

M. Straßburg, O. Schulz, U.W. Pohl and D. Bimberg (*Technische Universität Berlin, Institut für Festkörperphysik, Sekr. PN 5-2, Hardenbergstr. 36, Berlin, Germany*)

S. Itoh, K. Nakano and A. Ishibashi (*Sony Corporation, Shinagawa-Ku, Tokyo 141-0001, Japan*)

E-mail: matthias@sol.physik.tu-berlin.de

References

- KATO, E., NOGUCHI, H., NAGAI, M., OKUYAMA, H., KIJIMA, S., and ISHIBASHI, A.: 'Significant progress in II-VI blue-green laser diode lifetime', *Electron. Lett.*, 1998, **34**, (3), pp. 282-284
- ISHIBASHI, A.: 'II-VI blue-green light emitters', *J. Cryst. Growth*, 1996, **159**, pp. 555-565
- KIJIMA, S., OKUYAMA, H., SANAKA, Y., KOBAYASHI, T., TOMIYA, A., and ISHIBASHI, A.: 'Optimized ZnSe:N/ZnTe:N contact structure of ZnSe-based II-VI laser diodes', *Appl. Phys. Lett.*, 1998, **73**, pp. 235-237
- HOLONYAK, N. Jr.: 'Semiconductor device fabrication with disordering elements introduced into active region'. U.S. Patent 4 511 408, 1985
- DEPPE, D.G., GUIDO, L.J., HOLONYAK, N., HSIUEH, K.C., BURNHAM, R.D., THORNTON, R.L., and PAOLI, T.: 'Stripe-geometry quantum well heterostructure Al_xGa_{1-x}As-GaAs lasers defined by defect diffusion', *Appl. Phys. Lett.*, 1986, **49**, pp. 510-512
- YOKOGAWA, T., MERZ, J., LUO, H., FURDYNA, J., JAU, S., KUTTLER, M., and BIMBERG, D.: 'Disordering of CdZnSe/ZnSe strained superlattices by ion implantation', *Jpn. J. Appl. Phys.*, 1995, **34**, pp. 1159-1161
- KUTTLER, M., STRABURG, M., TÜRCK, V., ENGLISHARDT, R., POHL, U.W., BIMBERG, D., BEHRINGER, M., HOMMEL, D., NÜRNBERGER, J., and LANDWEHR, G.: 'Efficient lateral index guiding of II-VI laser structures by implantation-induced disordering', *J. Cryst. Growth*, 1998, **184/185**, pp. 566-570
- KUTTLER, M., STRABURG, M., TÜRCK, V., POHL, U.W., BIMBERG, D., KURTZ, B., LANDWEHR, G., and HOMMEL, D.: 'Laterally structured ZnCdSe/ZnSe superlattices by diffusion induced disordering', *Appl. Phys. Lett.*, 1996, **69**, pp. 2647-2649
- KUTTLER, M., STRABURG, M., STIER, O., HEITZ, R., POHL, U.W., BIMBERG, D., KURTZ, B., NÜRNBERGER, J., BEHRINGER, M., and HOMMEL, D.: 'Doping dependent ZnCdSe/ZnSe-superlattices disordering', *Appl. Phys. Lett.*, 1997, **71**, pp. 243-245
- STRABURG, M., KUTTLER, M., POHL, U.W., BIMBERG, D., BEHRINGER, M., and HOMMEL, D.: 'Lateral index guided ZnCdSe-based lasers by selective implantation-induced disordering'. Proc. 2nd ISBLLED, 1998, pp. 425-428
- KATAYAMA, K., YAO, H., NAKANISHI, F., DOI, H., SAEGUSA, A., OKUDA, N., YAMADA, T., MATSUBARA, R., IRIKURA, M., MATSUOKA, T., TAKEBE, T., NISHINI, S., and SHIRAKAWA, T.: 'Lasing characteristics of low threshold ZnSe-based blue/green laser diodes grown on conductive ZnSe substrates', *Appl. Phys. Lett.*, 1998, **73**, pp. 102-104
- AGRAWAL, G.P., and DUTTA, N.K.: 'Semiconductor lasers' (van Nostrand Reinhold, New York, 1993)

Approaching low rate Shannon limit using concatenated Hadamard codes

W.K. Leung, K.S. Ho and Li Ping

A family of concatenated Hadamard codes is presented, which have performances close to the low rate Shannon limit. A BER of $\sim 10^{-5}$ at an $E_b/N_0 \approx -1.15$ dB has been observed using an interleaver length of 60000.

