Simple Iterative Chip-by-Chip Multiuser Detection for CDMA Systems†

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Abstract-**This paper examines several low-cost iterative chip-by-chip multiuser detection algorithms for chip-interleaved code-division multiple-access (CDMA) systems in multipath and multiple antenna environments. The complexities (per user) of these algorithms are very low and increase either linearly or quadratically with path number. It is shown that a system employing a rate-1/2 16-state convolutional code and a length-8 spreading sequence can support more than 100 users with two receive antennas.**

I. Introduction

The performance of code-division multiple-access (CDMA) systems is mainly limited by multiple access interference (MAI). Turbo-type iterative multiuser detection (MUD) is a promising technique to mitigate MAI so as to enhance performance. Various MUD techniques [1-7] have been studied and significant progress has been made. In particular, it has been shown that MAI and intersymbol interference (ISI) can be jointly treated with soft cancellation or minimum mean-square error (MMSE) methods [3, 8-10].

The potential performance improvement by assigning different interleavers to different users has been demonstrated in [10-15]. Traditionally, interleaving in a CDMA system is performed on symbols, i.e., the signals before spreading. Recently, chip-level interleaved CDMA has been studied [10, 14, 15]. Reference [10] considered uncoded chip interleaved CDMA systems in multipath channels. References [14, 15] studied a coded scheme employing chip-level interleaving as the sole means for user separation (referred to as interleave-division multiple-access (IDMA)).

In this paper we examine several options for mitigating MAI and ISI in chip-interleaved CDMA systems. We will consider a simple soft rake (SR) algorithm and a more sophisticated joint Gaussian (JG) algorithm. Their complexities are $O(L)$ and $O(L^2)$ respectively, where L is the number of path taps. This is compared with the maximal ratio combining (MRC) algorithm presented in [10] with a complexity of $O(KL)$, where K is the number of users. The SR and MRC algorithms are suited to situations where system loading is relatively light, whereas the JG algorithm may be advantageous in heavily loaded situations. The complexities (per user) of the SR and JG algorithms are independent of the number of users. These algorithms are directly applicable to multiple antenna systems. The JG algorithm can also be applied to equalization for systems without spreading (e.g., TDMA systems).

II. Channel Models

A. The Basic Multipath Channel Model

Consider a multiple access system with K simultaneous users over quasi-static multipath channels with L tap-coefficients. After sampling at the chip rate, the signal at the receiver can be represented by

$$\boldsymbol{r} = \sum_{k=1}^{K} \boldsymbol{y}_k + \boldsymbol{n} \tag{1a}$$

where

$$\begin{bmatrix} y_k(1) \\ y_k(2) \\ \vdots \\ \\ y_k(J+L-1) \end{bmatrix} = \begin{bmatrix} h_{k,0} & & & \boldsymbol{0} \\ h_{k,1} & h_{k,0} & & \\ \vdots & h_{k,1} & \ddots & \\ h_{k,L-1} & & \ddots & h_{k,0} \\ & h_{k,L-1} & \ddots & h_{k,1} \\ & & \ddots & \vdots \\ \boldsymbol{0} & & & h_{k,L-1} \end{bmatrix} \begin{bmatrix} x_k(1) \\ x_k(2) \\ \vdots \\ x_k(J) \end{bmatrix} \tag{1b}$$

or simply

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k . \tag{1c}$$

In (1), $\boldsymbol{r}$ is the received sequence, $\boldsymbol{y}_k$ the contribution in $\boldsymbol{r}$ from user-k, $\boldsymbol{n}$ a vector of additive white Gaussian noise (AWGN) with zero-mean and variance $\sigma^2 = N_0/2$ per dimension, $\{h_{k,0}, \cdots, h_{k,L-1}\}$ the fading coefficients related to user-k, and $\boldsymbol{x}_k$ a length-J sequence transmitted by user-k.

