A Hybrid ARQ Scheme Based on CT-TCM Codes

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***Abstract*— A hybrid ARQ scheme exploiting the concatenated two-state trellis-coded modulation (CT-TCM) coding gain is proposed in this paper. This scheme involves multiple simple component TCM codes that are heavily punctured before transmission. The different punctured parts are used for retransmission and provide improved coding gain after being combined at the receiver. We demonstrate by simulations that our CT-TCM-based hybrid ARQ can achieve higher efficiency than existing schemes.**

***Index Terms*— Automatic-repeat-request (ARQ), hybrid ARQ, trellis-coded modulation (TCM).**

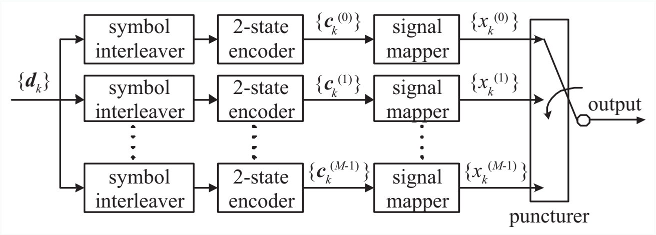

Fig. 1. The encoder structure of the CT-TCM algorithm.

I. INTRODUCTION

Automatic Repeat reQuest (ARQ), forward error correction (FEC) or hybrid schemes have been widely used in modern data communication systems to prevent error or packet loss. For enhanced performance, type-II hybrid ARQ (HARQ) schemes have been investigated [1]. In type-II HARQ schemes, the incremental redundancy bits can be used in retransmission instead of repeating the same codeword of the original data in all retransmissions. At the receiver, these bits are combined with the bits received in previous retransmissions to form a codeword in a FEC code. If this FEC code is carefully designed, a type-II HARQ scheme can achieve much higher coding gain than a type-I scheme.

However, it is difficult to combine a type-II HARQ with a bandwidth efficient TCM scheme. This is because, due to the nature of TCM code, the information bits and redundancy bits cannot be separated. This issue has been investigated in [2] where a different HARQ scheme based on supplementary TCM codes was proposed. The scheme in [2] works best for two transmissions (the original one plus a retransmission). After two transmissions, it switches to a repetition mode, which is not an optimized solution regarding coding gain.

In this paper, we propose an efficient HARQ scheme using concatenated two-state TCM (CT-TCM) codes [3]. A CT-TCM code in [3] involves multiple simple component TCM codes that are heavily punctured before transmission. In our HARQ scheme, we propose to use different punctured parts of a CT-TCM code in retransmission. Each separate retransmission in our scheme can be *self-decoded* at the receiver. Moreover, after combining the received parts from different retransmissions, steadily improved coding gain is achieved as in a type-II HARQ. We will demonstrate by simulations that, our proposed CT-TCM based HARQ can achieve significantly improved performance compared with the existing schemes.

II. THE PROPOSED HYBRID ARQ SCHEME

A. Coding Scheme

Fig. 1 shows the structure of a CT-TCM encoder which comprises M component encoders concatenated in parallel by M symbol interleavers. Let a binary n-tuple $\boldsymbol{d}_k = (d_{k,0}\ ,\ldots,d_{k,n-1})$ be an information symbol. Let $\boldsymbol{d} = \{\boldsymbol{d}_k,\ k \geq 0\}$ be an input sequence to the encoder. For component encoder m, $0 \leq m < M$, a sequence of coded symbols $\{\boldsymbol{c}_k^{(m)},\ k \geq 0\}$ is produced. Each coded symbol $\boldsymbol{c}_k^{(m)}$ includes a parity-check bit and is mapped to a signal constellation of size 2^{n+1}, producing a modulated symbol $x_k^{(m)}$. For spectral efficiency, puncturing is performed on the modulated symbols of all component codes, such that one and only one modulated symbol carrying the same $\boldsymbol{d}_k$ is transmitted, and that the punctured symbols are uniformly distributed in each component code.

