Power Allocation for Multiple Access Systems with Practical Coding and Iterative Multi-User Detection

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***Abstract*—This paper is concerned with the power allocation problem for practically coded interleave-division multiple-access (IDMA) systems with iterative multi-user detection (MUD) over multiple access channels (MACs). Both Additive White Gaussian Noise (AWGN) and fading channels are considered. For AWGN channels, two power allocation methods based on linear programming are discussed and compared with an interior-point method (IPM). These techniques are extended to power allocation over fading channels. Numerical results show that, with power optimization, IDMA can achieve significant performance improvements over TDMA in fading environments.**

Keywords—interleave-division multiple-access (IDMA), multi-user detection (MUD), power allocation, linear programming, multiple access channels (MACs).

I. INTRODUCTION

It is well known that, with single-user detection and equal power control, the throughput of a conventional code-division multiple-access (CDMA) system is interference limited [1][2]. Iterative multi-user detection (MUD) [3][4][5] is a solution to this problem and has been widely investigated. However, the complexity of MUD has always been a serious concern for practically coded CDMA systems.

Interleave-division multiple-access (IDMA) [6][7] is a special case of CDMA that solely relies on chip-level interleaving for user separation. IDMA allows the use of low rate forward error correction (FEC) codes to maximize coding gain. The chip-by-chip (CBC) MUD algorithm [6] for IDMA is an iterative soft-cancellation technique for treating multiple access interference (MAI). Its computation cost is very low, being independent of the total number of users when normalized to each user. With equal power control, it has been shown in [7] that IDMA together with CBC algorithm can achieve performance close to the theoretical limits. Unequal power allocation [7] is a technique to further enhance system performance. In this case, finding the optimal power profiles is a difficult issue for practically coded systems. In [7], a linear programming technique is employed for power optimization of IDMA systems over additive white Gaussian noise (AWGN) multiple access channels (MACs). This technique involves approximation and is only effective for systems with a large number of users. For a multiple access system with a small number of users, linear programming may not be efficient [7]. In [8], an interior-point method (IPM) is proposed as an alternative solution, which can be applied to both AWGN and fading channels. However, the computation cost of IPM grows rapidly with the number of users.

In this paper, we present an improved linear programming method for the power optimization problem for IDMA systems over AWGN channels. This method can also be applied to fading channels when combined with an ordered-matching technique. Numerical results illustrate that the proposed method can provide better solutions than the linear programming method discussed in [7], while having complexity much lower than IPM [8].

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II. IDMA SYSTEMS

A. System Model

Consider the K-user IDMA system shown in Fig. 1. At the transmitter for user-k, the information bit stream $\boldsymbol{d}_k$ is first encoded by an FEC encoder (ENC_k) and then interleaved and transmitted over a multiple access channel (MAC) with proper power control factor p_k as indicated in Fig. 1.

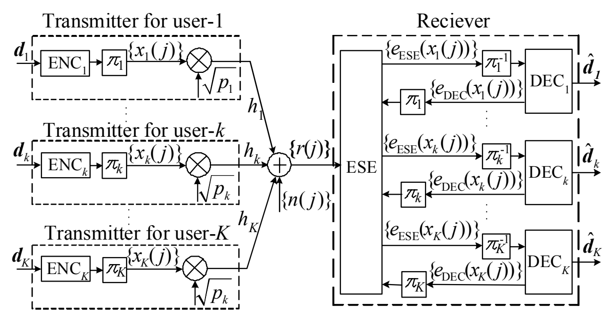

Fig. 1. The system model of an IDMA multiple access scheme with iterative multi-user detection.

Denote by $x_k(j) \in \{+1, -1\}$ the j-th chip of the length-J signal sequence after encoding and interleaving at the transmitter for user-k. Assuming BPSK modulation and using the equivalent discrete channel model, we can write the received signal as

$$r(j) = \sum_{k=1}^{K} \sqrt{p_k}\, h_k x_k(j) + n(j), \qquad j = 1, 2, \ldots, J \qquad (1)$$

where h_k is the channel coefficient for user-k, and $\{n(j)\}$ are samples of an additive white Gaussian noise (AWGN) process with zero-mean and variance $\sigma^2 = N_0/2$.

