

Zigzag Codes and Concatenated Zigzag Codes

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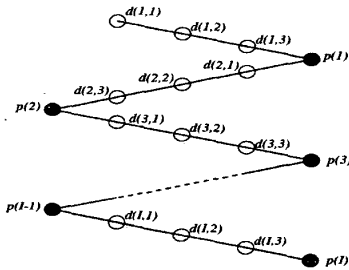
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ABSTRACT

This paper introduces a family of error-correcting codes called *zigzag codes*. A zigzag code is described by a highly structured zigzag graph. Due to the structural properties of the graph, very low-complexity soft-in, soft-out decoding rules can be implemented. We present a decoding rule, based on the Max-Log-MAP (MLM) formulation, which requires a total of only 20 addition-equivalent-operations per information bit, per iteration. Simulation of a rate-1/2, four-dimensional concatenated zigzag code with interleaver length 65536 yields a bit error rate (BER) of 10^{-5} at 0.9 dB and 1.4 dB away from the Shannon theoretical limit by optimal (MAP) and low-cost sub-optimal (MLM) decoders, respectively. Furthermore, these codes appear to have lower error floors than the comparable two-dimensional turbo codes.

DESCRIPTION OF THE CODE

A zigzag code can be described graphically as shown below. Here, white nodes represent information bits: $\{d(i, j)\}, i = 1, 2, \dots, I, j = 1, 2, \dots, J$. Black nodes represent parity bits: $\{p(i)\}, i = 1, 2, \dots, I$. We call $\{p(i-1), d(i, 1), d(i, 2), \dots, d(i, J), p(i)\}$ a *segment*. The parity bits are chosen such that each segment on the graph contains an even number of ones.



EFFICIENT SOFT-IN/SOFT-OUT DECODING

Let $D = \{d(i, j)\}$ be an $I \times J$ array of information bits and let $P = \{p(i)\}$ be the $I \times 1$ parity vector. Let $Z = (D, P)$ be the $\{+1, -1\}$ -modulated codeword ("zero" $\mapsto +1$, "one" $\mapsto -1$) and $\tilde{Z} = (\tilde{D}, \tilde{P})$ be the noisy received vector. The MLM of each information bit is given by:

$$L[d(i, j)] \triangleq \frac{1}{4} \left\{ \min_{d(i, j)=-1} \|Z - \tilde{Z}\|^2 - \min_{d(i, j)=+1} \|Z - \tilde{Z}\|^2 \right\}.$$

To efficiently implement the above, we define the *forward* MLM of the parity bits as follows:

$$F[p(i)] \triangleq \frac{1}{4} \left\{ \min_{p(i)=-1} \|Z_1^i - \tilde{Z}_1^i\|^2 - \min_{p(i)=+1} \|Z_1^i - \tilde{Z}_1^i\|^2 \right\},$$

where Z_1^i (\tilde{Z}_1^i) represents the partially transmitted codeword (received vector) from segment 1 up to segment i . The *backward* MLM, $B[p(i)]$, is similarly defined for segment $i + 1$ up

to segment I . It can be verified that forward and backward MLM can be solved recursively as:

$$\begin{aligned} F[p(i)] &= \tilde{p}(i) + W(F[p(i-1)], \tilde{d}(i, 1), \tilde{d}(i, 2), \dots, \tilde{d}(i, J)), \\ B[p(i-1)] &= \tilde{p}(i-1) + W(\tilde{d}(i, 1), \tilde{d}(i, 2), \dots, \tilde{d}(i, J), B[p(i)]), \end{aligned}$$

where $F[p(0)] = +\infty$, $B[p(I)] = \tilde{p}(I)$, and

$$W(a_1, a_2, \dots, a_n) \triangleq \left[\prod_{j=1}^n \text{sign}(a_j) \right] \min_{1 \leq j \leq n} |a_j|.$$

The MLM of the information bits can be determined as:

$$\begin{aligned} L[d(i, j)] &= \tilde{d}(i, j) + W(F[p(i-1)], \tilde{d}(i, 1), \tilde{d}(i, 2), \dots, \\ &\quad \tilde{d}(i, j-1), \tilde{d}(i, j+1), \dots, \tilde{d}(i, J), B[p(i)]). \end{aligned}$$

CONCATENATED ZIGZAG CODES

A concatenated zigzag code is described by a triplet (I, J, K) . Let $D_k = \pi_k(D)$ be an interleaved version of the data matrix D , $k = 1, 2, \dots, K$. For each D_k , we form an $I \times 1$ parity column vector P_k according to the figure. The transmit codeword consists of $[D, P_1, P_2, \dots, P_K]$ and the overall code rate is $J/(J + K)$. It can be shown that the overall decoding cost for a K -dimensional concatenated zigzag code is $K(4 + 4/J)$ arithmetic equivalent operations per information bit per iteration (AEO/IB/Iter), e.g., with $K = J = 4$, the decoding cost is 20 AEO/IB/Iter. As comparison, the complexity cost of a "standard" turbo code is 192 floating point operations per information bit per iteration.

Simulation results of a rate-1/2 concatenated zigzag code with interleaver length of 400 ($(I, J, K) = (100, 4, 4)$) are presented below, and is compared with 16-state turbo code. It is seen that the performance of the turbo code is slightly better at relatively high BER. However, the zigzag code surpasses the turbo code for BER at about 10^{-5} . This implies that the proposed code is useful for data communication systems where very low BER is required.

