][9]. The APP decoding is a standard function [9][10], so we only consider the ESE function below.

C. The ESE Function

For simplicity, we only consider binary-phase-shift-keying (BPSK) signaling and real channel coefficients. The generalization to situations without these restrictions can be found in [7].

We first assume quasi-static single-path channels. After chip-matched filtering, the received signal from K users can be written as

$$r(j) = \sum_{k=1}^K h_k x_k(j) + n(j), \quad j = 1, 2, \dots, J \quad (1)$$

where h_k is a coefficient for user- k representing the combined effect of power control and path loss, and $\{n(j)\}$ are samples of an additive white Gaussian noise (AWGN) process with variance $\sigma^2 = N_0/2$. We assume that $\{h_k\}$ are known *a priori* at the receiver.

The chip-level interleavers allow us to adopt a chip-by-chip (CBC) estimation technique [7] in the ESE. Rewrite (1) as

$$r(j) = h_k x_k(j) + \zeta_k(j) \quad (2a)$$

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$$\text{where } \zeta_k(j) \equiv \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j) \quad (2b)$$

is the distortion contained in $r(j)$ with respect to user- k . From the central limit theorem, $\zeta_k(j)$ is approximately Gaussian, so $x_k(j)$ can be estimated from (2a) provided that the mean and variance of $\zeta_k(j)$ are available [7]. Denote by $E(\cdot)$ and $\text{Var}(\cdot)$ the mean and variance functions, respectively. The main operations involved in the CBC algorithm are listed below, where $e_{ESE}(x_k(j))$ and $e_{DEC}(x_k(j))$ denote the extrinsic log-likelihood ratios (LLRs) of $x_k(j)$ generated in the ESE and DECs, respectively.

$$(i) \text{ Initialization: Set } e_{DEC}(x_k(j)) = 0, \quad \forall k, j.$$

(ii) Main operations:

$$E(x_k(j)) \leftarrow \tanh(e_{DEC}(x_k(j))/2), \quad \forall k, j. \quad (3a)$$

$$\text{Var}(x_k(j)) \leftarrow 1 - (E(x_k(j)))^2, \quad \forall k, j. \quad (3b)$$

$$E(\zeta_k(j)) \leftarrow \sum_{k' \neq k} h_{k'} E(x_{k'}(j)), \quad \forall k, j. \quad (4a)$$

$$\text{Var}(\zeta_k(j)) \leftarrow \sum_{k' \neq k} |h_{k'}|^2 \text{Var}(x_{k'}(j)) + \sigma^2, \quad \forall k, j. \quad (4b)$$

$$e_{ESE}(x_k(j)) \leftarrow \frac{2h_k}{\text{Var}(\zeta_k(j))} (r(j) - E(\zeta_k(j))), \quad \forall k, j. \quad (5)$$

The APP decoding in the DECs is performed at this stage to update $\{e_{DEC}(x_k(j))\}$. Then go back to (3a) for the next iteration.

It can be verified that the above algorithm is an extremely simplified form of that derived in [2] (i.e., when the spreading sequences are all of length-1).

The normalized computational cost in (3) – (5) (excluding the APP decoding of C) is only a few additions and multiplications per chip per user [7]. The cost per information bit per user is independent of the user number K .

D. The ESE Function in Asynchronous Channels

We now discuss the calculation of $\{e_{ESE}(x_k(j))\}$ in asynchronous channels with memory. Consider a quasi-static multipath fading channel with L tap-coefficients. Let $\{h_{k,0}, \dots, h_{k,L-1}\}$ be the coefficients for user- k combining the effects of power control and path loss. After chip-matched filtering, the received signal can be represented by

$$r(j) = \sum_{k=1}^K \sum_{l=0}^{L-1} h_{k,l} x_k(j-l) + n(j), \quad j=1, \dots, J+L-1. \quad (6)$$

In this case, each $x_k(j)$ is observed on L successive received samples $\{r(j), \dots, r(j+L-1)\}$. The information of $x_k(j)$ from all these samples should be combined to generate $e_{ESE}(x_k(j))$, and we adopt a LLR combining (LLRC) technique for this purpose,

$$e_{ESE}(x_k(j)) = \sum_{l=0}^{L-1} e_{ESE}(x_k(j))_l \quad (7)$$

where $e_{ESE}(x_k(j))_l$ is the extrinsic LLR of $x_k(j)$ based on $r(j+l)$ using (5). The complexity (per chip per user) is $O(L)$.