We assume that the interleavers $\{\pi_k\}$ in Fig. 1 are all randomly and independently generated. In this case, the users can be solely distinguished by interleavers, hence the name "IDMA". The use of random interleavers greatly simplifies the system analysis task, since it avoids the need to model the correlation among spreading sequences [6][7].

The receiver in Fig. 1 consists of an elementary signal estimator (ESE) and a bank of K single-user *a posteriori* probability (APP) decoders (DECs), operating iteratively. We follow the iterative detection principle of the CBC algorithm developed in [6], as outlined below. For simplicity, we only consider real $\{h_k\}$ here, but the results can be easily extended to complex systems.

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Focusing on user-k, we can rewrite (1) as

$$r(j) = \sqrt{p_k} h_k x_k(j) + \xi_k(j) \tag{2}$$

where

$$\xi_k(j) \equiv r(j) - \sqrt{p_k} h_k x_k(j) = \sum_{i \neq k} \sqrt{p_i} h_i x_i(j) + n(j) \tag{3}$$

is the noise-plus-interference component in $r(j)$ in (1) with respect to $x_k(j)$.

Based on the assumption that $\xi_k(j)$ is a Gaussian random variable with mean $\mathrm{E}(\xi_k(j))$ and variance $\mathrm{Var}(\xi_k(j))$, we can obtain the output of the ESE for user-k as follows.

$$e_{\mathrm{ESE}}(x_k(j)) = \frac{2\sqrt{p_k} h_k \left(r(j) - \mathrm{E}(\xi_k(j))\right)}{\mathrm{Var}(\xi_k(j))}. \tag{4}$$

Using $\{e_{\mathrm{ESE}}(x_k(j))\}$ as the *a priori* information, DEC_k generates extrinsic log-likelihood ratios (LLRs) $\{e_{\mathrm{DEC}}(x_k(j))\}$, based on which $\{\mathrm{E}(\xi_k(j))\}$ and $\{\mathrm{Var}(\xi_k(j))\}$ can be updated [6]. Then the operation defined by (4) is performed again. This iterative process continues for a preset number of times before a hard decision is made to produce the final output. The details can be found in [6].

B. Performance Evaluation

The performance of the above iterative process can be quickly evaluated using the following signal to noise-plus-interference ratio (SNIR) evolution technique [7]. Denote by $\{\gamma_k^{(l)}\}$ the average SNIR for the outputs of the ESE after the l-th iteration. Let $f_k(\gamma_k^{(l)})$ be the average variance of the outputs of DEC_k driven by an input sequence with SNIR $\gamma_k^{(l)}$. We can approximately track $\gamma_k^{(l)}$ using the following recursion:

$$\gamma_k^{(l+1)} = \frac{p_k |h_k|^2}{\sum_{i \neq k} p_i |h_i|^2 f_i(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \cdots, L-1 \tag{5}$$

where L is the maximum number of iterations.

At the beginning of the iteration, we set $\gamma_k^{(0)} = 0$, for all k, implying no feedback from DECs. By repeating (5), we can track the SNIR evolution for all users during the iterative process in the CBC algorithm. After the final iteration, we can estimate the final bit error rates (BERs) of all users according to their final SNIR values $\{\gamma_k^{(L)}\}$ and the codes used.

The $f(\cdot)$ function (with subscript i omitted here) in (5) is determined by the code used. For a rate-R ideal code, the $f(\cdot)$ function is obtained based on the Shannon capacity formula as

$$f(\gamma) = \begin{cases} 1, & \text{if } \gamma < 2^{2R} - 1 \\ 0, & \text{if } \gamma \geq 2^{2R} - 1 \end{cases}. \tag{6}$$

Here we assume that if the input SNIR $\gamma < 2^{2R}$-1, the decoder outputs are randomly binary errors and so the variance is 1 (assuming BPSK format). Otherwise the output is error free and so the variance is 0.

For a practical code, the $f(\cdot)$ function can be obtained by the Monte-Carlo method [7]. Fig. 2 shows some examples of $f(\cdot)$ functions. In practice, the $f(\cdot)$ function so obtained is monotonically decreasing with $f(0) = 1$ and $f(+\infty) = 0$. Note that $\gamma = 0$ means the decoder inputs are all noise, so the outputs are random binary errors with variance $f(0) = 1$. Also the input SNIR $\gamma = +\infty$ leads to error free decoding so that variance $f(+\infty) = 0$.

