Multi-Layer Turbo Space-Time Codes For High-Rate Applications[1]

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***Abstract*- We study a multi-layer space-time code based on random interleaving and proper power allocation. We show that, theoretically, the proposed code can achieve capacity for any transmission rate by superimposing many ideal low-rate codes. We also show that, with practically available codes, the proposed scheme can achieve performance close to the theoretical limit.**

I. Introduction

High-rate transmission schemes have been extensively studied for multiple-input-multiple-output (MIMO) systems. In particular, the Bell Laboratories Layered Space Time (BLAST) code [1] and the related schemes [2][3] offer efficient solutions. In general, multiple receive antennas are required to support such schemes. On the other hand, space-time (ST) coding techniques have been investigated for multiple-input-single-output (MISO) systems. It is relatively easy to design good ST codes with low-to-median rates [4]-[6] (say, rate ≤ 2 bits per channel use), but it is still a challenging issue to design high-rate ST codes with performance close to the theoretical limits.

Recently, transmission schemes combining the features of both BLAST and ST codes have been explored. In [7][8], linear dispersion codes are proposed where emphasis is given on the performance optimization rather than efficient decoding techniques. An alternative is the interleave-division-multiplexing-ST (IDM-ST) coding scheme studied in [9], where randomly interleaved codewords are transmitted simultaneously from all antennas. This approach is conceptually very simple. The use of random interleaving allows a low-cost turbo-type iterative decoding technique [10].

In this paper, we will discuss a family of multi-layer (ML) IDM-ST codes constructed by superimposing several layers of IDM-ST codes. This can lead to greatly enhanced total transmission rate. For performance optimization, we examine two power allocation strategies among layers. Theoretically, the ML-IDM-ST code can achieve capacity for any transmission rate using an ideal low-rate forward error correction (FEC) code. (***Notes:*** (i) Here "ideal" corresponds to a conventional AWGN channel. (ii) With the multi-layer structure, high overall rates are achieved by superimposing many low-rate codes.) Simulation results show that with practical FEC coding, the proposed schemes can achieve near-capacity performance at high rates for MISO systems.

II. ML-IDM-ST Coding Scheme

A. Transmitter Principle

We first outline the basic concepts and properties related to the ML-IDM-ST codes [9][11]. Consider a system with N transmit antennas and one receive antenna (an N×1 system) in a quasi-static fading channel. The transmitter structure is shown in Fig.1. The inputs are K equal-length information sequences $\{\boldsymbol{d}_k,\ k = 1, \ldots, K\}$, each encoded individually using a binary FEC code C, generating $\boldsymbol{c}_k=\{c_{k,i}\}$ ($c_{k,i} \in \{+1, -1\}$). Signals from the same encoder are referred to as a layer.

With binary-phase-shift-keying (BPSK) modulation, each $\boldsymbol{c}_k$ is independently interleaved N times, producing $\{\boldsymbol{x}_k^{(n)}, n=1,\cdots,N\}$, where $\boldsymbol{x}_k^{(n)} = \{x_{k,j}^{(n)}, j=1,\cdots,J\}$ with J the frame length. With quadrature-phase-shift-keying (QPSK) modulation, each interleaved version of $\boldsymbol{c}_k$ is divided into two equal-length sequences to form the real and imaginary parts of $\boldsymbol{x}_k^{(n)}$. The signals in $\{\boldsymbol{x}_k^{(n)}, n=1,\cdots,N\}$ are scaled by a common amplitude factor $\sqrt{p_k}$ (see Fig.1) before distributed to the N transmit antennas. For the nth transmit antenna, the transmitted signal is $\sum_{k=1}^{K}\sqrt{p_k}\,\boldsymbol{x}_k^{(n)}$. The signal received at time j is

$$y_j = \sum_{n=1}^{N}\alpha^{(n)}\sum_{k=1}^{K}\sqrt{p_k}\,x_{k,j}^{(n)} + n_j \tag{1}$$

where $\alpha^{(n)}$ is the fading coefficient for the nth transmit antenna, $\{n_j\}$ are samples of an additive white Gaussian noise (AWGN) process with zero-mean and variance $\sigma^2=N_0/2$ per dimension. Assume the same FEC code C with rate R_C for all layers. The overall rate $R = KR_C$ for BPSK and $R = 2KR_C$ for QPSK.

