A UNIFIED APPROACH TO THE DESIGN OF CANONICAL INTEGRATED LADDER FILTERS

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ABSTRACT

A method is introduced to design stable, active integrated ladder filters with canonic numbers of opamps from a class of hitherto problematic transfer functions including bandstop and even order elliptic functions. A variety of familiar parasitic-insensitive ladder structures can be produced by matrix methods retaining the low sensitivity properties expected of passive simulations. The method is applicable to active-RC, switched-capacitor and digital technologies.

INTRODUCTION

The advantages of adopting low sensitivity ladder simulation in integrated circuit realisation have always been compromised by their complicated design procedures and associated implementation cost [1]. Various design techniques must be used in ladder design to meet different specifications. This difficulty is due to two problems; first the prototype ladder itself cannot be synthesised from certain types of transfer functions and second the standard simulation methods cannot be applied efficiently to certain forms of prototype.

A common example occurs in the realisation of even order elliptic functions. Passive ladder networks must have open or short circuit characteristics (implying full or zero transmission) at zero or infinite frequency respectively [2]. Therefore, lowpass or bandpass functions with finite (non-zero) stopband transmission at these extreme frequencies cannot be synthesised as passive ladders. 'Pure' even order elliptic functions and their frequency-transformed versions belong to this category. To obtain a realiseable function, a finite transmission zero must be shifted to infinite frequency [3]. This has the dual penalty of degraded filter performance and non-uniform passive ladder structure between odd and even order design, reflected also in the simulation by integrated circuits. For this reason, such transfer functions are practically undesirable for ladder simulation, since they are so close in cost to their related higher odd order function. Highpass or bandstop functions cause a special problem associated with their transmission at infinite frequency. Although a ladder prototype can be synthesized without difficulty, its simulation by a bilinear switched-capacitor circuit would be unstable [4] necessitating more complicated design approaches [5].

In this paper, a general method is introduced to design canonical active ladder circuits realising transfer functions of the above-mentioned classes. There is a uniform progression of filter structure with order and no extra cost of realisation. The simulation methods are *exact* and so the realised transfer function is identical to the designed one. The techniques can be applied to all present ladder simulation methods; LUD, leapfrog and coupled-biquad structures [6-8], in both active-RC and switched-capacitor (SC) implementations. The principles apply equally for LUD digital filters [9].

MODIFIED TRANSFER FUNCTION

Let H(s) be a transfer function with all its zeros on the imaginary axis or at infinity. If the order of H(s) is m, then it can be realised by a canonical ladder prototype, with m/2 nodes (for m even) or (m+1)/2 nodes (for m odd), provided that certain constraints on the parity of the numerator are satisfied [10]. By matrix methods, a circuit with m operational amplifiers can then be designed to simulate the nodal voltages of the prototype. In the cases when the constraints are not met so that the numerator N(s) has the wrong parity, or where a prototype cannot be synthesized or simulated, a simple manipulation of the transfer function can be made to solve the problem. Consider three possibilities,

i) N(s) is a constant
ii) N(s) contains a single root at $w_i=0$
iii)N(s) contains a pair of imaginary roots at $\pm j\omega_i$

To change the parity

for i) let $H'(s) = H(s)s$
for ii) let $H'(s) = H(s)s$ or $H'(s) = H(s)/s$
for iii) let $H'(s) = H(s)1/(s+\omega_i^2 s^{-1})$

H'(s) now posesses zero transmission at both zero and infinite frequency and can be synthesised as a passive prototype of a standard form [1]. It should be re-scaled to have maximum transmission at a point in the passband to ensure optimal passive sensitivity.

SWITCHED-CAPACITOR LADDER DESIGN

A passive prototype synthesised from H'(s) can be described by the following matrix nodal equation

$$(sC + s^{-1}\Gamma + G)V = J \qquad (1)$$

where C, Γ and G represent the contributions of capacitors, inductors and conductors respectively. For convenience, V is defined by $[v_1,-v_2,v_3,-v_4,\ldots]^T$ to ensure that the entries of C, Γ and G are positive. For case iii) a system realising the original transfer function H(s) can be obtained as

$$(sC + s^{-1}\Gamma + G)V = (s + \omega_i s^{-1})J \qquad (2)$$

Cases i) and ii) follow by dropping either the s or s^{-1} term on the RHS of (2). Bilinearly transforming (2) (at a sampling frequency of T=2s for simplicity)

$$\left[\frac{1 - z^{-1}}{1 + z^{-1}}C + \frac{1 + z^{-1}}{1 - z^{-1}}\Gamma + G\right]V = \frac{1 - z^{-1}}{1 + z^{-1}}J + \frac{1 + z^{-1}}{1 - z^{-1}}\omega_i^{2}J \qquad (3)$$

