Joint Power and Rate Allocation for DF Two-Path Relay Systems
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Abstract—In this letter, we investigate the power and rate allocation problem for decode-and-forward two-path relay systems. To deal with the inter-relay interference at the relays, two power and rate allocation schemes based on feasible power region analysis are proposed. Numerical results show that the proposed schemes can provide better power and outage performances compared with the other existing alternatives. Besides, we also discuss the performance difference between the two proposed schemes for different rate requirements. We find that the suboptimal but simpler successive-interference-cancellation-based scheme can offer near-optimal performance.

Index Terms—Two-path relay system, decode-and-forward, feasible power region, power and rate allocation, successive interference cancellation.

I. INTRODUCTION

The conventional two-hop cooperative schemes [1]–[4] involve two phases. The relays receive in one phase and transmit in another. Such use of two phases suffers from rate loss, since the source and relay cannot transmit in the same phase under the common half-duplex mode.

A strategy studied in [5] and [6] mitigates the problem by employing two relays and allowing the source to transmit in both phases, which is referred to as two-path relay strategy. In this strategy, the relays take turn to receive data from the source in one phase and then forward it to the destination in the next phase. In this way, the transmit and receive phases of the two relays are staggered, which provides a continuous information flow. Both amplify-and-forward (AF) [7]–[9] and decode-and-forward (DF) [10]–[13] modes have been studied for this strategy. In this letter, we focus on DF mode.

A problem in the two-path relay strategy is the inter-relay interference (IRI) when one relay is transmitting while the other is receiving. Some power or rate control methods are studied to improve the system performance. For AF two-path relay systems, Kim et al. [9] proposed a power allocation scheme to deal with the IRI. In [12], optimal transmit power was given for no fading symmetrical DF systems. Successive-interference-cancellation (SIC)-based rate adaption was studied in [13] but without power allocation. Both the techniques in [12] and [13] are benefit for the DF two-path relay systems. However, joint power and rate control has not been well studied.

To further improve the performances of DF two-path relay systems, we discuss a joint power and rate allocation technique in this letter. According to the detection techniques at the relays, we consider two different feasible power regions (FPRs), based on which the optimization problem is formulated and power and rate are jointly designed. We will provide numerical results to show that the proposed schemes noticeably outperform the other existing alternatives. We will also show that the simple SIC-based scheme can achieve near-optimal performance.

II. SYSTEM MODEL

A. Two-Path Relay Model

Fig. 1 shows a two-path relay system involving a source S, a destination D and two relays R1 and R2. The following basic assumptions are similar to [10] and [12].

1. S always transmits and D always receives.
2. There is no direct path between S and D.
3. The whole time span is divided into several slots. R1 receives data from S in an odd slot and then forwards it to D in the next even slot. Similarly, R2 receives in an even slot and forwards in the next odd slot.
4. The signals between R1 and R2 are regarded as interference. (Section III. This is a suboptimal treatment, but it greatly simplifies the problem.)
5. The received data packet at each node is either perfectly decoded or completely discarded. The interference between R1 and R2 may be also decoded if this can help the detection of the wanted signal from S. The interference, even correctly detected, will be discarded and not be forwarded to D.

B. Power and Rate Allocation Problem

We assume quasi-static fading, i.e., the channel condition remains unchanged in all the slots during a transmission. We further assume the following periodic protocol.

- Two fixed rate-power pairs, denoted by \((r_{S2}, p_{S2})\) and \((r_{R1}, p_{R1})\) respectively, are used at S and R1 in all the even slots.
Two fixed rate-power pairs, denoted by \((r_{S1}, p_{S1})\) and \((r_{R2}, p_{R2})\) respectively, are used at S and R2 in all the odd slots.