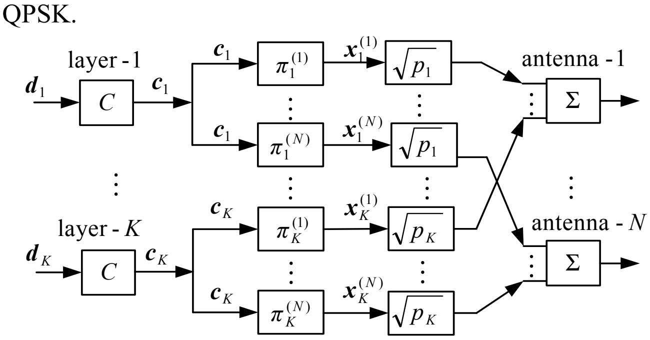

Figure 1. The transmitter structure of a N-antenna, K-layer ML-IDM-ST code, where $\pi_k^{(n)}$ is the interleaver for layer-k on the nth transmit antenna.

B. Decoding Principle

At the receiver side, we employ a sub-optimal iterative decoder [9], which consists of an elementary signal estimator (ESE) module and K *a posteriori* probability (APP) decoders (DECs), operating iteratively [10]. Fig. 2 illustrates a part of the receiver structure, in which only the DEC for layer-k (denoted by DEC-k) is shown. The DECs for other layers are connected to the ESE in the same way as DEC-k.

For the ESE operation, we first rewrite (1) as

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$$y_j = \alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} \tag{2}$$

where $\zeta_{k,j}^{(n)} = \sum_{(n',k')\neq(n,k)} \alpha^{(n')}\sqrt{p_{k'}}\, x_{k',j}^{(n')} + n_j$. Using the central limit theorem, we approximate $\zeta_{k,j}^{(n)}$ as a Gaussian random variable with mean $\mathrm{E}(\zeta_{k,j}^{(n)})$ and variance $\mathrm{Var}(\zeta_{k,j}^{(n)})$. For simplicity, we assume BPSK modulation and real $\{\alpha^{(n)}\}$. (The treatment for QPSK modulation and complex $\{\alpha^{(n)}\}$ can be found in [9]). Then the log-likelihood ratios (LLRs) for each transmitted bit $x_{k,j}^{(n)}$ can be computed based on (2) as [9]:

$$Ext(x_{k,j}^{(n)}) = \frac{2\alpha^{(n)}\sqrt{p_k}}{\mathrm{Var}(\zeta_{k,j}^{(n)})}\cdot\left(y_j - \mathrm{E}(\zeta_{k,j}^{(n)})\right) \qquad \forall k,n,j\ . \tag{3}$$

The technique to generate $\mathrm{E}(\zeta_{k,j}^{(n)})$ and $\mathrm{Var}(\zeta_{k,j}^{(n)})$ can be found in [9]. Let $S(c_{k,i})$ be the index set of N replicas in $\{x_{k,j}^{(n)}, \forall n, j\}$ related to $c_{k,i}$ (for all (n, j) combinations). We compute the LLR for each $c_{k,i}$ as:

$$\tilde{L}(c_{k,i}) = \sum_{(n,j)\in S(c_{k,i})} Ext(x_{k,j}^{(n)}) , \qquad \forall k,i\ . \tag{4}$$

These LLRs constitute the inputs to the DECs, see Fig. 2.

Each DEC-k carries out an APP decoding. Its outputs are *a posteriori* LLRs $\{L(c_{k,i}),\forall i\}$ for $\{c_{k,i}, \forall i\}$, which are used to update *a priori* LLRs $\{\tilde{L}(x_{k,j}^{(n)}),\forall n,j\}$ of $\{x_{k,j}^{(n)},\forall n,j\}$ as (assuming $(n,j)\in S(c_{k,i})$)

$$\tilde{L}(x_{k,j}^{(n)}) = L(c_{k,i}) - Ext(x_{k,j}^{(n)}) , \qquad \forall k,n,j\ . \tag{5}$$

Then $\{\tilde{L}(x_{k,j}^{(n)})\}$ are used to update $\{\mathrm{E}(\zeta_{k,j}^{(n)})\}$ and $\{\mathrm{Var}(\zeta_{k,j}^{(n)})\}$ in the ESE in the next iteration. The detailed discussion on the iterative decoding process can be found in [9][11].