Introduction: It is well known that the ultimate capacity of an additive white Gaussian noise (AWGN) channel is $E_b/N_0 = \ln 2 \approx -1.59$ dB (the Shannon limit). In this Letter, we present a family of turbo-type [1] concatenated Hadamard codes that have a performance which approaches the Shannon limit. We propose to adopt constituent two-state trellis codes with a large number of parallel branches associated with codewords from a Hadamard code [2]. This makes full advantage of the inherent coding gain of Hadamard codes as well as the interleaving gain of turbo-type concatenated codes. The new method also has a very modest decoding complexity. This property is due to the simple trellis structure

used, as well as a very efficient *a posteriori* probability (APP) decoding algorithm for the Hadamard code [2].

We show that the new scheme can achieve a BER $\approx 10^{-5}$ at an $E_b/N_0 \approx -1.15$ dB with an interleaving length $L = 60000$. It is only ~ 0.44 dB away from the ultimate Shannon limit. Apart from the theoretical interest, such codes may also find applications in CDMA systems.

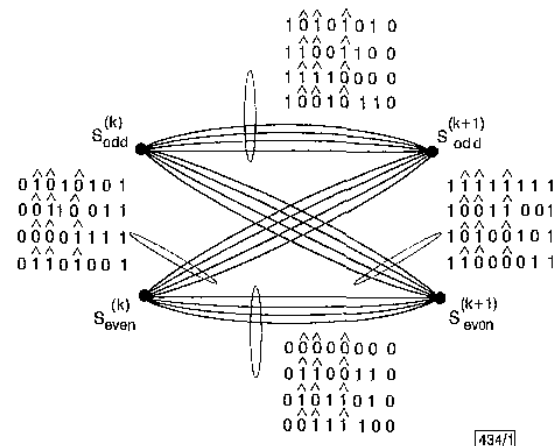


Fig. 1 *k*th section (*k*th depth) of trellis concatenation of 2^m Hadamard codes

$m = 3$

Sequences associated with branches shown, referred to as branch outputs, form a Hadamard code

Code structure: (i) *trellis concatenation of Hadamard codes:* Fig. 1 shows the *k*th trellis section of the scheme, associated with a length 2^m bi-orthogonal Hadamard code [3] with 2^{m+1} codewords. A simple two-state trellis is used, with an even and odd state in every section. There are 2^{m-1} parallel branches between any two consecutive states. Altogether there are 2^{m+1} branches in every section, each labelled by a unique Hadamard codeword. The trellis always starts at an even state. The input information bits are divided into groups of m bits each. Given a particular state, the next group of m information bits determines a branch along the coding path and so the next state. The Hadamard code used is encoded systematically, with the information positions indicated by the hats in Fig. 1.

A key strategy here is to divide the set of Hadamard codewords into four subsets. Two of these subsets are called even subsets and the other two are called odd subsets. Each of the two even (odd) subsets contains an even (odd) number of information bits at a value of 1. The even subsets are assigned to the branch between the states of the same parity (i.e. even-even or odd-odd connections). The odd subsets are assigned to the branch between states with opposite parities (i.e. even-odd or odd-even connections). The all-zero codeword is always assigned to a branch between two even-states.

This trellis structure has a recursive nature similar to that of the recursive convolutional code commonly used for turbo codes. This property can be seen by examining the codewords corresponding to a weight-1 information sequence (i.e. a sequence containing only one bit of value 1). For a weight-1 sequence, the encoding path in Fig. 1 will deviate from an even state to an odd state at the input block containing the only bit-1 and will stay with the odd states thereafter. The encoder will then continuously produce nonzero outputs. Therefore, a weight-1 input sequence has a very high probability of generating a codeword with a high weight (unless the bit-1 appears near the end of the trellis, but this is a small probability event). Based on the analysis of [4], it can be shown that this property will lead to an interleaving gain, similar to that of ordinary turbo codes.