According to the DF relay protocol, the forwarded signal from a relay should have the same rate as its received signal. Hence,

\[
\begin{align*}
r_{S1} &= r_{R1}, \quad (1a) \\
r_{S2} &= r_{R2}. \quad (1b)
\end{align*}
\]

We assume that the system is constrained by the average rate requirement:

\[
r_{AV} = \frac{r_{S1} + r_{S2}}{2} \quad (2)
\]

and our target is to minimize the total transmission power:

\[
\min \ p_{TOTAL} = p_{S1} + p_{R1} + p_{S2} + p_{R2}. \quad (3)
\]

**C. Decomposed Power Minimization Problem**

We first consider to minimize the total power for a given rate pair \((r_{S1}, r_{S2})\). The problem in (3) can be decomposed into two power minimization problems (see Figs. 1(a) and (b)):

\[
\begin{align*}
\min & \quad p_{S2} + p_{R1} \text{ in even time slots,} \quad (4a) \\
\min & \quad p_{S1} + p_{R2} \text{ in odd time slots.} \quad (4b)
\end{align*}
\]

When \(r_{S1}\) and \(r_{S2}\) are given, the above two problems are independent and can be solved separately. Thus for each given \((r_{S1}, r_{S2})\), we solve (4a) and (4b). The original problem in (3) can be solved by using one dimensional searching over \(r_{S1}\) (with corresponding \(r_{S2}\) constrained by (2)).

**III. POWER MINIMIZATION PROBLEM**

Continuing from Section II-C, we now consider solving (4) under the assumption that the \((r_{S1}, r_{S2})\) pair is given. Due to the symmetry of the problems, we will only focus on (4a) in this section. Refer to Fig. 1(a) for the discussions below.

**A. Feasible Power Region (FPR)**

Let the received signals at R2 and D in an even time slot be given respectively by

\[
\begin{align*}
y_{R2} &= h_{S-R2}\sqrt{p_{S2}}x_{S} + h_{R1-R2}\sqrt{p_{R1}}x_{R1} + w_{R2}, \quad (5a) \\
y_{D} &= h_{R1-D}\sqrt{p_{R1}}x_{R1} + w_{D}. \quad (5b)
\end{align*}
\]

where \(x_{S}\) and \(x_{R1}\) are the signals transmitted from S and R1 respectively, \(w_{R2}\) and \(w_{D}\) the additive white Gaussian noise (AWGN) with power \(N_{0}\), and \(h_{M-N}\) the channel coefficient between nodes M and N.

**For the given rate pair \((r_{S1}, r_{S2})\), the FPR is defined as the set of all the \((p_{R1}, p_{S2})\) pairs with which R2 can correctly decode \(x_{S}\) and D can correctly decode \(x_{R1}\). There are two FPR cases for different system states as shown in Fig. 2. The lines in Fig. 2 are defined as:**

\[
\begin{align*}
\text{L1:} & \quad |h_{R1-R2}|^{2}p_{R1} = (2^{r_{S1}} - 1)N_{0}, \quad (6a) \\
\text{L2:} & \quad |h_{S-R2}|^{2}p_{S2} = (2^{r_{S2}} - 1)N_{0}, \quad (6b) \\
\text{L3:} & \quad |h_{S-R2}|^{2}p_{S2} + |h_{R1-R2}|^{2}p_{R1} = (2^{2r_{AV}} - 1)N_{0}, \quad (6c) \\
\text{L4:} & \quad \frac{|h_{S-R2}|^{2}p_{S2}}{|h_{R1-R2}|^{2}p_{R1} + N_{0}} = 2^{r_{S2}} - 1, \quad (6d) \\
\text{L5:} & \quad |h_{R1-D}|^{2}p_{R1} = (2^{r_{S1}} - 1)N_{0}, \quad (6e) \\
\text{L6:} & \quad p_{S2} = k(p_{R1} - \frac{(2^{2r_{AV}} - 2^{r_{S2}})N_{0}}{|h_{R1-R2}|^{2}} - \frac{(2^{r_{S2}} - 1)N_{0}}{|h_{S-R2}|^{2}}) \quad (6f)
\end{align*}
\]

where \(k = \frac{(2^{2r_{S1}}-1)(2^{2r_{S2}}-1)|h_{R1-R2}|^{2}/|h_{S-R2}|^{2}|h_{R1-D}|^{2}}{(2^{r_{S1}}-1)|h_{R1-R2}|^{2}-(2^{r_{S2}}-1)|h_{R1-D}|^{2}}\) in (6f).