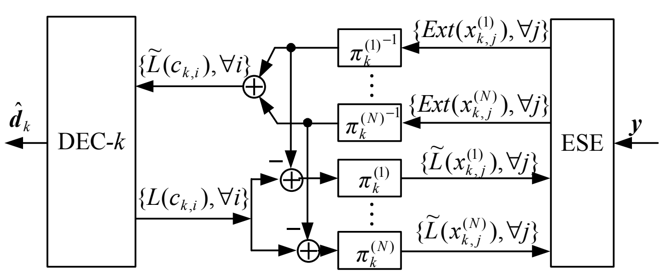

Figure 2. A part of the receiver structure of the ML-IDM-ST code related to layer-k, where $\mathbf{y} \equiv \{y_j\}$. The DECs for other layers are connected to the ESE in the same way as DEC-k.

III. Performance Analysis

A. Performance Analysis with Fixed Fading Coefficients

We first consider the performance analysis of ML-IDM-ST codes with real and fixed $\boldsymbol{\alpha} \equiv [\alpha^{(1)}\cdots\alpha^{(N)}]$. Define

$$V_{\zeta_k^{(n)}} \equiv \sum_{(n',k')\neq(n,k)} \left|\alpha^{(n')}\right|^2 p_{k'} V_{c_{k'}} + \sigma^2 \tag{6}$$

where V_{c_k} is the average variance of the feedback LLRs from DEC-k [11]. Approximating $\mathrm{Var}(\zeta_{k,j}^{(n)})$ by $V_{\zeta_k^{(n)}}$, (3) can be written as:

$$Ext(x_{k,j}^{(n)}) = \frac{2\alpha^{(n)}\sqrt{p_k}}{V_{\zeta_k^{(n)}}}\cdot\left(\alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)})\right). \tag{7}$$

(***Note:*** Using (7) leads to certain performance loss compared with using (3), but it makes the subsequent discussions easier.) In (7), $\alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)}$ and $\zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)})$ represent signal and distortion components, respectively. Approximate the average power of $\{\zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)}),\forall j\}$ by $V_{\zeta_k^{(n)}}$. The average signal-to-noise ratio (SNR) of $\{Ext(x_{k,j}^{(n)}),\forall j\}$, denoted by $snr_k^{(n)}$, is

$$snr_k^{(n)} = \frac{\mathrm{E}\left(\left|\alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)}\right|^2\right)}{\mathrm{E}\left(\left|\zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)})\right|^2\right)} \approx \frac{\left|\alpha^{(n)}\right|^2 p_k}{\sum_{(n',k')\neq(n,k)} \left|\alpha^{(n')}\right|^2 p_{k'} V_{c_{k'}} + \sigma^2}\ . \tag{8}$$

Substituting (7) into (4), we have

$$\tilde{L}(c_{k,i}) = \sum_{(n,j)\in S(c_{k,i})} \frac{2\alpha^{(n)}\sqrt{p_k}}{V_{\zeta_k^{(n)}}}\cdot\left(\alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)})\right). \tag{9}$$

Besides a scaling factor of 2, (9) can be regarded as a maximum ratio combining (MRC) of N independently distorted signals: $\left\{\alpha^{(n)}\sqrt{p_k}\, x_{k,j}^{(n)} + \zeta_{k,j}^{(n)} - \mathrm{E}(\zeta_{k,j}^{(n)}) \mid (n,j)\in S(c_{k,i})\right\}$. Using (8), (9) and the basic property of MRC [12], the average SNR for $\{\tilde{L}(c_{k,i}),\forall i\}$, denoted by snr_k, is given by

$$snr_k = \sum_{n=1}^{N} snr_k^{(n)} = \sum_{n=1}^{N} \frac{\left|\alpha^{(n)}\right|^2 p_k}{\sum_{(n',k')\neq(n,k)} \left|\alpha^{(n')}\right|^2 p_{k'} V_{c_{k'}} + \sigma^2}\ . \tag{10}$$