Another feature of the scheme is that every branch (associated with a Hadamard codeword) now carries multiple information bits. This explores the inherent coding gain of the Hadamard code for error protection. It also implies lower normalised decoding costs measured by the number of operations per information bit.

(ii) *Overall code:* The code in Fig. 1 can be concatenated again in a parallel manner as shown in Fig. 2, referred to as a Hadamard-

turbo code. The coding rate of the proposed code is $m/(m + M(2^m - m))$, where M is the number of dimensions. The principle is similar to that of multidimensional turbo codes and related schemes studied in [5 – 7].

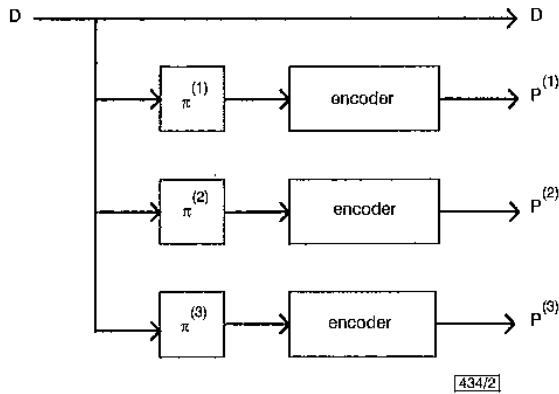


Fig. 2 Encoder for three-dimensional Hadamard-turbo code

D is the information sequence. Each constituent encoder here has same structure based on trellis section shown in Fig. 1, except that only parity bits are collected which form the vector $P^{(i)}$, $i = 1, 2, 3$. $\pi^{(i)}$, $i = 1, 2, 3$ stands for interleavers. Overall codeword is $\{D, P^{(1)}, P^{(2)}, P^{(3)}\}$.

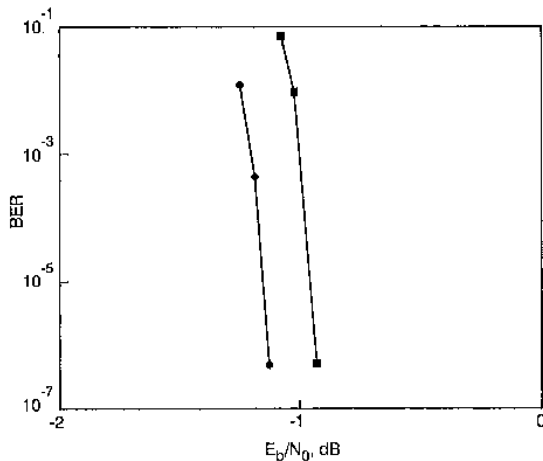


Fig. 3 Performances of concatenated Hadamard-turbo codes with interleaver length of 60000

m is order of Hadamard code (i.e. its length is 2^m) and M is the number of dimensions. Iteration number = 30.

- ◆ $m = 7$ and $M = 3$, rate = $7/1370$
- $m = 5$ and $M = 4$, rate = $5/113$

Decoding method: The global decoder for the code in Fig. 2 follows the general iterative turbo-type decoding principle studied in [6]. An efficient APP decoding procedure for the constituent codes is listed below.

- Step (i): Perform a fast Hadamard transform (FHT) on the codeword in a segment-by-segment manner. This is used to generate the likelihood ratios of the Hadamard codewords associated with the branches, based on the observations of individual branch outputs.
- Step (ii): Apply the forward and backward searches of the maximum *a posteriori* (MAP) algorithm to the trellis in Fig. 1. This generates the *a posteriori* likelihood ratios for all the branches in the trellis. (These ratios are also the *a posteriori* likelihood ratios for Hadamard codewords associated with the branches).
- Step (iii): Generate the *a posteriori* likelihood ratios of every information bit using the fast algorithm presented in [2].