The system can be re-arranged as

$$\left[A + \frac{4z^{-1}}{(1 - z^{-1})^2}\Gamma + \frac{2z^{-1}}{1 - z^{-1}}G\right]V = (1 + \omega_i^2)J + \frac{4z^{-1}}{(1 - z^{-1})^2}J \qquad (4)$$

with

$$A = C + \Gamma + G \qquad (5)$$

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and for simplicity can be rewritten as

$$(\mathbf{A} + \Psi\Phi 4\mathbf{\Gamma} + \Psi 2\mathbf{G})\mathbf{V} = -(1+ \omega_i^2)\mathbf{J}' - 4\Psi\Phi\omega_i^2\mathbf{J}' \quad (6)$$

with $\mathbf{J}'=-\mathbf{J}$ and

$$\Psi = z^{-1}/(1 - z^{-1}) \quad (7a)$$

$$\Phi = 1/(1 - z^{-1}) \quad (7b)$$

Where Ψ and Φ constitute the transfer functions of a pair of LDI integrators. System (6) can now be expressed in realisable form following techniques presented in [11].

Right-ULD design

Let

$$4\mathbf{\Gamma} = \mathbf{UL} \quad (8)$$

then (2) can be decomposed as

$$\mathbf{AV} = -\Phi\mathbf{UX} - 2\Psi\mathbf{GV} - (\omega_i^2 + 1)\mathbf{J}' \quad (9a)$$

$$\mathbf{IX} = \Psi(\mathbf{LV} + 4\omega_i^2\mathbf{U}^{-1}\mathbf{J}') \quad (9b)$$

Left-ULD design

Let

$$\mathbf{A} = \mathbf{UL} \quad (10)$$

then the system can be decomposed as

$$\mathbf{LV} = \Psi\mathbf{IX} - (\omega_i^2 + 1)\mathbf{U}^{-1}\mathbf{J}' \quad (11a)$$

$$\mathbf{UX} = -\Phi 4\mathbf{\Gamma V} - 2\mathbf{GV} - 4\omega_i^2\Phi\mathbf{J}' \quad (11b)$$

Right-direct design (coupled-biquads)

Decompose the system by direct factorisation of $\mathbf{\Gamma}=\mathbf{\Gamma I}$

$$\mathbf{AV} = -\Phi\mathbf{IX} - 2\Psi\mathbf{GV} - (\omega_i^2 + 1)\mathbf{J}' \quad (12a)$$

$$\mathbf{IX} = 4\Psi(\mathbf{\Gamma V} + \omega_i^2\mathbf{J}') \quad (12b)$$

Left-direct design (coupled-biquads)

Decompose the system by direct factorisation of $\mathbf{A}=\mathbf{AI}$

$$\mathbf{AV} = \Psi\mathbf{IX} - (\omega_i^2 + 1)\mathbf{J}' \quad (13a)$$

$$\mathbf{IX} = -\Phi 4\mathbf{\Gamma V} - 2\mathbf{GV} - 4\omega_i^2\Phi\mathbf{J}' \quad (13b)$$

Note that the well known LU factorisation is replaced by its dual operation UL decomposition. This is to avoid forming $\mathbf{L}^{-1}$ which would be a full lower triangular matrix causing a large number of feed-in branches. $\mathbf{U}^{-1}\mathbf{J}$ on the other hand is a vector with one element implying a single feed-in branch. Two inputs are generally required, one to the variable set $\mathbf{V}$ and the other to variable set $\mathbf{X}$. This result can be further generalised to ensure that all ladder filters have a pair of input branches to either variable set. Active-RC circuits can be generated in a similar manner by replacement of Ψ and Φ by s^{-1} and $-s^{-1}$ representing inverting and noninverting Miller integrators.

STABILITY AND SENSITIVITY

It can be theoretically verified that the new structures are stable provided the transfer functions are stable. This has been checked by time domain simulation of the SC circuits. Because of the zeros realised at both zero and infinite frequency by the main system, the instability problem is avoided for highpass or bandstop implementations.