Recall that V_{c_k} is the variance of the output of DEC-k with input SNR given by snr_k. We can express V_{c_k} as a function of snr_k as

$$V_{c_k} = f(snr_k). \tag{11}$$

In general, there is no closed-form expression for $f(\cdot)$, but it can be approximated by the Monte Carlo simulation results for C in an AWGN channel with specified input SNRs. Similarly we can define the frame-error-rate (FER) performance for DEC-k, denoted by FER_k, as a function of snr_k,

$$\mathrm{FER}_k = g(snr_k) \tag{12}$$

which can also be obtained by simulation. Substituting (11) into (10), we have

$$snr_{k\_\mathrm{new}} = \sum_{n=1}^{N} \frac{\left|\alpha^{(n)}\right|^2 p_k}{\sum_{(n',k')\neq(n,k)} \left|\alpha^{(n')}\right|^2 p_{k'} f(snr_{k'\_\mathrm{old}}) + \sigma^2} \tag{13}$$

where snr_{k_new} and snr_{k_old} are snr_k values after and before one iteration. At the start, we initialize $f(snr_{k\_\mathrm{old}}) = 1$ for $\forall k$, implying no feedback from DECs. After the final iteration, we can estimate the FER performance of layer-k using the final value of snr_k obtained through (13) as:

$$\mathrm{FER}_k = g(snr_{k\_\mathrm{final}}). \tag{14}$$

The above technique provides a fast and reasonably accurate ways to assess the performance of a ML-IDM-ST scheme [11]. The result can also be generalized to QPSK modulation and complex $\boldsymbol{\alpha}$ (following the discussion in [13]).

B. Performance Analysis in Quasi-Static Rayleigh Fading Channels

The discussion in III.A is for fixed $\boldsymbol{\alpha}$. We now consider quasi-static Rayleigh fading channels where $\boldsymbol{\alpha}$ remain unchanged in one frame and change independently from frame to frame. Taking all possible $\boldsymbol{\alpha}$ into consideration, we have

$$\mathrm{FER}_{k\_\mathrm{fading}} = \int g(snr_{k\_\mathrm{final}})\, p(\boldsymbol{\alpha}) d\boldsymbol{\alpha}\,, \tag{15}$$

where $p(\boldsymbol{\alpha}) \equiv p(\alpha^{(1)}) \cdots p(\alpha^{(N)})$ is the joint probability density function (pdf) of $\{\alpha^{(1)}, \cdots, \alpha^{(N)}\}$. (***Note:*** snr_{k_final} is a function of $\boldsymbol{\alpha}$.) The computation of (15) is relatively complicated since it involves an N-fold multiple integral. To avoid this difficulty, we introduce a bounding technique. Denote $\lambda \equiv \sum_n |\alpha^{(n)}|^2 = \boldsymbol{\alpha}\boldsymbol{\alpha}^H$ the total power gain of the channel. From (10), snr_k can be lower-bounded as [11]

$$snr_k \ge \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} V_{c_{k'}} - \lambda p_k V_{c_k} / N + \sigma^2} \tag{16}$$

where the equality holds when $|\alpha^{(1)}|^2 = \ldots = |\alpha^{(N)}|^2 = \lambda/N$, i.e., the uniform-fading situation. Consequently, $\mathrm{FER}_{k\_\mathrm{fading}}$ can be upper-bounded as

$$\mathrm{FER}_{k\_\mathrm{fading}} \le \int_0^{+\infty} g(\overline{snr}_{k\_\mathrm{final}})\, p(\lambda) d\lambda\,, \tag{17}$$

where $p(\lambda) = \lambda^{N-1} e^{-\lambda} / (N-1)!$ is the pdf of λ in a quasi-static Rayleigh fading channel, and $\overline{snr}_{k\_\mathrm{final}}$ is calculated using the following iteration

$$\overline{snr}_{k\_\mathrm{new}} = \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} f(\overline{snr}_{k'\_\mathrm{old}}) - \lambda p_k f(\overline{snr}_{k\_\mathrm{old}}) / N + \sigma^2}\,, \tag{18}$$

with $f(\overline{snr}_{k\_\mathrm{old}})$ initialized to 1 for $\forall k$. Although several approximations are involved in this bounding technique, it is quite close to the simulated performance (see Fig. 4 in [11]), and can be used for fast performance assessment.