Denote the jth column of $\boldsymbol{H}_k$ by

$$\boldsymbol{h}_k(j) \equiv [\underbrace{0, \cdots, 0}_{(j-1)\text{ zeros}}, h_{k,0}, h_{k,1}, \cdots, h_{k,L-1}, 0, \cdots, 0]^T . \tag{2}$$

Then (1a) can be rewritten as

$$\boldsymbol{r} = \sum_{k=1}^{K} \sum_{j=1}^{J} \boldsymbol{h}_k(j) x_k(j) + \boldsymbol{n} . \tag{3}$$

B. Complex, Multipath and Multiple Antenna Systems

The channel model in (1) can actually be applied to more complicated situations. For example, let a, b, and c be complex scalars. Multiplication $a = b \cdot c$ can be expressed in matrix form

$$\begin{bmatrix} \mathrm{Re}(a) \\ \mathrm{Im}(a) \end{bmatrix} = \begin{bmatrix} \mathrm{Re}(b) & -\mathrm{Im}(b) \\ \mathrm{Im}(b) & \mathrm{Re}(b) \end{bmatrix} \begin{bmatrix} \mathrm{Re}(c) \\ \mathrm{Im}(c) \end{bmatrix} \tag{4}$$

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where Re(·) and Im(·) denote the real and imaginary part respectively. We call a system with N transmit antennas and M receive antennas an $N\times M$ system. Denote

$$\boldsymbol{x}_k^n(j)\equiv\begin{bmatrix}\mathrm{Re}(x_k^n(j))\\ \mathrm{Im}(x_k^n(j))\end{bmatrix}\qquad \boldsymbol{y}_k^m(j)\equiv\begin{bmatrix}\mathrm{Re}(y_k^m(j))\\ \mathrm{Im}(y_k^m(j))\end{bmatrix} \tag{5a}$$

$$\boldsymbol{h}_{k,l}^{m\leftarrow n}\equiv\begin{bmatrix}\mathrm{Re}(h_{k,l}^{m\leftarrow n}) & -\mathrm{Im}(h_{k,l}^{m\leftarrow n})\\ \mathrm{Im}(h_{k,l}^{m\leftarrow n}) & \mathrm{Re}(h_{k,l}^{m\leftarrow n})\end{bmatrix} \tag{5b}$$

where $x_k^n(j)$ is the complex signal of user-k transmitted on the nth antenna at time j, $y_k^m(j)$ the received signal by the mth antenna from user-k at time j and $h_{k,l}^{m\leftarrow n}$ is the lth tap-coefficient of the channel from the nth transmit antenna of user-k to the mth receive antenna. Using the sub-blocks defined in (5), a complex 2×2 multipath system with 2 tap-coefficients (ignoring additive noise) can be expressed in real matrix form as below. The principle can be extended to a general $N\times M$, L-tap system.

$$\begin{bmatrix}\boldsymbol{y}_k^1(1)\\ \boldsymbol{y}_k^2(1)\\ \boldsymbol{y}_k^1(2)\\ \boldsymbol{y}_k^2(2)\\ \\ \vdots \\ \\ \end{bmatrix}=\begin{bmatrix}\boldsymbol{h}_{k,0}^{1\leftarrow 1} & \boldsymbol{h}_{k,0}^{1\leftarrow 2} & & & & & \\ \boldsymbol{h}_{k,0}^{2\leftarrow 1} & \boldsymbol{h}_{k,0}^{2\leftarrow 2} & & & \mathbf{0} & & \\ \boldsymbol{h}_{k,1}^{1\leftarrow 1} & \boldsymbol{h}_{k,1}^{1\leftarrow 2} & \boldsymbol{h}_{k,0}^{1\leftarrow 1} & \boldsymbol{h}_{k,0}^{1\leftarrow 2} & & & \\ \boldsymbol{h}_{k,1}^{2\leftarrow 1} & \boldsymbol{h}_{k,1}^{2\leftarrow 2} & \boldsymbol{h}_{k,0}^{2\leftarrow 1} & \boldsymbol{h}_{k,0}^{2\leftarrow 2} & & & \\ & & \boldsymbol{h}_{k,1}^{1\leftarrow 1} & \boldsymbol{h}_{k,1}^{1\leftarrow 2} & \boldsymbol{h}_{k,0}^{1\leftarrow 1} & \boldsymbol{h}_{k,0}^{1\leftarrow 2} & \\ & & \boldsymbol{h}_{k,1}^{2\leftarrow 1} & \boldsymbol{h}_{k,1}^{2\leftarrow 2} & \boldsymbol{h}_{k,0}^{2\leftarrow 1} & \boldsymbol{h}_{k,0}^{2\leftarrow 2} & \\ & & & & \ddots & & \ddots \\ & \mathbf{0} & & & & \ddots & & \ddots\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_k^1(1)\\ \boldsymbol{x}_k^2(1)\\ \boldsymbol{x}_k^1(2)\\ \boldsymbol{x}_k^2(2)\\ \vdots \\ \\ \end{bmatrix} \tag{6}$$