The sensitivity behaviour of the new structures must be examined as they are no longer strictly ladder simulation and seem to depart from Orchard's low-sensitivity criterion [1]. From the many examples studied by computer simulation, the sensitivity for the new structures has been confirmed to be much better than their biquad counterparts, and very close to traditional ladder simulations.

DESIGN EXAMPLE

A bandpass filter, with passband edges of 9kHz and 11kHz, 1dB passband ripple and 50dB stopband attenuation, is designed by the PANDDA filter compiler [12]. Two forms of symmetric 8th order elliptic approximation are illustrated in Fig.1. One is a 'pure' elliptic function with non-zero transmission at 0Hz and infinite frequency. Since this cannot be synthesized as a passive ladder it is normally modified by shifting one notch to 0Hz and another to infinity. The penalty is a loss of ~5dB in the stopband. The 'pure' elliptic function has been realised as a biquadratic cascade and as canonical left-ULD and right-ULD ladder simulations (Fig.2a,b). The sensitivities of the various filter realisations are shown in Fig.3 indicating that the ladder realisations are better than the biquad. Table 1 gives a set of component values for the left-ULD simulation which has a comparable total capacitance to a biquad realisation.

The number of components required to implement each of the four ladder circuits and the biquadratic cascade is very similar. The component spread of the ULD circuit is found to be best, particularily for extremely wide or narrow band filters. For bandpass examples the sensitivities of the ladder filters are all below those of the biquad. In the lowpass case, the leapfrog is the best choice as the other ladder simulations will exhibit higher sensitivity at low frequency [8].

CONCLUSIONS

It has been shown that a transfer function with finite transmission at high frequency can be realised by canonical ladder structures. The design technique is applicable to a wide variety of active-RC and switched-capacitor ladder simulation methods. The filter structures are extended regularily from lower order forms, with the cost of only a few extra components.

ACKNOWLEDGEMENT

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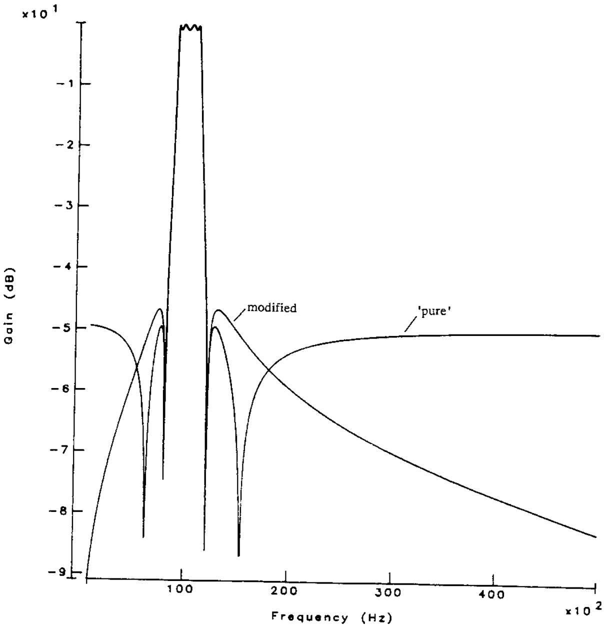