IV. POWER ALLOCATION

From previous work [3], proper power allocation can improve the system performance. We now show that the SNR evolution techniques developed above can be used to devise power allocation strategies for performance optimization.

A. Power Allocation with Ideal FEC Coding

With ideal FEC coding, we employ the successive cancellation technique [14][15] at the receiver. For each layer-k, assume that the contributions from layers $k' > k$ have been removed from the received signal, and treat the interference from layers $k' \le k$ in the same way as the additive noise.

We first assume that at the transmitter, λ is known *a priori* but the detailed values of $\{\alpha^{(n)}\}$ are unknown. We generate $\{p_k\}$ recursively as follows [11]:

$$\frac{\lambda p_k}{\lambda \sum_{k'<k} p_{k'} + \lambda(N-1) p_k / N + \sigma^2} = 2^{2R_C} - 1\,. \tag{19}$$

If C is capacity achieving, it can be shown [11] that a ML-IDM-ST code designed based on (19) can achieve reliable communication. The total power required can be calculated as

$$P_{\mathrm{IDM}} = \sum_k p_k = \frac{\sigma^2}{\lambda}\left(\left(\frac{2^{2R_C} - 1 + N}{N - (2^{2R_C} - 1)(N-1)}\right)^{\frac{R}{2R_C}} - 1\right). \tag{20}$$

(Here we consider complex signaling so that $R = 2KR_C$.)

The capacity of the channel considered is [16]

$$CAP = \log_2(1 + \boldsymbol{\alpha}\boldsymbol{\alpha}^H \rho / N) = \log_2(1 + \lambda P / \sigma^2) \tag{21}$$

where

$$\rho = NP/\sigma^2 \tag{22}$$

is the average SNR per receive antenna. For any given R, the minimum required value of $P \equiv \sum_{k=1}^{K} p_k$ for reliable transmission can be calculated from (21) by setting $CAP = R$ as

$$P_{\min} = (2^R - 1)\sigma^2 / \lambda\,. \tag{23}$$

In general, there is a gap between P_{IDM} and $P_{\min}$ even when C is capacity achieving. However, the following equation shows that using a very low-rate code C can narrow this gap [11].

$$\lim_{R_C \to 0}(P_{\mathrm{IDM}} - P_{\min}) = \frac{\sigma^2}{\lambda} \lim_{R_C \to 0}\left(\left(\frac{2^{2R_C} - 1 + N}{N - (2^{2R_C} - 1)(N-1)}\right)^{\frac{R}{2R_C}} - 2^R\right) = 0\,. \tag{24}$$

Hence the ML-IDM-ST code can indeed achieve capacity provided that an ideal low-rate code C is used.

When λ is a random variable unknown at the transmitter but the distribution of λ is known at the transmitter, we use the so-called outage capacity β_{out} [6][16] as the performance criterion,

$$\beta_{\mathrm{out}} \equiv \Pr(CAP < R)\,. \tag{25}$$

It is verified in [11] that if (24) holds, a ML-IDM-ST code can approach the outage capacity in quasi-static fading channels with ideal low-rate FEC coding.

B. Power Allocation with Non-Ideal FEC Coding

With non-ideal FEC coding, we use the iterative decoding procedure introduced in Section II.B at the receiver. In this case, since there is no closed-form power allocation strategy (as (19)) known to us, we search for a distribution of $\{p_k\}$ leading to the optimal system performance under the constraint of $\sum_k p_k = P$. In the following, we discuss two searching strategies.