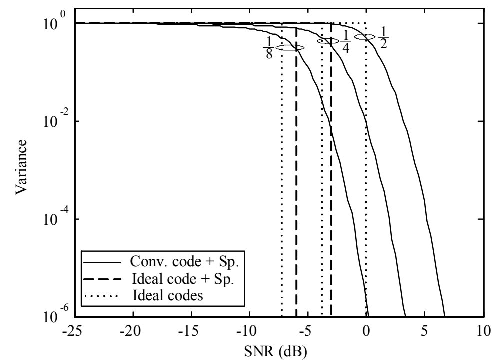


Fig. 2. Some examples of $f(\cdot)$ functions. The corresponding codes are a rate-1/2 convolutional code (generators: $(23, 35)_8$) with length-1, 2 and 4 spreading, rate 1/2, 1/4 and 1/8 ideal codes and a rate-1/2 ideal code with length-2 and 4 spreading. The numbers marked beside the curves are coding rates.

III. Power Allocation over AWGN Channels

This section is concerned with the power optimization problem for IDMA systems over AWGN channels (fading channels is discussed in the next section).

A. Problem Formulation

For simplicity, we assume that, for all users, the same FEC code is used and the same BER performance is required. For AWGN channels, we have $|h_k|^2 = 1$, $\forall k$. Then recursion (5) reduces to the following form.

$$\gamma_k^{(l+1)} = \frac{p_k}{\sum_{i \neq k} p_i f(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \cdots, L-1. \tag{7}$$

Our objective now is to minimize total power $\sum_k p_k$ while achieving the required performance $\gamma_k^{(L)} \geq \Gamma$, $\forall k$, for pre-specified Γ. This power optimization problem can be formulated as follows.

<u>Power optimization over AWGN channels:</u> Find the distribution $\{p_k\}$ that minimizes

$$\Phi = \sum_{k=1} p_k \tag{8a}$$

subject to

$$\gamma_k^{(L)} \geq \Gamma, \forall k \tag{8b}$$

where $\{\gamma_k^{(L)}\}$ are obtained through the following SNIR evolution process with initialization $\gamma_k^{(0)} = 0$, for all k.

$$\gamma_k^{(l+1)} = \frac{p_k}{\sum_{i \neq k} p_i f(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \cdots, L-1. \tag{8c}$$

Problem (8) is generally nonlinear and non-convex. We call a power profile feasible if it satisfies the system requirements and define the feasible region as the set of all feasible power profiles. Then the problem considered above is to find the minimum sum-power solution within the feasible region.

We define the total interference after the l-th iteration of recursion (7) as

$$I^{(l)} = \sum_{k=1} p_k f(\gamma_k^{(l)}) + \sigma^2. \tag{9}$$

Then (7) can be rewritten as

$$\gamma_k^{(l+1)} = \frac{p_k}{I^{(l)} - p_k f(\gamma_k^{(l)})}, \forall k, l = 0,1,\cdots,L-1. \quad (10)$$

B. Properties of $\{\gamma_k^{(l)}\}$ *and* $\{I^{(l)}\}$

The two monotonic properties about $\{\gamma_k^{(l)}\}$ and $\{I^{(l)}\}$ below will be useful in later derivation. They follow the fact that $f(\cdot)$ is monotonically decreasing:

Property I: $\{\gamma_k^{(l)}\}$ is a monotonically increasing sequence for any k, i.e.,

$$\gamma_k^{(l+1)} \ge \gamma_k^{(l)}, \forall k, l. \quad (11)$$

This property can be verified by induction as follows: for $l = 0$, we have

$$\gamma_k^{(1)} = \frac{p_k}{\sum_{i \ne k} p_i f(\gamma_i^{(0)})} = \frac{p_k}{\sum_{i \ne k} p_i} > 0 = \gamma_k^{(0)}. \quad (12)$$

Suppose that (11) holds for some l. Then we have

$$\gamma_k^{(l+2)} = \frac{p_k}{\sum_{i \ne k} p_i f(\gamma_i^{(l+1)})} \ge \frac{p_k}{\sum_{i \ne k} p_i f(\gamma_i^{(l)})} = \gamma_k^{(l+1)}. \quad (13)$$

Thus (11) holds for all k and l.