E. Generalization to Multiple Receive Antenna Systems

The above principles can be easily generalized to channels with multiple receive antennas. The signal from each receive antenna can be treated as that from a set of independent paths. The LLRC technique discussed above can be directly applied.

F. An Example

Let N_{info} be the number of information bits in a frame, K the number of simultaneous users, L the tap number of a multipath channel, N_r the number of receive antennas, It the iteration number, R_C the rate of each user, and $K \times R_C$ the system throughput that is a measurement of the overall bandwidth efficiency. QPSK signaling is always assumed.

We consider a simple system model. Construct C using a rate-1/2 (23, 35)₈ convolutional code followed by (i.e., in serial concatenation with) a length- S repetition code with alternative signs, i.e., $[+1, -1, +1, -1, \dots]$ ($R_C=1/(2S)$). The repetition coding can be viewed as a kind of spreading, except that all users use the same sequence. Two independent chip interleavers are employed by each user to produce the in-phase and quadrature parts of the transmitted sequence.

Fig. 2 shows the BER performance of the above system with $S=8$ ($R_C=1/16$) and equal (transmission) power allocation in quasi-static Rayleigh fading multipath channels with different L and N_r . The corresponding single-user performance is also included for reference. It is observed that the system can achieve $K \times R_C=3$ bits/chip for $K=48$ using one receive antenna and $K \times R_C=6$ bits/chip for $K=96$ using two receive antennas with performance close to the single-user performance at $\text{BER}=10^{-4}$. Such throughputs are rather high, recalling that with TDMA we may require a 128-QAM trellis coded modulation scheme to achieve similar throughput and performance.

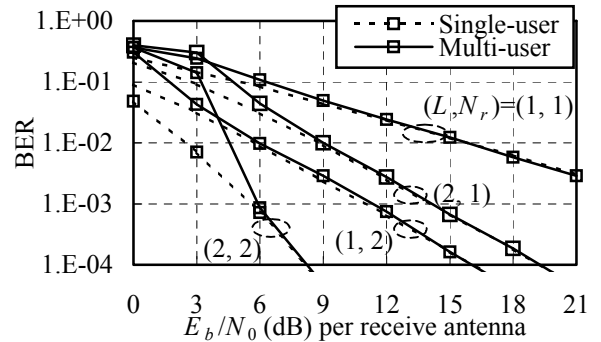


Figure 2. Performance of a convolutionally coded IDMA system in quasi-static Rayleigh fading multipath channels. $K=48$ for $N_r=1$, $K=96$ for $N_r=2$. $It=10$, $N_{\text{info}}=128$.

It is interesting to examine the diversity degree defined as $L \times N_r$, i.e., the total number of paths considering all the antennas. The asymptotic slopes of the curves in Fig. 2 show their diversity degrees. Clearly, a higher diversity degree results in better performance. For $(L, N_r) = (2, 1)$ and $(1, 2)$, the diversity degrees are both 2, and the asymptotic slopes for the corresponding curves are the same. The performance for $(L, N_r) = (1, 2)$ has a 3 dB gain since two receive antennas are used. We can see that the IDMA scheme together with the CBC algorithm can efficiently exploit the space diversity provided by the channel.

III. PERFORMANCE ANALYSIS

The performance analysis for a conventional CDMA multi-user detection scheme requires the knowledge of the correlation

characteristics among signature sequences. It can be a quite complicated issue and sophisticated large random matrix theory has been used in the past to tackle the problem [11].

IDMA does not involve signature sequences, which greatly simplifies the problem. In the following, we will derive a simple and efficient performance assessment technique. The method is semi-analytical since some of the functions involved (related to the FEC codes) are pre-calculated by simulation (similar to the methods in [12][13]).