Method 1: Exhaustive Search

For a small K, an exhaustive search is feasible. Eqn. (15) can be used for performance evaluation in the search, but the complexity involved is high. Alternatively, we can approximate the performance using the upper bound in (17). The exhaustive search becomes very time-consuming when K is large.

Method 2: Layer-by-Layer Search

The following is a low-complexity alternative. Suppose that we have obtained a sub-optimal solution for a (K-1)-layer system, denoted by $\{p_1^{(K-1)}, \ldots, p_{K-1}^{(K-1)}\}$. We search for $\{p_1^{(K)}, \cdots, p_{K-1}^{(K)}, p_K^{(K)}\}$ for a K-layer system assuming that the relative power ratios for the lower K-1 layers are preserved, i.e., $p_k^{(K)} = \beta p_k^{(K-1)}$ for k=1, …, K-1. In this way we only need to search for the best $(\beta, p_K^{(K)})$ pair for the K-layer system. This method is much faster than the exhaustive search.

V. A Modified ML-IDM-ST Coding Scheme

Power allocation results in fast convergence for the layers with stronger power levels. This will in turn reduce the interference to other layers and accellerate their convergence. In the sheme shown in Fig.1, we assume that the signals in a layer are transmitted continuously on every antenna. Then the signals in the same layer transmitted from different antennas will interfer each other. Such interference does not benefit from power allocation since all signals from the same layer have the same power level.

We now introduce a modified ML-IDM-ST coding scheme to treat the above problem. The basic principle is the same as that of the code in Fig.1: The data sequence of each layer is encoded and randomly interleaved. Each coded bit is transmitted from all N antennas using proper power levels. The differences between the modified code and the code in Fig. 1 are as follows:

(i) Signals from different layers are transmitted alternatively on each antenna in a time-division-multiplexing manner.

(ii) At any time instance, the signal from any layer is transmitted on a unique (and only one) antenna.

To achieve property (ii), a cyclic shift is adopted (but other methods are also possible), as shown in Fig. 3. Clearly, in the modified scheme, the interference experienced by one layer comes from outside the layer. The total rate of this scheme is $R = R_C K/N$ for BPSK and $R = 2R_C K/N$ for QPSK.

time index	0	1	2	3	4	5	6	7
antenna-1	1+5	4+8	3+7	2+6	1+5	4+8	3+7	2+6
antenna-2	2+6	1+5	4+8	3+7	2+6	1+5	4+8	3+7
antenna-3	3+7	2+6	1+5	4+8	3+7	2+6	1+5	4+8
antenna-4	4+8	3+7	2+6	1+5	4+8	3+7	2+6	1+5

Figure 3. The transmitter structure of a modified ML-IDM-ST code with K = 8 and N = 4. The symbol transmitted on antenna-1 at time 0 contains the signals in the 1st and 5th layers. The symbol transmitted on antenna-1 at time 1 contains the signals in the 4th and 8th layers, etc.

The modified ML-IDM-ST scheme can be regarded as a generalization of the diagonal BLAST code [1]. There are two key differences.

- Random interleaving is a crucial operation in the ML-IDM-ST scheme.
- In both Figs. 1 and 3, each coded bit is repeated over all antennas. This repetition operation contributes to the diversity property of the ML-IDM-ST scheme and distinguishes it from the turbo-BLAST scheme [2].

We now proceed to consider a fast performance assessment technique for the modified ML-IDM-ST scheme. Similar to the discussion in Section III, denote by snr_k the average SNR for the coded bits in layer-k. From Fig. 3, the signals in layer-k are transmitted on antenna-n at time j only when

$$n = ((j+k-1) \bmod N)+1.$$

Similar to the derivation of (13), we can show that

$$snr_{k\_\text{new}} = \sum_{n=1}^{N} \frac{\left|\alpha^{(n)}\right|^2 p_k}{\sum_{k'\neq k} \left|\alpha^{(((k'-k+n-1) \bmod N)+1)}\right|^2 p_{k'} f(snr_{k'\_\text{old}}) + \sigma^2} . \quad (26)$$

Comparing (26) with (13), we can see that the denominator in (13), which represents the interference-plus-noise power for layer-k, contains the contribution of layer-k itself (from antenna-n', with $n' \neq n$). This situation is avoided in (26).