Property II: $\{I^{(l)}\}$ is a monotonically decreasing sequence, i.e.,

$$I^{(l+1)} \le I^{(l)}, \quad l = 0,1,\cdots L-1. \quad (14)$$

This property can be easily seen from Property I as

$$I^{(l+1)} = \sum_k p_k f(\gamma_k^{(l+1)}) + \sigma^2 \le \sum_k p_k f(\gamma_k^{(l)}) + \sigma^2 = I^{(l)}. \quad (15)$$

C. Linear Programming Method I (LPM-I)

In problem (8), the constraints (8b) and (8c) are nonlinear with respect to the optimization variables $\{p_k\}$, which makes the problem complicated. We therefore consider a linearization technique to solve this problem [7].

We quantize the power values $\{p_k\}$ into M+1 discrete levels: $\{p(m), m=0,1,\ldots M\}$ with $p(m\text{-}1)<p(m)$, $m=1,2,\ldots M$, and partition the K users into M+1 groups according to their power levels. Letting $\lambda(m)$ denote the number of users assigned with power level $p(m)$. Then the total power (8a) is rewritten as

$$\Phi = \sum_m \lambda(m) p(m) \quad (16)$$

and the number of users K can be represented as

$$K = \sum_m \lambda(m). \quad (17)$$

Denote by $\gamma^{(l)}(m)$ the SNIR of the ESE outputs for the users in the group with power $p(m)$ after the l-th iteration and assume that user-k has power $p(m)$. Then (9) and (10) can be rewritten as follows, respectively.

$$I^{(l)} = \sum_m \lambda(m) p(m) f(\gamma^{(l)}(m)) + \sigma^2, \quad (18)$$

$$\gamma^{(l+1)}(m) = \frac{p(m)}{I^{(l)} - p(m) f(\gamma^{(l)}(m))}, \forall k, l = 0,1,\cdots,L-1. \quad (19)$$

Substituting (18) into (14) and introducing a decay factor δ ($0<\delta<1$) to control the convergence speed of the iterative detection, we have

$$\sum_m \lambda(m) p(m) f(\gamma^{(l+1)}(m)) + \sigma^2 \le (1-\delta) I^{(l)}, l = 0,1,\cdots L-1. \quad (20)$$

Now, we can use $\{\lambda(m)\}$ as optimization variables. In this case, both the target function (16) and the constraints (17) and (20) appear linear with respect to $\{\lambda(m)\}$. However, we should note that $\gamma^{(l+1)}(m)$ in (20) is dependent on $\{\lambda(m)\}$. Next, we present a technique by which the $\{\gamma^{(l+1)}(m)\}$ can be treated as pre-calculated parameters when linear programming is applied.

Using an approximation

$$\gamma^{(l+1)}(m) \approx \gamma^*(m) = \frac{p(m)}{I^{(l)}}, \quad (21)$$

we can rewrite (20) as

$$\sum_m \lambda(m) p(m) f(\frac{p(m)}{I}) + \sigma^2 \le (1-\delta) I, I_{\min} \le I \le I_{\max} \quad (22)$$

where I_{max} and I_{min} specify the total interference at the beginning and end of the iterative detection and the iteration index l for $I^{(l)}$ is omitted. We evaluate (22) at a set of quantized values of I: $\{I(n), n = 0,1,\ldots N\}$. By pre-calculating $\{f_{m,n}=f(p(m)/I(n))\}$ and using $\{f_{m,n}\}$ as constants, we obtain a group of linear constraints with respect to $\{\lambda(m)\}$. In summary, we have the following.

Linear programming method I (LPM-I): Find the distribution $\{\lambda(m)\}$ that minimizes

$$\Phi = \sum_m \lambda(m) p(m) \quad (23a)$$

subject to

$$\sum_m \lambda(m) = K, \quad (23b)$$

$$\lambda(m) \ge 0, m = 0,1,\cdots M, \quad (23c)$$

$$\sum_m \lambda(m) p(m) f(\frac{p(m)}{I}) + \sigma^2 \le (1-\delta) I, I_{\min} \le I \le I_{\max}. \quad (23d)$$

D. Linear Programming Method II (LPM-II)

LPM-I is suboptimal due to the approximation in (21). The related performance loss is significant when K is small. In the following, we introduce a better approximation for $\{\gamma^{(l+1)}(m)\}$.