For simplicity, we only consider BPSK signaling, real $\{h_k\}$, one receive antenna and synchronous channels. The principle can be directly generalized to situations without these restrictions. We first redefine $e_{ESE}(x_k(j))$ in (5) as

$$e_{ESE}(x_k(j)) = \frac{2h_k}{V_{\zeta_k}}(h_k x_k(j) + \zeta_k(j) - E(\zeta_k(j))) \quad (8)$$

$$\text{with } V_{\zeta_k} \equiv \sum_{k' \neq k} |h_{k'}|^2 V_{x_{k'}} + \sigma^2, \quad (9a)$$

$$\text{and } V_{x_k} \equiv \frac{1}{J} \times \sum_{j=1}^J \text{Var}(x_k(j)). \quad (9b)$$

Notes: $\text{Var}(x_k(j))$ is the variance for a particular $x_k(j)$ obtained from a feedback $e_{DEC}(x_k(j))$ using (3). V_{x_k} and V_{ζ_k} are averages of $\{\text{Var}(x_k(j), \forall j)\}$ and $\{\text{Var}(\zeta_k(j), \forall j)\}$ respectively, which can be different for different k due to unequal power allocation.

In our simulation, we observed that (8) leads to slightly poorer performance compared with (5), since $\text{Var}(\zeta_k(j))$ carries more information about $\zeta_k(j)$ (for a particular j) than V_{ζ_k} . Therefore, replacing (5) by (8) is a pessimistic approximation. However, this replacement simplifies the analysis issue for the CBC algorithm.

In (8), $h_k x_k(j)$ and $\zeta_k(j) - E(\zeta_k(j))$ represent signal and distortion components, respectively. Since $x_k(j) = \pm 1$, signal power $E(h_k x_k(j))^2 = |h_k|^2$. We approximate the average noise power after soft cancellation (for a fixed k) by its sample mean,

$$E(|\zeta_k(j) - E(\zeta_k(j))|^2) \approx V_{\zeta_k}. \quad (10)$$

The coefficient $2h_k/V_{\zeta_k}$ in (8) is a constant factor that will not affect the SNR. The SNR for $\{e_{ESE}(x_k(j))\}$ averaged over j , denoted by snr_k , is thus given by

$$snr_k = \frac{E(|h_k x_k(j)|^2)}{V_{\zeta_k}} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 V_{x_{k'}} + \sigma^2}. \quad (11)$$

We assume that $\{e_{ESE}(x_k(j)), \forall j\}$ can be approximately treated as LLRs of $\{x_k(j), \forall j\}$ generated from the observations of an AWGN channel with SNR equal to snr_k . This requires that $\{e_{ESE}(x_k(j)), \forall j\}$ are uncorrelated, which is approximately true when the chip-level interleavers are random and their length $\rightarrow \infty$.

Recall that $\text{Var}(x_k(j))$ in (3b) is calculated based on $e_{DEC}(x_k(j))$, so V_{x_k} in (9b) is a function of snr_k ,

$$V_{x_k} = f(snr_k). \quad (12)$$

In general, there is no closed form expression for $f(\cdot)$, but it can be easily obtained by the Monte Carlo method. This only involves simulating a single-user APP decoder for C in an AWGN channel with specified SNRs. We assume that all users use the same FEC code, so $f(\cdot)$ is the same for every user- k . In

addition, we define the bit-error-rate (BER) performance for the k th DEC as a function of snr_k ,

$$\text{BER} = g(snr_k) \quad (13)$$

which can also be obtained by simulation. Combining (11) and (12), we have

$$snr_{k_new} = \frac{|h_k|^2}{\sum_{k' \neq k} |h_{k'}|^2 f(sn_{k'_old}) + \sigma^2}, \quad \forall k \quad (14)$$

where snr_{k_new} and snr_{k_old} are, respectively, snr_k values after and before one iteration. At the start, we initialize $f(sn_{k_old}) = 1$ for all k , implying no feedback from the DECs. Repeating (14), we can track the SNR evolution for the iterative process. During the final iteration, we can estimate the BER performance of all users as $g(sn_{k_final})$ from (13).

IV. POWER ALLOCATION

We now employ the above SNR evolution technique as the searching tool in performance optimization. Based on previous work [14][15], proper power allocation can improve the system performance. The intuition is that, with high probability, strong signals can be correctly detected first so that their interference to weak signals can be correctly cancelled, which in turn facilitates the detection of weak signals. The overall performance can be potentially improved in this way.