The details are as below.

We first consider the case of \(|h_{R1-D}|^{2} > |h_{R1-R2}|^{2}\) when L5 is at the left of L1, as shown in Fig. 2(a). From (5b), D can correctly decode \(x_{R1}\) provided that the power pair \((p_{R1}, p_{S2})\) falls on the right hand side of L5. According to the definition of FPR, we only need to focus on the right hand side of L5 to find the regions in which R2 can correctly decode \(x_{S}\).

Notice that in (5a), R2 receives signals of both \(x_{R1}\) and \(x_{S}\), but only needs to decode \(x_{S}\). There are three different situations as shown in Fig. 2(a).

- In region A bounded by L1, L2 and L3, R2 decodes both \(x_{R1}\) and \(x_{S}\). This is the standard multiple access channel (MAC) power region.
- In region B bounded by L1, L4 and L5, R2 can only decode \(x_{S}\) with treating \(x_{R1}\) as noise.
In this subsection, we assume only low-cost SIC is allowed at R2. We will show that the performance loss due to the SIC is a simple but suboptimal technique for a multi-user system. In Section III-A, we assume optimal decoding at R2.

B. SIC-FPR

SIC is a simple but suboptimal technique for a multi-user system. In Section III-A, we assume optimal decoding at R2. In this subsection, we assume only low-cost SIC is allowed at R2. We will show that the performance loss due to the SIC-FPR is small. Two cases will be discussed as illustrated in Fig. 3. We will show that the performance loss due to the SIC-FPR is small. Two cases will be discussed as illustrated in Fig. 3.

Fig. 3(a) is for the case when L5 does not intersect with region H bounded by L2 and L7, where L2 is given in (6b) and Fig. 3.

Fig. 3(b) shows the case when L5 intersects with region H. Then the FPR is now formed by regions H’ and G’. The basic principles are similar to that for Fig. 3(a), except that now correct decoding at D demands a relatively large $p_{R1}$.

C. Problem Formulation and Solution

We now rewrite the problem in (4a) by adding the power constraints discussed above:

$$\min \ p_{S2} + p_{R1},$$

subject to $$(p_{R1}, p_{S2}) \in \text{FPR}. \quad (8a)$$

In practice, the transmission power for each node can also be limited as:

$$0 \leq p_{R1} \leq p_{\text{MAX}}, \quad 0 \leq p_{S2} \leq p_{\text{MAX}}. \quad (8c)$$

The FPR used for (8b) depends on the type of detection at R2, i.e., ideal detection in Section III-A or SIC detection in Section III-B. Since (8) is linear, its solution is achieved at a corner point of the region constrained by (8b) and (8c). This solution can be obtained by comparing the values of $p_{S2} + p_{R1}$ at all the corner points. Some corner points can be excluded without computing $p_{S2} + p_{R1}$, but we will not discuss in details. The procedure is straightforward.

Since maximum power constraints are considered, outage may occur. Notice that outage minimization can be also achieved through power minimization. (The reverse is not necessarily true.) In the next section, we will show the advantages of the proposed schemes regarding through power and outage performances.

IV. NUMERICAL RESULTS

We now provide some numerical results based on the following settings: the source S is located at $(0, 0.5)$ of a two-dimensional coordinate plane and the destination D at $(0, -0.5)$. Two relays R1 and R2 are randomly located in a square with four vertexes: $(0.5, 0.5), (0.5, -0.5), (-0.5, 0.5)$ and $(-0.5, -0.5)$. We consider both path loss and small scale fading for the channels. The path loss factor $\beta$ is set to 3. The small scale fading is Raleigh fading with averaged power gain 1. Average transmission rate $r_{AV}$ in (2) is set to 3.