Returning to (19), we define

$$F(x) = \frac{p(m)}{I^{(l)} - p(m) f(x)}, \quad x \in [\gamma^{(l)}(m), +\infty). \quad (24)$$

$F(x)$ is monotonically decreasing since for any $\Delta > 0$,

$$F(x+\Delta) = \frac{p(m)}{I^{(l)} - p(m) f(x+\Delta)} \le \frac{p(m)}{I^{(l)} - p(m) f(x)} = F(x), \forall x \ge 0. \quad (25)$$

Using $F(x)$, we rewrite (19) as

$$\gamma^{(l+1)}(m) = F(\gamma^{(l)}(m)) \quad (26)$$

From (11), (25) and (26), we can see that $x = F(x)$ has a unique solution within $[\gamma^{(l)}(m), +\infty)$. Denoting this unique solution by $\gamma^{**}(m)$, we have

$$\gamma^{(l)}(m) \le \gamma^{**}(m) < +\infty. \quad (27)$$

Using (27) and the monotonicity of $F(x)$, we obtain

$$F(\gamma^{(l)}(m)) \ge F(\gamma^{**}(m)) > F(+\infty) \quad (28)$$

or equivalently (since from (21) and $f(+\infty) = 0$, we have $\gamma^*(m) = F(+\infty)$)

$$\gamma^{(l+1)}(m) \geq \gamma^{**}(m) > \gamma^*(m). \quad (29)$$

This means that $\gamma^{**}(m)$ is a better approximation for $\gamma^{(l+1)}(m)$ than $\gamma^*(m)$.

Based on (14) (16), (17), (18) and using the approximation

$$\gamma^{(l+1)}(m) \approx \gamma^{**}(m), \quad (30)$$

we obtain an improved linear programming method as follows.

Linear programming method II (LPM-II): Find the distribution $\{\lambda(m)\}$ that minimizes

$$\Phi = \sum_m \lambda(m) p(m) \quad (31a)$$

subject to

$$\sum_m \lambda(m) = K, \quad (31b)$$

$$\lambda(m) \geq 0, m = 0,1,\cdots M, \quad (31c)$$

$$\sum_m \lambda(m) p(m) f(\gamma^{**}(m)) + \sigma^2 \leq (1-\delta)I, I_{\min} \leq I \leq I_{\max} \quad (31d)$$

where $\gamma^{**}(m)$ is the solution of

$$x = \frac{p(m)}{I - p(m) f(x)}. \quad (31e)$$

In both LPM-I and LPM-II, we do not require the optimized values of $\{\lambda(m)\}$ to be integers, so after optimization we need to quantize them to obtain a valid power profile. This can be done as follows: we divide $\{\lambda(m)\}$ into two sets, one is a fixed set and the other is unfixed. At the beginning, we put all of the $\{\lambda(m)\}$ into the unfixed set and leave the fixed set empty. After optimization, we choose the $\lambda(m)$ from the unfixed set that is the nearest to an integer and round it to this integer. Then we move this $\lambda(m)$ to the fixed set, repeat the optimization for the new unfixed set and continue this process until the unfixed set is empty.

E. Interior-Point Method (IPM)

The interior-point method (IPM) is an alternative approach to the power optimization problem defined above. We modify problem (8) to the following form.

Interior-point method (IPM): Find the distribution $\boldsymbol{p}=\{p_k\}$ that minimizes

$$\Phi = \sum_k p_k + \alpha^l \cdot \varphi(\boldsymbol{p}), \quad l = 0,1,\cdots. \quad (32)$$

In (32), the constraints of (8) are incorporated in the barrier function $\varphi(\boldsymbol{p})$ discussed in more detail below. The parameter α^l ($0<\alpha<1$) is used to control the impact of the barrier function. When $l\to+\infty$ ($\alpha^l = 0$), the target functions in (8) and (32) are the same. For any given l, we can find an optimized solution for (32) by simply using the standard steepest descent method [9] provided that a feasible starting point is available. By gradually increasing l, we can guide the solution of the problem (32) towards the solution of (8).