The effect of power allocation can also be seen from the fact that (14) is a function of the distribution of $\{h_k\}$ when $p \equiv \sum_k |h_k|^2$ is fixed. Suppose that we can adjust $\{h_k\}$ through power control. Then we can search for the distribution of $\{h_k\}$ that maximizes $\min_k(sn_{k_final})$. Since $g(\cdot)$ in (13) is monotonously decreasing (as a function of BER vs SNR), the above procedure is equivalent to minimizing the maximum BER among all users after a specified number of iterations. Due to space limitations, we omit the details of the optimization procedure but only provide some results.

Consider again the system model used in II.F. For simplicity, we assume one receive antenna and no multipath effects. We reduce the length of the repetition code S to 4 ($R_C = 1/8$), so as to avoid the situation of excessively large user numbers. Other parameters are the same as those used in II.F.

Tab. 1 shows the optimized power levels for the above system with $K=16, 32, 48$ and 64 , i.e., $K \times R_C = 2, 4, 6$ and 8 bits/chip. We adopted a quantization technique in optimization; therefore several users may be assigned the same power level. We can see from Tab. 1 that for $K \times R_C = 2$, equal power allocation is the best solution. For higher rates, however, unequal power allocation is necessary for good performance.

Fig. 3 shows the simulated BER performance (solid lines) for the users with the lowest power level. (The performance of other users is always better.) The performance curves estimated using the SNR evolution outlined in III with $It = 100$ (dashed lines) are also included. The capacity limits for their respective throughput and single-user performance are plotted for reference. The performance in Fig. 3 is considerably better than that reported in the literature for RW-CDMA systems [16, Fig. 5], especially in the high SNR region. The simulation and

evolution results are reasonably close, which confirms the viability of the performance analysis method and the power allocation scheme discussed above. The difference between the two methods may be due to several reasons: (i) Eqn. (8) is pessimistic. (ii) The inputs to the DECs become correlated during iterations with finite frame lengths.

Table 1. Relative power levels obtained by optimization.

	(power level (dB)) × (user number)
$K=16$	0×16
$K=32$	$0 \times 15, 3.311 \times 1, 3.725 \times 4, 4.139 \times 3, 6.623 \times 1, 7.037 \times 4, 7.451 \times 4$
$K=48$	$0 \times 15, 3.311 \times 3, 4.139 \times 3, 4.553 \times 3, 6.623 \times 2, 7.037 \times 2, 7.451 \times 3, 9.52 \times 1, 9.934 \times 3, 10.348 \times 3, 10.762 \times 1, 13.246 \times 4, 13.659 \times 5$
$K=64$	$0 \times 15, 3.311 \times 3, 3.725 \times 1, 4.139 \times 2, 4.553 \times 2, 6.209 \times 1, 6.623 \times 3, 7.037 \times 2, 7.451 \times 1, 8.692 \times 1, 9.106 \times 1, 9.52 \times 2, 9.934 \times 2, 10.762 \times 1, 11.176 \times 1, 11.59 \times 1, 12 \times 2, 12.832 \times 1, 13.246 \times 1, 13.66 \times 2, 14.487 \times 3, 16.143 \times 2, 16.557 \times 3, 16.971 \times 2, 19.454 \times 4, 19.868 \times 3, 20.282 \times 2$

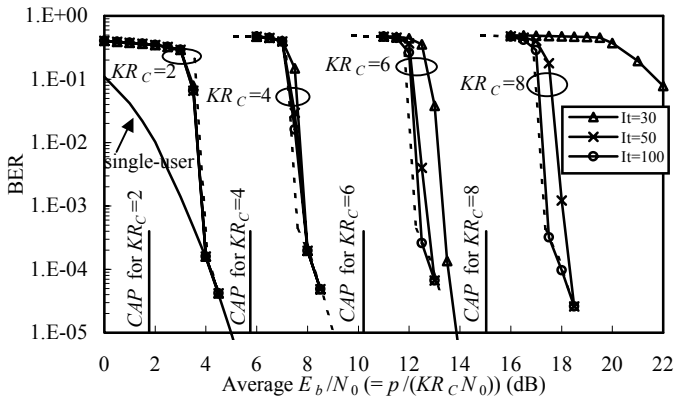


Figure 3. The lowest-power-users' BER performance obtained by simulation (solid lines) and SNR evolution (dashed lines) with $N_{\text{info}}=2048$ and $K \times R_c=2, 4, 6$ and 8 bits/chip. The capacity limits (abbreviated to CAP) for the corresponding throughput and single-user performance are plotted for reference. The performance of the users with higher power levels is always better.