For complexity analysis, the cost of MAP decoding in step (ii) is $\sim 1.5 \times 2^m/m$ multiplications and $2 \times 2^m/m$ additions per

information bit. The combined cost of steps (i) and (iii) is $\sim 2^{m+1}$ exponential functions and $9 \times 2^m/m$ additions per information bit [2]. For simplicity, we assume that the complexities of an exponential function and a multiplication are approximately the same. Then the overall decoding costs are $\sim 3.5 \times 2^m/m$ multiplications and $11 \times 2^m/m$ additions per information bit. For example, for $m = 7$, the cost is only ~ 64 multiplications and 201 additions, which is very modest. In comparison, MAP decoding for a standard 16-state convolutional code costs ~ 256 multiplications and 192 additions [8].

Fig. 3 shows the simulation results with an interleaver length $L = 60000$. With $m = 7$, the required $E_b/N_0 \approx -1.15$ for a BER $\approx 10^{-5}$ is only 0.44dB away from the Shannon limit of -1.59 dB. In comparison, the best performance achievable by the low-rate turbo codes in [9] (with 16 states) is a BER $\approx 10^{-5}$ at -0.85 dB.

Conclusion: We have introduced a modified turbo-type concatenated Hadamard code. The proposed scheme can achieve a performance only 0.44dB away from the ultimate Shannon limit with a modest decoding complexity. The applications of the new scheme in CDMA communication systems are currently under investigation.

© IEE 2000

Electronics Letters Online No: 20000114
DOI: 10.1049/el:20000114

10 November 1999

W.K. Leung, K.S. Ho and Li Ping (Department of Electronic Engineering, City University of Hong Kong, Hong Kong)

E-mail: celiping@cityu.edu.hk

References

- 1 BERROU, C., GLAVIEUX, A., and THITIMAJISHIMA, P.: 'Near Shannon limit error-correcting coding and decoding: turbo-codes'. IEEE Proc. ICC-93, 1993, Vol. 2, pp. 1064–1070
- 2 YING, L., and CHAN, S.: 'Iterative decoding of concatenated Hadamard codes'. IEEE Proc. ICC '98, 1998, Vol. 1, pp. 136–140
- 3 MacWILLIAMS, F.M., and SLOANE, N.J.A.: 'The theory of error-correcting codes' (North-Holland, Amsterdam, 1977), Vols. 1 and 2
- 4 BENEDETTO, S., and MONTORSI, G.: 'Unveiling turbo codes: some results on parallel concatenated coding schemes', IEEE Trans. Inf. Theory, 1996, 42, pp. 409–428
- 5 DIVSALAR, D., and POLLARA, F.: 'Multiple turbo codes'. IEEE Proc. Milcom-95, 1995, Vol. 1, pp. 279–285
- 6 PING, L., CHAN, S., and YEUNG, K.L.: 'Iterative decoding of multidimensional concatenated single parity check codes'. IEEE Proc. ICC '98, 1998, Vol. 1, pp. 131–135
- 7 PING, L.: 'Modified turbo codes with low decoding complexity', Electron. Lett., 1998, 23, pp. 2228–2229
- 8 ROBERTSON, P., VILIBRUN, E., and HOEHER, P.: 'A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain', IEEE Proc. ICC '95, 1995, Vol. 2, pp. 1009–1013
- 9 KOMULAINEN, P., and PIIRONEN, K.: 'Performance evaluation of superorthogonal turbo codes in AWGN and flat Rayleigh fading channels', IEEE J. Sel. Areas Commun., 1998, 16, pp. 196–205

Class of turbo code interleavers based on divisibility

F. Qi, M.Z. Wang, A.U.H. Sheikh and D.R. Shao

A class of interleavers for turbo codes is proposed. The interleavers possess properties that enable weight spectrum thinning for the resultant turbo codes. Simulation results show that turbo codes based on the proposed interleavers outperform those based on conventional interleavers in terms of the bit error rate.

Introduction: Turbo codes achieve near Shannon-limit error correction performance [1] and have been proposed for W-CDMA systems. The interleaver of a turbo code plays an important role in enhancing the bit error rate (BER) performance. This is because the interleaver structure affects the distance spectrum of the resulting turbo code. The main function of the interleaver is to permute the incoming information sequence with a low Hamming weight in