To confine the search within the feasible region, the barrier function $\varphi(\boldsymbol{p})$ can be used to establish a "lofty wall" on the border of the feasible region. Such a function is constructed as follows.

$$\varphi(\boldsymbol{p}) = \sum_k \varphi_k(\gamma_k^{(L)}) \quad (33a)$$

where $\{\gamma_k^{(L)}\}$ is obtained by the recursion (7) and

$$\varphi_k(\gamma_k^{(L)}) = \begin{cases} \log\left(\dfrac{\gamma_k^{(L)}}{\gamma_k^{(L)} - \Gamma}\right), & \gamma_k^{(L)} > \Gamma \\ +\infty, & \gamma_k^{(L)} \leq \Gamma \end{cases}. \quad (33b)$$

In the interior of the feasible region, $\gamma_k^{(L)} > \Gamma$, $\forall k$. If $\boldsymbol{p}$ is in the area far away from the border of the feasible region, $\gamma_k^{(L)} >> \Gamma$, $\forall k$, and so $\varphi(\boldsymbol{p}) \approx 0$. However, if $\boldsymbol{p}$ is close to the border, i.e., $\gamma_k^{(L)} \to \Gamma$ for some k, then $\varphi(\boldsymbol{p}) \to \infty$. Thus introducing $\varphi(\boldsymbol{p})$ in (34) ensures that the search is confined to the feasible region, so long as the starting point is inside the feasible region. Such a starting point can be obtained using the linear programming methods introduced above. Thereafter the optimized power profile from iteration l can be used as the starting point for l+1. This process continues until the difference between the results of two successive iterations is small enough.

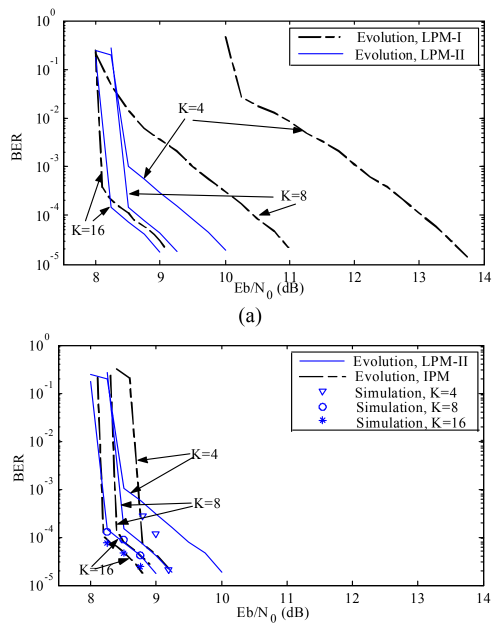
A disadvantage of IPM is its high computation cost. Also, we have no guarantee about its convergence.

F. Simulation Examples

We now compare the efficiency of LPM-I, LPM-II and IPM using numerical results. Consider a K-user IDMA system with QPSK modulation, where a rate-1/2 convolutional code with different spreading lengths (the corresponding $f(\cdot)$ functions are shown in Fig. 2) is used by all users. The system sum-rate is fixed to be 4 bits/symbol and the system requirement Γ is selected to ensure $\text{BER}_k \leq 10^{-4}$, $\forall k$.

Fig. 3(a) compares the SNIR evolution performance of the power profiles obtained by LPM-I and LPM-II, from which we can see that LPM-II significantly outperforms LPM-I for small K. The difference reduces with increasing K. Fig. 3(b) compares the SNIR evolution performance of the power profiles obtained with LPM-II and IPM. For IPM, the optimized results are sensitive to the value of α and the curves shown in Fig. 3(b) are obtained by setting α = ½ experimentally. From Fig. 3(b) we can see that LPM-II is slightly inferior to IPM. However, it should be noticed that LPM-II is much faster than IPM, especially when K is large.

We have also carried out simulations for the power profiles obtained using IPM to verify the evolution results. The corresponding relative power values are listed in Table I. The data length is 4096. The simulation performance shown in Fig 3(b) is for the user with the lowest power level, and that of the other users is always better since they have higher power levels. We can see from Fig. 3(b) that the evolution and simulation results are in good agreement. The minor difference is due to the fact that an infinitely long code length is assumed in the evolution results, while a finite code length is used in the simulations. This difference reduces as the code length increases.



(a)

(b)

Fig. 3. Performance comparison of LPM-I, LPM-II and IPM. L = 30 and the system sum-rate is fixed at 4 bits/symbol. The spreading lengths are 1, 2 and 4 for K = 4, 8 and 16, respectively.