C. Equivalence between (1b) and (6)

The similarity between (1b) and (6) is clear. Therefore in the following, we will concentrate on the channel model in (1b) with real signaling and real channel coefficients. The results can be directly extended to the more complicated systems modeled by (6).

III. Overall Transmitter and Receiver Principle

A. Transmitter Structure

We consider a coded system with K users in multipath channels with AWGN. The upper part of Fig. 1 shows the transmitter structure of the proposed scheme. The input data sequence $\boldsymbol{d}_k$ of user-k is encoded based on a low-rate code C into $\boldsymbol{c}_k$. The spreading operation, if presents, is incorporated in C. A chip-level interleaver π_k is then used, producing $\boldsymbol{x}_k$. Following the convention, we call the elements in $\boldsymbol{c}_k$ and $\boldsymbol{x}_k$ "chips". The signals $\{x_k(j)\}$ are transmitted in a multipath channel defined in (1).

The key principle here is that the interleavers $\{\pi_k\}$ should be different for different users. We assume that the interleavers are generated independently and randomly. These interleavers disperse chips related to a common bit. Therefore the adjacent chips are approximately uncorrelated, which facilitates the simple chip-by-chip detection schemes discussed below.

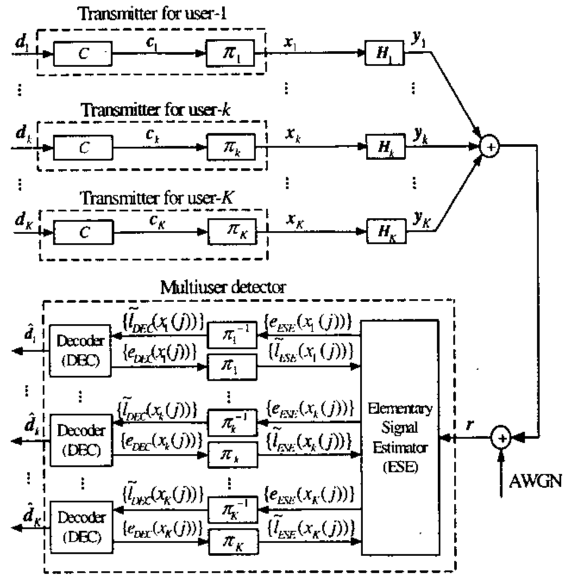

Fig. 1. Transmitter and receiver structures of the scheme under consideration.

B. Decoder Structure

The receiver operation is based on two constraints: (i) the multipath multiple access channel constraint and (ii) the coding constraint of C. Finding an optimal solution is usually prohibitively complicated. An alternative sub-optimal receiver structure is illustrated by the lower part of Fig. 1. Overall, it follows the well-known turbo multiuser detection principles [3-9]. However, the chip-level interlevavers used in the transmitters allows a low-cost chip-by-chip detection strategy [10, 14, 15] as detailed below.

The receiver in Fig. 1 consists of an elementary signal estimator (ESE) and K single-user *a posteriori* probability (APP) decoders (DECs). The two constraints are considered separately in the ESE and in the DECs, and then the results are combined using a turbo-type iterative process [3-9, 16]. This greatly reduces the complexity involved.

For simplicity, we will only consider BPSK signaling, i.e., $x_k(j)\in\{+1,-1\}$, $\forall\, k, j$. Denote $\{\tilde{l}_{ESE}(x_k(j))\}$ and $\{\tilde{l}_{DEC}(x_k(j))\}$ two sets of *a priori* log-likelihood ratios (LLRs). At the start, $\{\tilde{l}_{ESE}(x_k(j))\}$ are initialized to zeros.