Based on the above two constraints, a number of puncture patterns can be obtained. Each puncture pattern can be represented as an $M \times M$ indication matrix Γ, according to which the puncturing process is performed as follows. At time instant k ($k = 0, 1, 2, ...$), $x_k^{(m)}$ from component encoder m is selected for transmission if the (k mod M, m)th entry of Γ is 1, otherwise it is punctured. Table I exemplifies the puncture patterns, denoted as $\Gamma_j, 0 \leq j \leq 3$, which satisfy the above two constraints in the case of 8PSK-based CT-TCM with $M = 4$. They are referred to as 1/4 puncture patterns hereafter, since they select only one of four components to transmit for each input symbol. When Γ_0, for example, is used, $x_k^{(0)}$, $x_{k+1}^{(1)}$, $x_{k+2}^{(2)}$, $x_{k+3}^{(3)}$, $x_{k+4}^{(0)}$, $x_{k+5}^{(1)}$, $\ldots$ will be chosen as output sequence for transmission.

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TABLE I

PUNCTURE PATTERNS VERSUS CORRESPONDING OUTPUTS ($M = 4$)

	Γ_0				Γ_1				Γ_2				Γ_3			
(m)	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
x_k	**1**	0	0	0	0	0	**1**	0	0	**1**	0	0	0	0	0	**1**
x_{k+1}	0	**1**	0	0	0	0	0	**1**	0	0	**1**	0	**1**	0	0	0
x_{k+2}	0	0	**1**	0	**1**	0	0	0	0	0	0	**1**	0	**1**	0	0
x_{k+3}	0	0	0	**1**	0	**1**	0	0	**1**	0	0	0	0	0	**1**	0

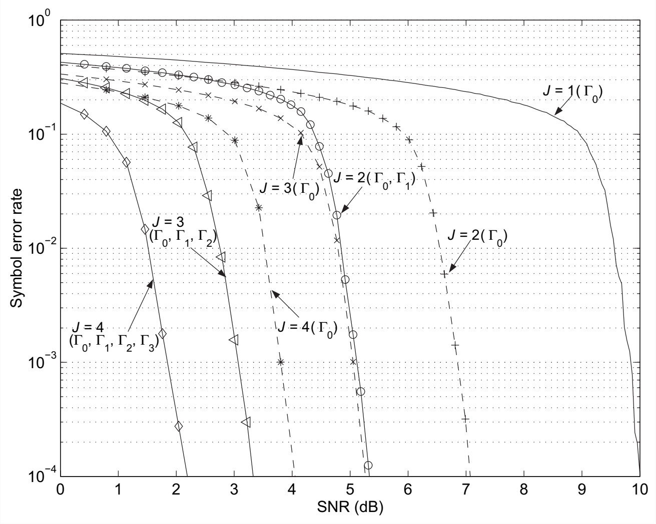

Fig. 2. SER performance comparison between our proposed HARQ and the repetition method.

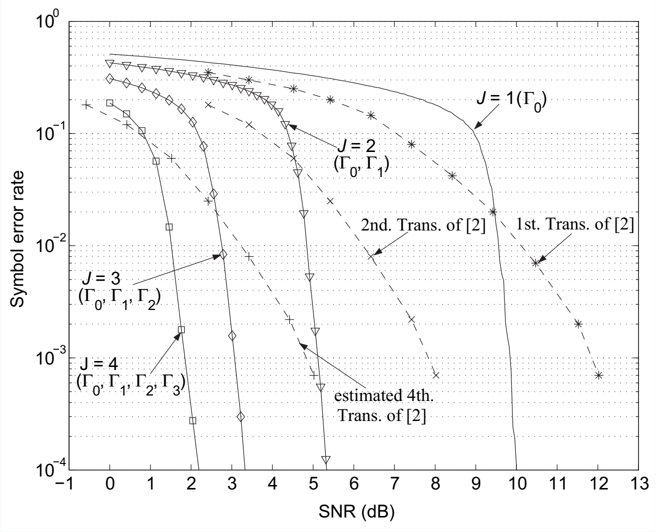

Fig. 3. SER performance comparison of our proposed HARQ and [2]

B. ARQ Scheme

As mentioned above, the nature of traditional TCM codes limits the only choice of HARQ to be simple repetition of the same TCM code in all retransmissions, like the schemes of [2], [5], [6]. However, as we will demonstrate later, such a method leads to inefficient spectral utilization. In this paper, based on the puncturing feature of CT-TCM codes, we propose a different but still simple HARQ scheme using different punctured parts of a CT-TCM code in retransmission.