Using the same principle, systems with even higher throughputs can also be designed. It appears that (provided that the frame length is sufficiently large) the throughput is only limited by SNR, not by the number of users (as in traditional CDMA systems [16]).

V. THE USE OF LOW-RATE CODES

Interestingly, IDMA is closely related to the low-rate coded CDMA scheme discussed in [15]. To see the advantage of introducing low-rate codes, return to a conventional CDMA system with both FEC coding and spreading. Let R be the rate of the FEC code and N the length of the signature sequence. Assume the same R and N for all users. After multiple access by K users, on average $R \times K/N$ information bits are transmitted in a chip duration, so we refer to $R \times K/N$ as the system throughput. Denote by η the maximum achievable value of $R \times K/N$ for a given R for reliable communication (η is called the spectral efficiency in [17]). When $K \rightarrow \infty$, the optimal spectral efficiency η of an RW-CDMA system in AWGN channels is (by substituting $SNR=2RE_b/N_0$ to (9) in [17])

$$\eta = \lim_{K \rightarrow \infty} \left(\frac{\beta}{2} \log_2 \left(1 + 2R \frac{E_b}{N_0} - \frac{F}{4} \right) + \frac{1}{2} \log_2 \left(1 + 2\beta R \frac{E_b}{N_0} - \frac{F}{4} \right) - \frac{F \log_2 e}{16R} \cdot \frac{N_0}{E_b} \right) \quad (15)$$

where $F = \left(\sqrt{2R \frac{E_b}{N_0} (1 + \sqrt{\beta})^2 + 1} - \sqrt{2R \frac{E_b}{N_0} (1 - \sqrt{\beta})^2 + 1} \right)^2$

and $\beta = K/N$.

Fig. 4 shows the η versus R curves according to (15). (We expect that they also shed light on cases with large finite K .) It is seen that η is maximized when R is minimized for any fixed E_b/N_0 . Some consequences of this observation are as follows.

- Fixing the bandwidth expansion factor ($=N/R$) for each user, minimizing R implies minimizing N . The minimum value of N is 1, meaning no spreading.
- RW-CDMA incurs no penalty on throughput when $R \rightarrow 0$, since it can be verified that η approaches the unconstrained AWGN channel capacity when $R \rightarrow 0$.
- There will be a non-negligible penalty on system throughput if R is not sufficiently small.

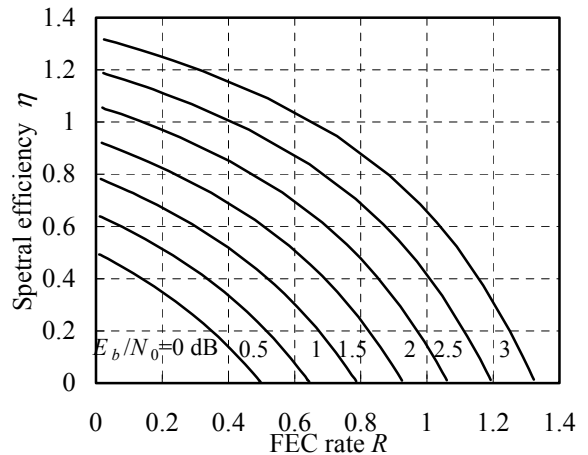


Figure 4. Spectral efficiency versus FEC rate in a real Gaussian MAC for $K \rightarrow \infty$.

We can also intuitively justify the above statements. Lower rate codes can provide higher coding gains, so they should be used in systems with bandwidth expansion. If we do not fully exploit redundancy related to bandwidth expansion using FEC coding, penalties on spectral or power efficiency result.