TABLE I. RELATIVE POWER PROFILES FOR THE SIMULATIONS IN FIG. 3(b)

K	Relative power levels (dB)
4	0, 0.0021, 5.4647, 7.5530
8	0, 0.0004, 0.0021, 0.0097, 4.0177, 5.7518, 6.8848, 7.7535
16	0, 0.0002, 0.0005, 0.0011, 0.0024, 0.0050, 0.0103, 0.0209, 3.0501, 4.9935, 5.1017, 5.3528, 5.5379, 7.0597, 7.5528, 7.8468

IV. POWER OPTIMIZATION OVER FADING CHANNELS

A. Problem Formulation

Power optimization for IDMA systems over fading channels is more complicated than that over AWGN channels. In this case, the problem can be formulated as follows.

Power optimization over fading channels: Find the distribution $\{p_k\}$ that minimizes

$$\Phi = \sum_k p_k \tag{34a}$$

subject to

$$\gamma_k^{(L)} \ge \Gamma \,,\ \forall k \tag{34b}$$

where $\{\gamma_k^{(L)}\}$ are obtained through the following SNIR evolution process with initialization $\gamma_k^{(0)} = 0$, for all k.

$$\gamma_k^{(l)} = \frac{p_k\,|h_k|^2}{\sum_{i\neq k} p_i\,|h_i|^2\, f_i(\gamma_i^{(l-1)}) + \sigma^2}\,,\ \forall k, l = 1:L\,. \tag{34c}$$

B. Two-Step Method (TSM)

The following two-step method (TSM) [7] produces a suboptimal solution to the power optimization problem in (34): Given the values of the channel coefficients $\{h_k\}$, we first solve (8) and obtain an optimized power profile. Then we use it as the received power profile and obtain the corresponding transmitted power profile by assigning a lower received power level to a user with lower channel gain. This ordered-matching approach is optimal for systems with ideal codes, but not for those with practical codes, which can be seen from the numerical results later.

C. Interior-point Method (IPM)

The IPM introduced in Section III-E can also be applied to power optimization for IDMA systems over fading channels. Similarly to (33), we can modify (35) by adding a logarithmic barrier to the original target function. The initial power profile can then be obtained using the two-step method.

D. Numerical Results

We consider a K-user IDMA system with QPSK modulation over a quasi-static fading channel. A rate-1/2 convolutional code with different spreading lengths is used by all users and the corresponding $f(\cdot)$ functions are shown in Fig. 2. User-specific chip-level interleavers are used, all generated randomly and independently. In this case, a simple repetition code can be used as the spreading sequence for all users without affecting performance. Without spreading, the basic rate per user is 1 bit/symbol with rate-1/2 convolutional coding and QPSK modulation. If rate higher than 1 bit/symbol is required for a user, we can assign two or more codes to each user. This follows the superposition-coding scheme [11].

In our simulation, the channel coefficients are modeled as $h_k = A(\rho_k^{-\nu} 10^{\xi_k/10})^{1/2} \chi_k$, where A is a constant, ρ_k ($0 \le \rho_k \le 1$) the normalized distance between user-k and the receiver, ν the path-loss exponent, ξ_k a zero-mean Gaussian random variable with variance σ_s^2 that characterizes shadow fading, and χ_k the complex Gaussian variable with unit power. For simplicity, we set A = 1, so the amplitude of h_k is just a relative value. We set ν = 4, σ_s= 8 and the values of $\{\rho_k\}$ are generated with the assumption that all users are uniformly located in a normalized circular cell area with unit radius.

With regard to deep fading, we allow transmission outage for the users with channel gains below a given threshold G_0. That is to say, if $|h_k|^2 < G_0$, an outage is declared for user-k and p_k is set to zero. Then power allocation is applied to the remaining active users. We define the outage probability $P_{\text{out}}(G_0) = \Pr(|h_k|^2 < G_0)$, $\forall k$. The Γ value for the active users is set to ensure $\text{BER}_k \le 10^{-4}$.

Fig. 4 shows the required average transmitted power versus the number of users K with TSM and IPM for the IDMA system shown in Fig. 1. The system sum-rates are fixed at 4 and 8 bits/symbol in Fig. 4(a) and (b), respectively. The outage probability $P_{\text{out}}(G_0)$ is set to be 10^{-2}. When rate higher than 1 bit/symbol is required for a user, we assign two or more codes to each user. For example, in the case of K = 2 in Fig. 4(a), two codes are assigned to each user to achieve a sum-rate of 4 bits/symbol.