The ESE performs a coarse chip-by-chip estimation. It uses $\{r_j\}$ and $\{\tilde{l}_{ESE}(x_k(j))\}$ as its inputs and computes the following extrinsic *a posteriori* LLRs about $\{x_k(j),\forall k, j\}$ based on the multiple access channel constraint $\boldsymbol{H}$ (without considering the coding constraint C) [15].

$$e_{ESE}(x_k(j))=\log\left(\frac{p(\boldsymbol{r}\mid x_k(j)=+1,\boldsymbol{H})}{p(\boldsymbol{r}\mid x_k(j)=-1,\boldsymbol{H})}\right) \tag{7}$$

The DECs compute the extrinsic *a posteriori* LLRs about $\{x_k(j),\forall k, j\}$ based on C [15]

$$e_{DEC}(x_k(j)) = \log\left(\frac{Pr(x_k(j)=+1 \mid C, (\tilde{L}_{DEC})_k \setminus \tilde{l}_{DEC}(x_k(j)))}{Pr(x_k(j)=-1 \mid C, (\tilde{L}_{DEC})_k \setminus \tilde{l}_{DEC}(x_k(j)))}\right) \quad (8)$$

where $(\tilde{L}_{DEC})_k \equiv \{\tilde{l}_{DEC}(x_k(j)), \forall j\}$ and $(\tilde{L}_{DEC})_k \setminus \tilde{l}_{DEC}(x_k(j))$ is obtained by setting $\tilde{l}_{DEC}(x_k(j)) = 0$ in $(\tilde{L}_{DEC})_k$. During the turbo-type iterative process, the extrinsic information generated by the ESE/DECs with appropriate interleaving is then used to update the *a priori* information, see Fig. 1.

The APP decoding in the DEC is a standard function [16], so we will not discuss it in detail. In the following, we will focus on the ESE.

IV. Chip-by-Chip Estimation in Synchronous Channels

We first consider a synchronous single-path channel. In this case, there is only one channel tap-coefficient for user-k, i.e., $h_{k,0}$, that is simply denoted by h_k. Assuming quasi-static fading, the received signal (refer to (3)) at time instant j is simplified to

$$r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j). \quad (9)$$

Based on (9) and applying a Gaussian approximation [15], (7) can be evaluated as

$$e_{ESE}(x_k(j)) = 2h_k \cdot \frac{r(j) - \mathrm{E}(r(j)) + h_k \mathrm{E}(x_k(j))}{\mathrm{Var}(r(j)) - |h_k|^2 \mathrm{Var}(x_k(j))} \quad (10)$$

where $\mathrm{E}(\cdot)$ and $\mathrm{Var}(\cdot)$ denote the mean and variance respectively.

The iterative detection can be carried out in either a parallel or a serial scheme [15]. For clarity, we list the serial detection algorithm below.

The Basic Gaussian (BG) algorithm with serial scheduling:

Initially, set $k = 1$, $\tilde{l}_{ESE}(x_k(j)) = 0$, $\forall k, j$, $\mathrm{E}(r(j)) = 0$ and $\mathrm{Var}(r(j)) = \sigma^2 + \sum_{k=1}^{K} |h_k|^2$, $\forall j$.

(i) Main operations:

$$e_{ESE}(x_k(j)) \Leftarrow 2h_k \cdot \frac{r(j) - \mathrm{E}(r(j)) + h_k \mathrm{E}(x_k(j))}{\mathrm{Var}(r(j)) - |h_k|^2 \mathrm{Var}(x_k(j))}, \quad \forall j. \quad (11)$$