Let an 8PSK-based CT-TCM code with $M = 4$ be used in our HARQ scheme, which involves a set of 1/4 puncture patterns. Initially, we select one 1/4 puncture pattern to generate the sequence for the first transmission of an original data frame. The punctured parts are stored in a buffer to be used for retransmission. If the initial transmission contains error after it is decoded, another sequence corresponding to another 1/4 puncture pattern is selected from the buffer for retransmission. At the receiver, if the retransmitted sequence cannot be correctly decoded separately, the received sequences from the initial transmission and subsequent retransmissions are combined and decoded together. These steps are repeated until the data is decoded correctly or all punctured parts have been transmitted. In the latter case, the data frame is re-encoded and the above transmission procedures are repeated. Let γ_i denote the puncture pattern used in the ith transmission, $i \geq 1$. By exhaustive search, we find that, among all possible 1/4 puncture patterns that satisfy the above-mentioned constraints, the best performance is provided by sequentially selecting the four puncture patterns shown in Table I for the initial transmission and subsequent retransmissions. That is, $\gamma_i = \Gamma_{(i-1) \bmod M}$ for the ith transmission of a data frame.

By simulation, we obtain the symbol error rate (SER) under different average signal-to-noise ratios (SNR)[1] when the data is transmitted J $(1 \leq J \leq 4)$ times by our proposed HARQ scheme using an 8PSK-based CT-TCM code. The results are plotted in Fig. 2 and Fig. 3, in which the size of the link layer data frame is 2048 bits.

For comparison, we include in Fig. 2 the SER obtained by repetition of the same sequence generated by Γ_0 in all retransmissions. We can observe that compared to the repetition method, significantly improved coding gain is achieved in our proposed HARQ by using a different puncture pattern per retransmission. For example, more than 7.5 dB gain is achieved at $J = 4$ (the curve designated as $J = 4(\Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3)$) over the initial transmission by Γ_0. However, there is only 6 dB gain achieved at $J = 4$ by repetition of Γ_0 (the curve designated as $J = 4(\Gamma_0)$). Thus, the proposed method can deliver an extra 1.5 dB over the repetition method.

For more comparison, we cite in Fig. 3 the SER obtained in the initial and second transmissions by the 8PSK-based TCM ARQ scheme of [2], which switches to repetition coding after two transmissions. We estimate its SER in the fourth transmission as the "+"-marked dash line in Fig. 3. We can

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[1] For one transmission, SNR is related to the energy-per-symbol(E_s)-to-noise-power-spectral-density-ratio (N_0) by SNR$= 2E_s/N_0$. If a symbol is transmitted for N times, the totally received E_s/N_0 is N times over that of one transmission. However, SNR is not affected by retransmissions.

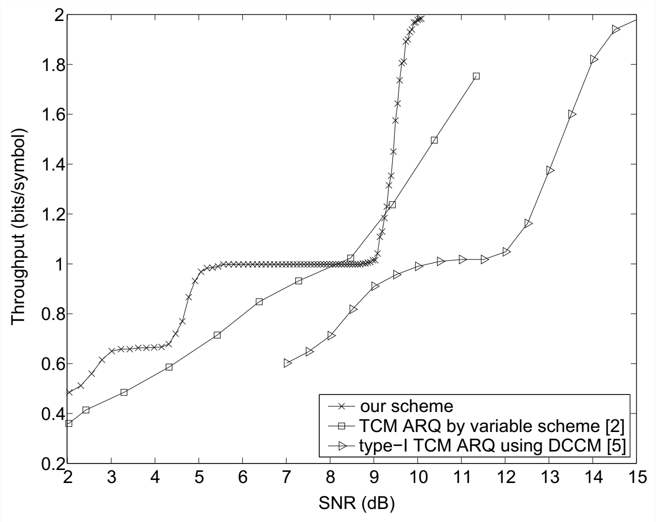

Fig. 4. Throughput efficiency comparisons.