Fig. 4 shows the average power consumptions of the proposed-optimal and proposed-SIC schemes, as discussed in Sections II and III. For reference, we consider several existing works on AF or DF modes. In conventional AF and DF schemes, the source broadcasts to the relays in the first time slot, and the relays forward in the following time slot. (For DF, a relay will not forward if its decoding is not successful.) The destination receives and decodes with maximal ratio combining (MRC). Power used in these two schemes are optimized in the simulations. AF scheme in [9] is a power and rate allocation scheme for AF two-path relay model. The scheme

Note that points M1, M2 and M3 in Fig. 2(a) fall in either A or B or both. Region C is feasible since any point in C can be expressed as a linear combination of M1, M2 and M3 with weighting coefficients between 0 and 1. Thus it can be realized by timesharing M1, M2 and M3.

The FPR is the union of regions A, B and C in this case.

Another case is $|h_{R1-D}|^2 \leq |h_{R1-R2}|^2$ when L5 is at the right of L1, as shown in Fig. 2(b). In this case, the FPR is region E bounded by L2, L3 and L5. Region E is a sub-region of A in Fig. 2(a), so both $x_S$ and $x_{R1}$ are decodable at R2. Notice that if $|h_{R1-D}|^2$ becomes smaller, L5 in Fig. 2(b) will further shift right and finally intersect with L2 instead of L3, which means E becomes a region bounded by only L2 and L5. It can be seen as a special case of Fig. 2(b) and we will not show it in a new figure.

Ideal multiuser detection or timesharing is required to achieve the FPR discussed above. These two techniques may cause difficulty in a practical system. We will discuss a more practical approach in the next subsection.

The path loss factor $\beta$ is set to 3.

$$\text{L7: } \frac{|h_{R1-R2}|^2 p_{R1}}{|h_{S-R2}|^2 p_{S2} + N_0} = 2^{r_{S1}} - 1. \quad (7)$$

Similar as the discussions in Section III-A, we only focus on the right hand side of L5 in which D can correctly decode $x_{R1}$. In region H, $x_{R1}$ is decoded first at R2 by treating $x_S$ as noise, and then $x_S$ is decoded after subtracting $x_{R1}$ from $y_{R2}$ in (5a). In region G bounded by L4 and L5 (given in (6d) and (6e)), $x_S$ is decoded directly at R2 by treating $x_{R1}$ as noise.

Fig. 3(b) shows the case when L5 intersects with region H. Then the FPR is now formed by regions H’ and G’. The basic principles are similar to that for Fig. 3(a), except that now correct decoding at D demands a relatively large $p_{R1}$.

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in [13] is a DF two-path relay scheme but without power allocation. It can be seen that, to achieve the same outage performance, the proposed schemes can save considerable system power compared with the no power allocation case [13]. They also noticeably outperform the conventional one-path relay schemes and AF scheme in [9].

In Fig. 5, outage probabilities of different cooperative schemes are represented. The abscissa is the maximum transmission signal-to-noise-ratio (SNR) $p_{\text{MAX}}/N_0$ of each node. Similar result can be found in terms of outage performance: the proposed schemes noticeably outperform the other alternatives. Therefore, according to Figs. 4 and 5, the proposed schemes have advantages with respect to either power consumption or outage performance.

More specifically, the results in Figs. 4 and 5 imply: 1) two-path relay schemes can achieve better system performance than the conventional one-path relay schemes if we deal with the IRI properly; 2) with power and rate allocation, DF mode has more potential for two-path relay model than AF mode; 3) a proper power allocation is necessary for the system to achieve good performance.

Besides, it is also observed that, both the power and outage performance loss of the proposed SIC scheme (compared with the proposed optimal scheme) is very small. To further specify the difference between the two schemes, we also simulate some other $r_{UV}$ values (not shown in the figures due to the space limitation) and find that the gap between them is always small. This means, with proper power and rate allocation, SIC is near-optimal.

V. CONCLUSION

This letter investigates the joint power and rate allocation problem in DF two-path relay systems. The feasible power regions with different detection techniques at the relays are analyzed, based on which the joint power and rate optimization problem is solved through examining a few corner points in the derived regions and one dimensional search. It is simple but effective. The numerical results show that the proposed schemes noticeably outperform the existing alternatives with respect to either power consumption or outage performance. Also, low-cost SIC detection at the relays can achieve almost the same performance as the optimal detection provided that proper power and rate allocation is adopted.

REFERENCES