Update $\{\tilde{l}_{DEC}(x_k(j)), \forall j\} \Leftarrow \{e_{ESE}(x_k(j)), \forall j\}$. Perform APP decoding for user-k to produce $\{e_{DEC}(x_k(j)), \forall j\}$. Then update:

$$\{\tilde{l}_{ESE}(x_k(j)), \forall j\} \Leftarrow \{e_{DEC}(x_k(j)), \forall j\}.$$

$$\mathrm{E}(x_k(j)) \Leftarrow \tanh(\tilde{l}_{ESE}(x_k(j))/2), \quad \forall j. \quad (12a)$$

$$\mathrm{Var}(x_k(j)) \Leftarrow 1 - (\mathrm{E}(x_k(j)))^2, \quad \forall j. \quad (12b)$$

$$\mathrm{E}(r(j)) \Leftarrow \mathrm{E}(r(j)) + h_k \Delta\mathrm{E}(x_k(j)), \quad \forall j. \quad (13a)$$

$$\mathrm{Var}(r(j)) \Leftarrow \mathrm{Var}(r(j)) + |h_k(j)|^2 \Delta\mathrm{Var}(x_k(j)), \quad \forall j. \quad (13b)$$

(ii) If $k = K$, reset $k = 1$. Otherwise increase k by 1. Go back to (i).

In (13), $\Delta\mathrm{E}(x_k(j))$ and $\Delta\mathrm{Var}(x_k(j))$ are defined as

$$\Delta\mathrm{E}(x_k(j)) = \mathrm{E}(x_k(j))|_{current\_iteration} - \mathrm{E}(x_k(j))|_{last\_iteration}$$

$$\Delta\mathrm{Var}(x_k(j)) = \mathrm{Var}(x_k(j))|_{current\_iteration} - \mathrm{Var}(x_k(j))|_{last\_iteration}$$

The normalized computational cost in (11)-(13) (excluding the APP decoding of C) is only about 8 additions, 7 multiplications and a tanh function per chip per user per iteration. The complexity per information bit per user increases linearly with spreading ratio but is independent of user number K.

V. Chip-by-Chip Estimation in Multipath Channels

We now consider MUD in an asynchronous multipath channel. A transmitted chip $x_k(j)$ is now observed on L successive samples $\{r(j), r(j+1), \ldots, r(j+L-1)\}$ in the received signal $\boldsymbol{r}$, see (1). The iterative principle in Fig. 1 and section III is still applicable, except the calculation of $e_{ESE}(x_k(j))$.

The MRC algorithm below performs a combining operation directly on channel outputs $\{r(j)\}$. The SR algorithm performs combining on soft chip estimates. Both do not consider the correlation among interfering signals caused by multipath reflection. The more sophisticated JG algorithm takes the correlation due to multipath reflection into account.

A. The Maximal Ratio Combining (MRC) Approach

In this approach [10], $\boldsymbol{r}$ is passed through a MRC filter matched to the L tap-coefficients for a particular user. A MMSE based chip detection is then applied to generate $\{e_{ESE}(x_k(j))\}$. Since K different L-tap matched filters are involved, the complexity is $O(KL)$ [10] per chip per user.

B. The Soft Rake (SR) Approach

The SR algorithm performs a combining operation

$$e_{ESE}(x_k(j)) = \sum_{l=0}^{L-1} e_{ESE}(x_k(j))_l \quad (14a)$$

where

$$e_{ESE}(x_k(j))_l = 2h_{k,l} \cdot \frac{r(j+l) - \mathrm{E}(r(j+l)) + h_{k,l}\mathrm{E}(x_k(j))}{\mathrm{Var}(r(j+l)) - |h_{k,l}|^2 \mathrm{Var}(x_k(j))} \quad (14b)$$

is the soft estimate of $x_k(j)$ based on $r(j+l)$ using (10). The complexity (per chip per user) of the SR algorithm is $O(L)$.

C. The Joint Gaussian (JG) Approach

In a multipath channel, the adjacent interfering signals are correlated. The joint Gaussian (JG) algorithm described below takes this into account. Its principle is similar to the MMSE multiuser detector developed in [3]. However, due to the chip-level interleavers used, we can ignore the correlation among adjacent chips caused by spreading and focus on the multipath effect only. This greatly reduces the dimension (and so the complexity) of the problem.