Inspired by (16), we may expect

$$snr_{k\_\text{new}} \geq \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} f(snr_{k'\_\text{old}})/N - \lambda p_k f(snr_{k\_\text{old}})/N + \sigma^2} . \quad (27)$$

Unfortunately, (27) does not hold since counter examples exist. Nevertheless, simulation results show that the performance estimated using the following approximation is reasonably accurate (see Section VI).

$$snr_{k\_\text{new}} \approx \frac{\lambda p_k}{\lambda \sum_{k'} p_{k'} f(snr_{k'\_\text{old}})/N - \lambda p_k f(snr_{k\_\text{old}})/N + \sigma^2} . \quad (28)$$

Therefore, we have used (28) as a fast performance assessment tool in the power allocation process. We are still working on a rigorous justification for (28).

VI. Numerical Results

In our simulation, QPSK modulation is always assumed. We apply different rotations $\{0, \pi/2K, \cdots, (K-1)\pi/2K\}$ to signals from different layers to make the interference from other layers more Gaussian-like. We call each $\boldsymbol{d}_k$ a frame and $\{\boldsymbol{d}_k, k=1, \ldots, K\}$ a super-frame. The frame error rate (FER) and super-frame error rate (SFER) are defined accordingly. Clearly, SFER≥FER.

We first compare the exhaustive and layer-by-layer search methods. Consider 2×1 and 4×1 ML-IDM-ST codes in Fig. 1 with R = 2 and 4 bits per channel use, corresponding to K = 3 and 6, respectively. A rate-1/3 turbo code with generator $G(x) = (1+x+x^3)/(1+x^2+x^3)$ is used for all layers. Table 1 illustrates the power levels for these codes obtained using the two methods. As we can see, the two methods lead to similar power levels.

Table 1. Power allocation for the original ML-IDM-ST code in Fig. 1.

Method	(N,R)	Power Levels
Exhaustive search	(2,2)	0.16P, 0.3P, 0.54P
	(4,2)	0.174P, 0.3P, 0.526P
	(2,4)	0.0231P, 0.0433P, 0.0798P, 0.1456P, 0.2574P, 0.4508P
	(4,4)	0.0246P, 0.0445P, 0.0808P, 0.147P, 0.2569P, 0.4462P
Layer-by-layer search	(2,2)	0.1617P, 0.3003P, 0.538P
	(4,2)	0.1737P, 0.2983P, 0.528P
	(2,4)	0.0242P, 0.045P, 0.0806P, 0.1439P, 0.2543P, 0.452P
	(4,4)	0.0268P, 0.0461P, 0.0809P, 0.1421P, 0.2521P, 0.452P

Fig. 4 shows the performance comparison between the original and modified ML-IDM-ST codes with N = 2 and 4, R = 2 and 4 bits per channel use. The rate-1/3 turbo code with generator $G(x) = (1+x+x^3)/(1+x^2+x^3)$ is used for both schemes. For the original scheme, the power levels obtained using the layer-by-layer search method are used. The power levels for the modified scheme are listed in Table 2, which are also obtained using layer-by-layer search. The values of K for the modified codes with different N and R are shown in the first column of Table 2. Compared with the original scheme, the modified scheme has better or comparable FER. It is also interesting to note that the SFER of the modified scheme is inferior to that of the original one with $N = 2$. (In practice, FER can be a more useful performance measurement than SFER, as in case of error it is only necessary to discard the erroneous frames, instead of the complete super-frame.)

Fig. 5 shows the comparison between the simulation results and the estimation results based on (28) for the modified ML-IDM-ST codes in Fig. 4(b). As we can see, the estimation results are quite close to the simulation results.