Fig. 1 Comparison of 'pure' and modified 8th order bandpass elliptic functions

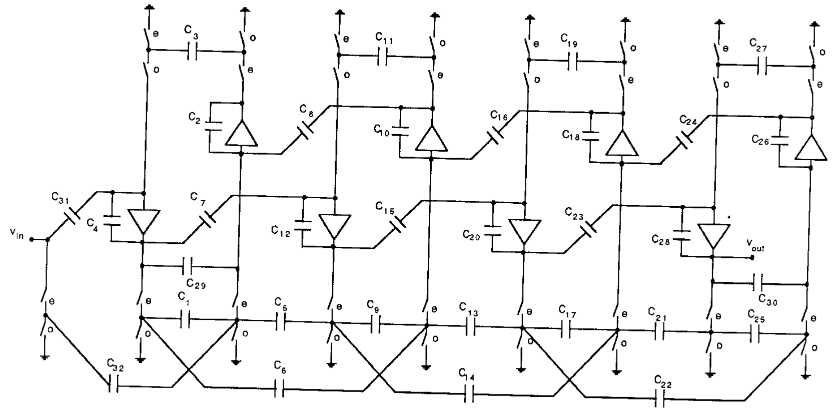

Fig. 2a. 8th order canonic left-ULD switched-capacitor ladder filter

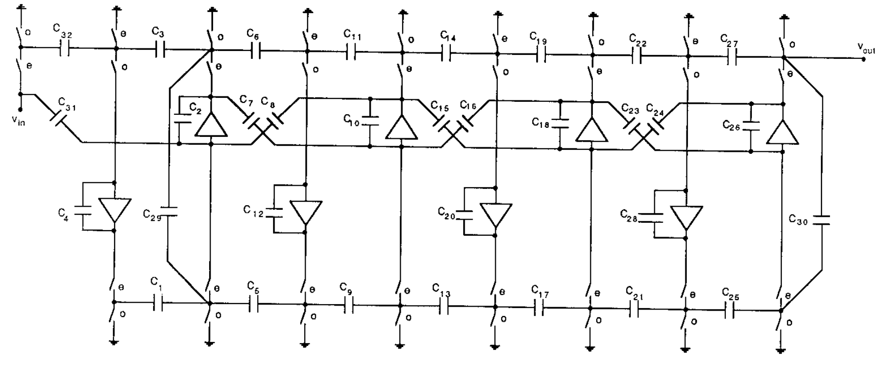

Fig. 2b. 8th order canonic right-ULD switched-capacitor ladder filter

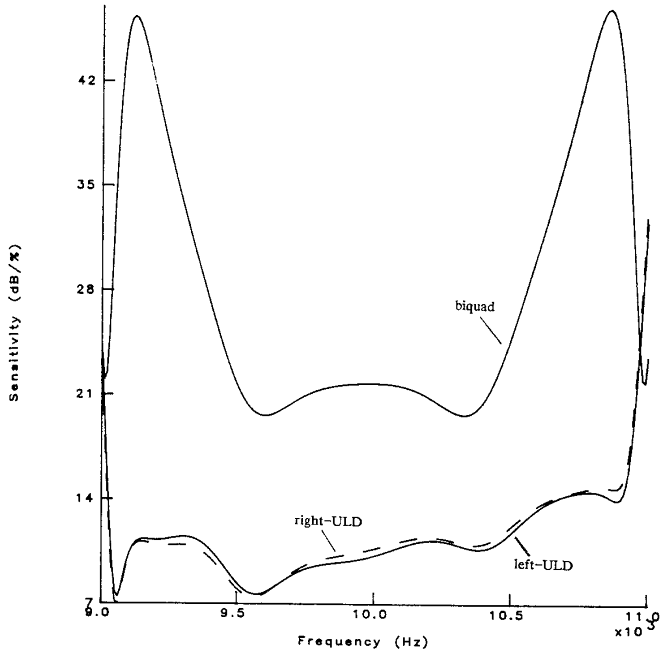

Fig. 3 Sensitivity comparison of 8th order elliptic filter realisations

Clock frequency = 200kHz
Passband ripple < 1dB
Lower passband edge = 9kHz
Stopband attenuation > 50dB
Upper passband edge = 11kHz

Comparison of canonical left-ULD ladder and biquad SC filters

ULD		Biquad	
Total capacitance	= 188.03	Total capacitance	= 193.91
Capacitance spread	= 57.38	Capacitance spread	= 41.54
Average capacitor	= 5.88	Average capacitor	= 6.46
Number of capacitors	= 32	Number of capacitors	= 30
Number of switches	= 34	Number of switches	= 36
Number of op-amps	= 8	Number of op-amps	= 8

Capacitor values of left-ULD SC ladder filter in Fig. 2a

C1 = 18.20	C2 = 57.38	C3 = 2.28	C4 = 7.69	C5 = 3.82
C6 = 1.05	C7 = 1.00	C8 = 4.80	C9 = 5.25	C10 = 14.09
C11 = 3.12	C12 = 10.72	C13 = 1.00	C14 = 1.12	C15 = 1.36
C16 = 5.36	C17 = 3.96	C18 = 12.66	C19 = 1.00	C20 = 2.87
C21 = 1.00	C22 = 2.01	C23 = 1.26	C24 = 3.21	C25 = 2.57
C26 = 11.05	C27 = 1.00	C28 = 2.16	C29 = 2.05	C30 = 1.00
C31 = 1.00	C32 = 1.00			

TABLE 1 DESIGN DATA FOR 8TH ORDER ELLIPTIC FILTER REALISATIONS