For user-k, we rewrite (3) as

$$\boldsymbol{r} = \boldsymbol{h}_k(j) x_k(j) + \boldsymbol{\zeta}_k(j) \quad (15a)$$

where

$$\boldsymbol{\zeta}_k(j) \equiv \sum_{(k',j') \neq (k,j)} \boldsymbol{h}_{k'}(j') x_{k'}(j') + \boldsymbol{n}. \quad (15b)$$

We make the hypothesis that $x_k(j)$ is either +1 or −1. We assume that $\boldsymbol{r}$ and so $\boldsymbol{\zeta}_k(j)$ in (15) can be approximated by vectors of joint multi-dimensional Gaussian random variables with mean vectors

$$\mathrm{E}(\boldsymbol{r}) = \sum_{k,j} \boldsymbol{h}_k(j)\mathrm{E}(x_k(j)) \tag{16a}$$

$$\mathrm{E}(\boldsymbol{\zeta}_k(j)) = \mathrm{E}(\boldsymbol{r}) - \boldsymbol{h}_k(j)\mathrm{E}(x_k(j)). \tag{16b}$$

Assuming that the correlation among $\{r(j)\}$ is caused only by multipath reflection, we calculate covariance matrices as

$$\boldsymbol{R} \equiv \mathrm{Cov}(\boldsymbol{r}) = \mathrm{E}(\boldsymbol{r}\boldsymbol{r}^T) - \mathrm{E}(\boldsymbol{r})\mathrm{E}(\boldsymbol{r}^T) = \sum_{k,j} \boldsymbol{h}_k(j)\boldsymbol{h}_k(j)^T \mathrm{Var}(x_k(j)) + \sigma^2 \boldsymbol{I} \tag{17a}$$

$$\mathrm{Cov}(\boldsymbol{\zeta}_k(j)) = \mathrm{E}(\boldsymbol{\zeta}_k(j)\boldsymbol{\zeta}_k(j)^T) - \mathrm{E}(\boldsymbol{\zeta}_k(j))\mathrm{E}(\boldsymbol{\zeta}_k(j)^T) = \boldsymbol{R} - \boldsymbol{h}_k(j)\boldsymbol{h}_k(j)^T \mathrm{Var}(x_k(j)). \tag{17b}$$

where $\boldsymbol{I}$ is an identity matrix with an appropriate size. Applying the matrix inversion lemma to (17b) yields

$$(\mathrm{Cov}(\boldsymbol{\zeta}_k(j)))^{-1} = \boldsymbol{R}^{-1} + \frac{\mathrm{Var}(x_k(j))\boldsymbol{R}^{-1}\boldsymbol{h}_k(j)\boldsymbol{h}_k(j)^T\boldsymbol{R}^{-1}}{1-\mathrm{Var}(x_k(j))\boldsymbol{h}_k(j)^T\boldsymbol{R}^{-1}\boldsymbol{h}_k(j)}. \tag{18}$$

Based on (15)-(18), we evaluate the LLR defined in (7) as,

$$\begin{aligned} e_{ESE}(x_k(j)) &= \log\left(\frac{p(\boldsymbol{r} \mid x_k(j)=+1, \boldsymbol{H})}{p(\boldsymbol{r} \mid x_k(j)=-1, \boldsymbol{H})}\right) \\ &= 2\boldsymbol{h}_k(j)^T(\mathrm{Cov}(\boldsymbol{\zeta}_k(j)))^{-1}(\boldsymbol{r}-\mathrm{E}(\boldsymbol{\zeta}_k(j))) \\ &= 2\cdot\frac{\boldsymbol{h}_k(j)^T\boldsymbol{R}^{-1}(\boldsymbol{r}-\mathrm{E}(\boldsymbol{r}))+\mathrm{E}(x_k(j))\boldsymbol{h}_k(j)^T\boldsymbol{R}^{-1}\boldsymbol{h}_k(j)}{1-\mathrm{Var}(x_k(j))\boldsymbol{h}_k(j)^T\boldsymbol{R}^{-1}\boldsymbol{h}_k(j)} \\ &= 2\cdot\frac{\boldsymbol{h}_k(j)^T\boldsymbol{w}+\mathrm{E}(x_k(j))\boldsymbol{v}_{k,j}^T\boldsymbol{v}_{k,j}}{1-\mathrm{Var}(x_k(j))\boldsymbol{v}_{k,j}^T\boldsymbol{v}_{k,j}} \end{aligned} \tag{19}$$

where $\boldsymbol{v}_{k,j} = \boldsymbol{F}^{-1}\boldsymbol{h}_k(j)$, $\boldsymbol{w} = \boldsymbol{R}^{-1}(\boldsymbol{r}-\mathrm{E}(\boldsymbol{r}))$ and $\boldsymbol{F}$ is a lower-triangle matrix obtained from the Cholesky factorization $\boldsymbol{R} = \boldsymbol{F}\boldsymbol{F}^T$. $\boldsymbol{R}$ is a band matrix with bandwidth $2L$-1.