A conventional CDMA scheme, e.g., IS-95, employs FEC coding rate at around $1/2 \sim 1/3$. Quite long signature sequences are required to support a large number of users. This is not an optimized approach (although perhaps a convenient one) according to the above discussion.

Clearly, some structural change is necessary if we want to make progress towards fully exploiting the available capacity. A key issue is how to implement multiple access without spreading (that, in a conventional sense, does not provide coding gain). It appears that IDMA is the right answer to this question.

We now show the advantage of low-rate coding through an example. We employ the turbo-Hadamard code studied in [10] that can achieve performance close to the ultimate Shannon limit in AWGN channels. The particular code used below is constructed by concatenating three convolutional-Hadamard codes [10] in parallel, each generated from a length-32 Hadamard code and a convolutional code with polynomial $G(x) = 1/(1+x)$. The information bits in all component codes except one are punctured. A random puncturing operation on parity bits is also adopted to make $R_C = 1/16$.

Fig. 5 illustrates the performance of IDMA systems with equal power allocation based on the turbo-Hadamard code (Scheme I) in AWGN channels. (We are currently working on power allocation for low-rate coded systems.) From Fig. 5, performance of $\text{BER} = 10^{-5}$ is observed at $E_b/N_0 \approx 1.4$ dB with $K = 16$, which corresponds to $K \times R_C = 1$ bit/chip. This is only about 1.4 dB away from the corresponding Shannon limit, which is $E_b/N_0 = 0$ dB for a throughput of 1 bit/chip, the same as that for a single-user AWGN channel [17].

For comparison, we have also included in Fig. 5 the performance of an IDMA system based on a standard turbo code (Scheme II), in which C is constructed using a rate-1/3 $(1, 35/23)_8$ turbo code followed by a length-6 repetition code. Puncturing is applied to make $R_C = 1/16$. The advantage of using a low-rate code is clearly seen. With $K = 16$, Scheme I demonstrates about 1dB performance advantage over Scheme II, due to the higher coding gain of the turbo-Hadamard code. The decoding costs of Schemes I and II are quite similar.

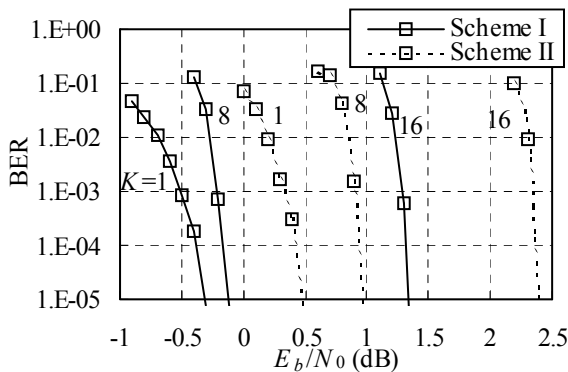


Figure 5. Performance of IDMA systems based on the turbo-Hadamard code [10] and turbo code over AWGN channels. $N_r = 1$, $l_t = 30$, $N_{\text{info}} = 4095$ for Scheme I and $N_{\text{info}} = 4096$ for Scheme II.

The power efficiency improvement illustrated above can be directly translated into spectral efficiency improvement in cellular systems. Since the performance of cellular systems is mainly limited by the interference among users. Lower transmission power from each user leads to reduced interference, and consequently a larger number of simultaneous users can be supported.

VI. CONCLUSIONS

The basic principle of the IDMA scheme is to employ interleavers as the only means for user separation. This has several consequences as listed below.

- A very low-cost CBC detection algorithm can be used in both synchronous and asynchronous channels.
- The performance of the CBC algorithm can be assessed by a simple semi-analytical technique. This technique is very efficient and we have used it to search for optimal power allocation. The resultant benefit is substantial as can be seen from Fig. 3. It appears that with sufficiently large frame lengths, the throughput of IDMA systems is only limited by SNR, not by the number of users as in traditional CDMA systems.
- Powerful low-rate FEC coding can be applied to further enhance the power efficiency of IDMA systems. Performance close to the theoretical limit in a Gaussian MAC has been demonstrated.

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