For comparison, we have included results for a practical TDMA scheme using trellis-coded modulation (TCM) with rate of 4 bits/symbol in Fig. 4(a). The rate-4 bits/symbol TCM-32QAM code is cited from [10]. The performance of a practical TDMA system at the rate of 8 bits/symbol is not included here since we cannot identify a suitable practical coding scheme from existing literatures. We have also included the TDMA capacities in Fig. 4, which are generated based on the assumption that all users transmit information at a fixed rate of 4 bits/symbol over mutually orthogonal time slots with equal length. Also shown are the capacity curves of fading MACs with rate constraints. These curves are the statistical mean of the sum-power for randomly generated channel realizations. For each channel realization, the sum-power is computed based on ideal coding with optimal power allocation at the transmitters and stripping decoding at the receiver [2][12].

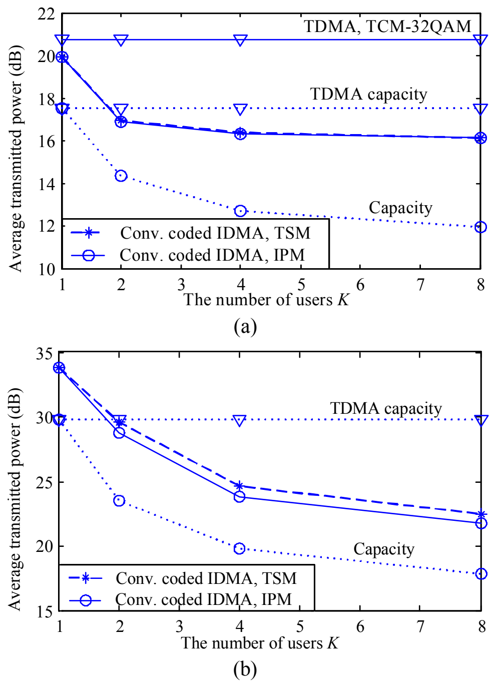


Fig. 4. The required average transmitted power versus the number of users K for an IDMA system over fading channels. $P_{out}(G_0) = 10^{-2}$, $A = 1$, $\nu = 4$, $\sigma_s = 8$ and the system sum-rates in (a) and (b) are 4 and 8 bits/symbol respectively.

From Fig. 4 we can make several interesting observations:

- At a sum-rate of 4 bits/symbol, both IPM and TSM achieve almost the same system performance for IDMA systems with practical coding. At a sum-rate of 8 bits/symbol, IPM outperforms TSM, but the difference between them is not significant. However, TSM is much faster.
- Unlike TDMA systems, the average transmitted power of convolutionally coded IDMA systems decreases with increasing number of users K. This is the so-called multi-user (MU) diversity gain discussed in [13, p. 253]. Also we can see from Fig. 4 that most MU gain is achieved with a small number of users K, e.g. two or four. Further increase in K yields only marginal improvements. When the system sum-rate increases, the achievable MU gain also increases. Potentially, very significant gains can be achieved using IDMA with iterative MUD at a high sum-rate.
- In Fig. 4(a), the average transmitted power of convolutionally coded IDMA systems for $K = 8$ is very close to that for $K = 4$. This is because the spreading operation for $K = 8$ does not produce any coding gain. The loss of coding gain due to spreading can also be seen in Fig. 2. Therefore, to maximize the MU gain by increasing K, we should employ low-rate codes with good coding gain instead of conventional spreading. IDMA [6] provides a good framework for this purpose.

V. Conclusions

In this paper, we have proposed a power optimization technique (LPM-II) for practically coded IDMA systems over AWGN channels. This technique is based on linear programming and is compared with the linear programming method (LPM-I) in [7] and an interior-point method (IPM) [8]. We have also compared the performance of TSM [7] and IPM in the context of power optimization over fading channels. The key findings are listed below.

- LPM-II outperforms LPM-I significantly.
- Compared with IPM, LPM-II has slightly lower performance but much lower complexity.
- For power optimization over fading channels, TSM can achieve similar performance to IPM at reduced computation cost.
- Based on the power allocation techniques discussed in this paper, significant performance improvements can be achieved by IDMA over TDMA in fading channels.

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