Consider the evaluation of (19). We write $\boldsymbol{v}_{k,j} = [v_{k,j}(1), \ldots, v_{k,j}(J+L-1)]^T$. Since the first j-1 entries in $\boldsymbol{h}_k(j)$ are all zero and $\boldsymbol{F}$ is a lower triangle band matrix, the first j-1 entries of $\boldsymbol{v}_{k,j}$ are always zero. Thus the inner product in (19) can be written as

$$\boldsymbol{v}_{k,j}^T\boldsymbol{v}_{k,j} = \sum_{i=j}^{J+L-1} v_{k,j}^2(i). \tag{20}$$

We can use the following partial sum to approximate (20).

$$\boldsymbol{v}_{k,j}^T\boldsymbol{v}_{k,j} \approx \sum_{i=j}^{j+W-1} v_{k,j}^2(i) \tag{21}$$

where W is a properly selected positive integer. Since $\boldsymbol{R}$ and $\boldsymbol{F}$ are both band matrices, the cost involved in (21) is $O(W\times L)$. Experimentally, we observed that the performance loss is marginal when setting $W \geq 2L$ so that the total cost becomes $O(L^2)$ per chip per user per iteration. This approximation is justified as follows. Let $\hat{\boldsymbol{r}}$ be a truncated vector consisting of the first $(j+W)$th entries of $\boldsymbol{r}$. Suppose that we generate $\hat{\boldsymbol{v}}_{k,j}$ and $\hat{e}_{ESE}(x_k(j))$ based on $\hat{\boldsymbol{r}}$. Then $\hat{e}_{ESE}(x_k(j))$ is an estimate using a reduced set of statistics and it should be a good approximation of $e_{ESE}(x_k(j))$ when W is large. It can be verified that $\hat{\boldsymbol{v}}_{k,j}$ actually consists of the first $(j+W)$ entries of $\boldsymbol{v}_{k,j}$, leading to (21).

D. A Complexity Comparison

SR always has the lowest complexity among the three algorithms discussed above. JG has lower complexity than MRC when K is very large. However, MRC can be more cost-effective when L is very large. In particular, if L is large and the number of non-zero tap-coefficients for each user is very small, then MRC is a good choice.

VI. Numerical Results

In Figs. 2-4, we consider a chip-interleaved CDMA system in which each user employs a rate-1/2 nonsystematic convolutional code with generator polynomials $(23, 35)_8$. The information block size is 128 and the coded frame length is 256+8, including 8 extra bits for termination. The coded bits are further spread with a spreading factor of S = 8 (thus the coding rate of each user is $R_C \approx 1/2\times1/8=1/16$). Due to the random interleavers used afterwards, the sequential order of +1 and −1 in the spreading sequences has no impact on system performance. Thus we let all the users employ the same spreading sequence [+1, −1, +1, −1, +1, −1, +1, −1] that contains a balanced number of +1 and −1. After spreading, the signal for each user is interleaved by two independent chip interleavers to produce two chip streams for the in-phase and quadrature parts. All the interleavers (length of 2112 = (256×2+8)×8) are generated randomly and independently. The total system throughput is $K\times R_C$ bits/chip that is a measurement of the overall bandwidth efficiency. One transmit antenna (N = 1) for each user is always assumed. Since interleaving is the only mechanism in this scheme for user separation, we refer to it as interleave-division multiple-access (IDMA) [14,15].

Fig. 2 shows the BER performance using the SR algorithm in quasi-static Rayleigh fading multipath channels with different numbers of tap-coefficients. It is seen that performance improves uniformly with increasing path number due to the improved diversity.