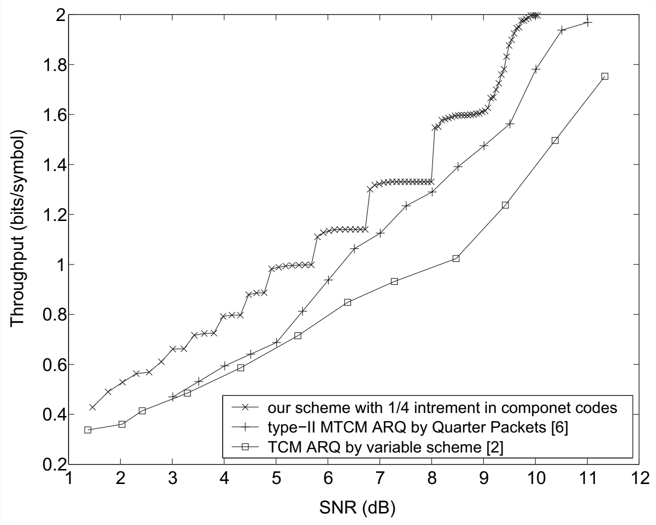

Fig. 5. Throughput efficiency comparisons.

observe that the proposed HARQ scheme outperforms the scheme of [2] in the following aspects. Firstly, the proposed HARQ scheme provides significantly improved coding gain over the scheme of [2] even in the first transmission. This simply results from the fact that the CT-TCM scheme is a more powerful code (measured at SER = 10^{-3}). Secondly, the coding gain can be further increased if we adopt the above-mentioned method with a different puncture pattern in each retransmission. In contrary, the scheme of [2] cannot provide further coding gain after two transmissions. Thirdly, as SNR increases, the CT-TCM algorithm provides faster increase in coding gain than the scheme of [2]. We will demonstrate later that this advantage results in fast throughput improvement, which is helpful for power conservation in high speed data services.

C. Link Layer Performance

A link layer simulation model is developed in OPNET [4] to evaluate the performance of the proposed HARQ. For simplicity, the feedback messages from the receiver are assumed to be always correctly received.

In Fig. 4, we compare the throughput of our scheme obtained by the 1/4 puncture patterns shown in Table I, with the results provided by the TCM-based ARQs in [2] and [5] [2], over the additive white Gaussian noise (AWGN) channel. The throughput is measured by the number of transmitted information-bits-per-symbol[3] (bits/symbol). We observe that considerable improvement is achieved by our scheme over the others, except for a small SNR range (8.2 dB $\sim$ 9.2 dB) where our performance is worse than that of [2]. This is a direct result of the discrete coding gains under the 1/4 puncture patterns shown in Fig. 2. However, we can modify our HARQ scheme to eliminate such stair-like phenomenon in throughput as follows. Assuming that Γ_0 is still used for the initial transmission. For the second transmission, we propose to transmit one quarter of the sequence generated by Γ_1, instead of the complete sequence as in our original HARQ scheme. At the receiver, the received sequences from the initial and second transmissions are combined for decoding. If the decoded data contains errors, the third transmission is invoked and another quarter of the sequence generated by Γ_1 is transmitted. Similar methods are repeated in the subsequent possible retransmissions. Such modification results in 1/4 increment for each retransmission.

Fig. 5 shows the throughput (in bits/symbol) of the modified HARQ scheme. It demonstrates that with incremental partial sequence in retransmission, the stair-like phenomenon in Fig. 4 is smoothed. We can also observe that the throughput achieved by our modified scheme outperforms the best of [6] under their type-II 8PSK TCM ARQ by quarter packets, and further outperforms that of [2].

III. CONCLUSION

We have proposed a new HARQ scheme based on the CT-TCM codes. The performance of the proposed scheme is evaluated by simulations. The results demonstrate that the proposed scheme can achieve higher throughput than other existing TCM-based ARQ schemes.

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[2] In this paper, the results from [2], [5] and [6] are cited by converting their E_b/N_0 and E_s/N_0 coordinates into corresponding SNR coordinates.

[3] With 8PSK-based TCM schemes, the maximal bits per symbol is 2.