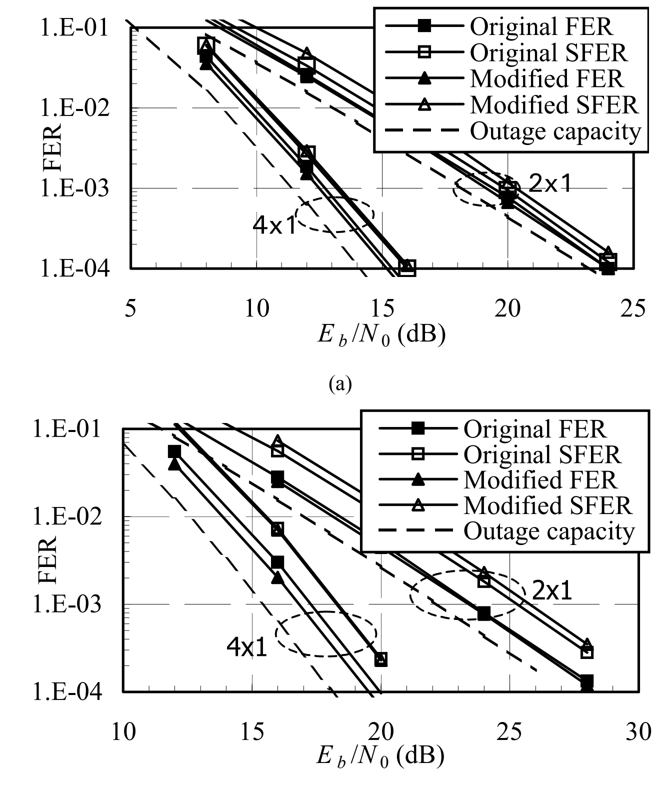

Figure 4. FER and SFER performance of 2×1 and 4×1 ML-IDM-ST codes with (a) $R = 2$ and (b) $R = 4$ bits per channel use over a quasi-static complex Rayleigh fading channel. QPSK modulation is used. The number of information bits per layer = 4096, iteration number = 30.

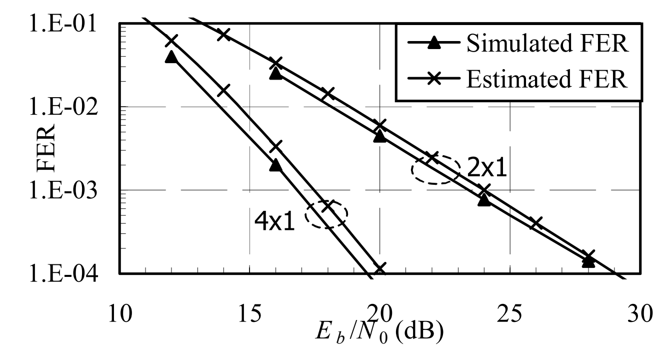

Figure 5. Comparison between the simulation results and estimation results based on eqn. (28) for the modified ML-IDM-ST codes in Fig. 4 (b).

Table 2. Power allocation for the modified ML-IDM-ST code in Fig. 3.

(N,R,K)	Power Levels
(2,2,6)	0.0642P, 0.0956P, 0.1287P, 0.1738P, 0.2297P, 0.308P
(4,2,12)	0.0376P, 0.0397P, 0.047P, 0.0559P, 0.0607P, 0.0695P, 0.0786P, 0.0901P, 0.1066P, 0.1149P, 0.1394P, 0.16P
(2,4,12)	0.0101P, 0.0146P, 0.0196P, 0.0274P, 0.0341P, 0.0471P, 0.0619P, 0.0819P, 0.1097P, 0.1442P, 0.1934P, 0.256P
(4,4,24)	0.006P, 0.0063P, 0.0075P, 0.0089P, 0.0096P, 0.011P, 0.0125P, 0.0143P, 0.0169P, 0.0182P, 0.0221P, 0.0254P, 0.0289P, 0.0331P, 0.0378P, 0.0428P, 0.0515P, 0.0565P, 0.0677P, 0.0777P, 0.0888P, 0.103P, 0.1175P, 0.136P

VII. CONCLUSIONS

High-rate multi-layer ST coding schemes have been designed based on proper power allocation. Fast performance estimation techniques have been proposed, based on which power allocation can be carried out efficiently. Both theoretical analysis and simulation results show that the proposed scheme can achieve performance close to the theoretical limits.

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