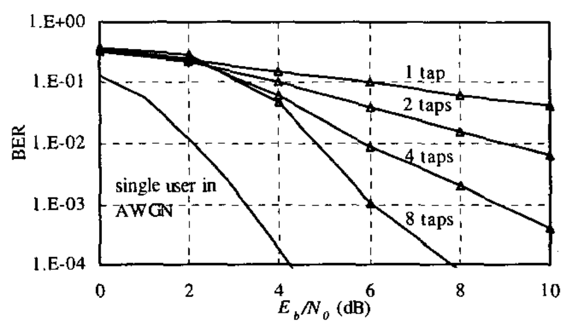

Fig. 2. Performance in quasi-static Rayleigh fading multipath channels. Iteration number = 5. K = 32. One receive antenna. S = 8.

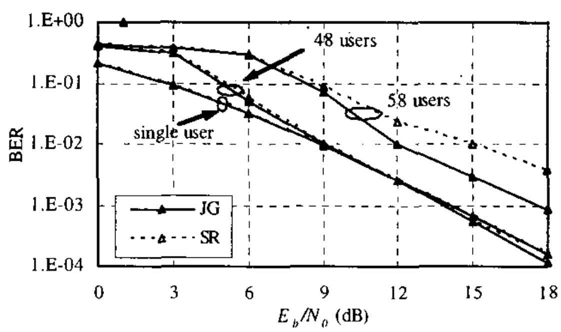

Fig. 3. Performance in quasi-static Rayleigh fading multipath channels with L = 2. Iteration number = 8. One receive antenna. S = 8.

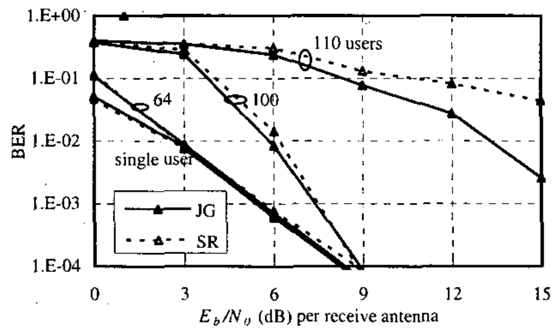

Fig. 4. Performance in quasi-static Rayleigh fading multipath channels with L = 2. Iteration number = 8. Two receive antennas. S = 8.

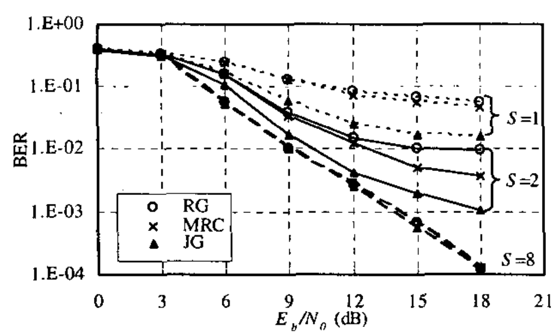

Fig. 5. Performance comparison of MRC, SR and JG algorithms. One receive antenna. Spreading length S = 1, 2 and 8. Throughput is fixed at 3 bits/chip, i.e., $K = 6S$. Iteration number = 8.

Figs. 3 and 4 compare the performance of the SR and JG algorithms in multipath channels with L = 2. One receive antenna is assumed in Fig. 3 and two in Fig. 4. SR performs well for small K values but the advantage of JG becomes noticeable for large K values. The system throughputs are 3 bits/chip for 48 users (Fig. 3) and more than 6 bits/chip for 100 users (Fig. 4). Such throughputs are rather surprising as the spreading length is only 8.

Fig. 5 compares the performance of MRC, SR and JG algorithms in multipath channels (L = 2) with a fixed throughput $K \times R_C$ = 3 bits/chip and different S (spreading length) and K (user number) values. Other parmeters are the same as those used in Figs. 2-4. From Fig. 5, JG always outperforms the other two alternatives. For a small S, MRC and SR have similar performance. For a large S, the performance of MRC and SR becomes closer to JG.

VII. Conclusions

We have examined low-cost MUD receiver algorithms in multipath multiple antenna channels. For very heavily loaded situations, the JG algorithm demonstrates better performance. The SR algorithm appears a good compromise between cost and performance, since its complexity is only $O(L)$ per user.

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