Multi-Layer Turbo Space-Time Codes

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***Abstract*— This letter describes a multi-layer turbo space-time coding scheme. Based on a carefully designed power allocation strategy, performance reasonably close to the theoretical limits can be achieved at a rate of two bits per channel use with very low complexity.**

***Index Terms*— Fading channels, iterative decoding, multiple antennas, space-time codes.**

I. INTRODUCTION

RECENTLY , much research effort has been devoted to space-time (ST) coding and significant progresses have been made [1]-[5]. However, approaching the theoretical limits [6] of multiple transmit antenna systems is still an open problem, especially for high-rate applications.

In this letter, we describe a multi-layer turbo ST coding scheme, in which interleaving is used as the only mechanism to separate signals from different antennas and layers. The proposed scheme, referred to as multi-layer interleave-division-multiplexing space-time (ML-IDM-ST) codes, is an improvement of the IDM-ST code discussed in [7]. Compared with the original IDM-ST code [7], the proposed code can achieve much higher bandwidth efficiency without increasing complexity. The key to the success is a carefully designed power allocation strategy. Simulation results show that performance reasonably close to the theoretical limits [6] can be achieved at a rate of two bits per channel use.

II. SYSTEM MODEL

Consider a system with N transmit antennas and one receive antenna (an $N\times 1$ system) in a quasi-static Rayleigh fading channel. Let $\alpha^{(n)}$ $(n = 1, \cdots, N)$ be the fading coefficient for the nth transmit antenna. In a quasi-static fading channel, $\boldsymbol{\alpha} \equiv \{\alpha^{(n)}\}$ remain unchanged during one frame and vary independently from frame to frame. We always assume no channel state information (CSI) at the transmitter and ideal CSI at the receiver.

A. Transmitter Principle

Fig. 1 shows the transmitter structure of a K-layer ML-IDM-ST code. The inputs are K equal-length sequences $\{\boldsymbol{d}_k, k = 1, \cdots, K\}$, each encoded individually using a binary forward error correction (FEC) code, generating $\boldsymbol{c}_k \equiv \{c_{k,i}\}$.

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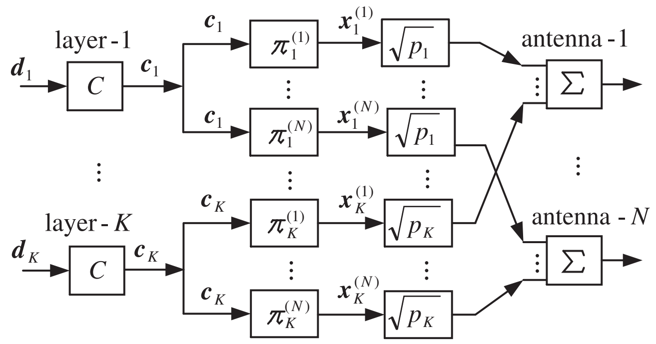

Fig. 1. The transmitter structure of a K-layer ML-IDM-ST code, where $\pi_k^{(n)}$ is the interleaver for layer-k on the nth transmit antenna.

Signals from the same encoder are referred to as a layer. We assume the same FEC code C for all layers.

For each layer-k, $\boldsymbol{c}_k$ is independently interleaved and modulated N times, producing $\{\boldsymbol{x}_k^{(n)}, n = 1, \cdots, N\}$. We first consider binary-phase-shift-keying (BPSK) signaling. In this case, each $\boldsymbol{x}_k^{(n)} \equiv \{x_{k,j}^{(n)}\}$ is a binary sequence, i.e., $x_{k,j}^{(n)} \in \{+1, -1\}$. These signals are then scaled by a common amplitude factor $\sqrt{p_k}$ (see Fig. 1) before distributed to the N antennas. For the nth transmit antenna, the transmitted signal is $\sum_{k=1}^{K} \sqrt{p_k}\boldsymbol{x}_k^{(n)}$. The signal received at time j is

$$y_j = \sum_{n=1}^{N} \alpha^{(n)} \sum_{k=1}^{K} \sqrt{p_k} x_{k,j}^{(n)} + n_j, \tag{1}$$

where $\{n_j\}$ are samples of an additive white Gaussian noise (AWGN) process with variance $\sigma^2 = N_0/2$. Denote by R_C the rate of C, and R the overall rate. We have $R = KR_C$.

A key property of the above scheme is the use of different power amplification factors $\{p_k, k = 1, \cdots, K\}$ for K layers. Notice that the concepts of "layering" here and "stacking" in [7] are similar, except that power distribution is not uniform among layers here.

B. Receiver Principle

At the receiver, we employ a sub-optimal iterative decoder [7], which consists of an elementary signal estimator (ESE) and K *a posteriori* probability (APP) decoders (DECs), operating iteratively [7]. Fig. 2 illustrates a part of the receiver structure in which only the DEC for layer-k (denoted by DEC-k) is shown. The DECs for other layers are connected to the ESE in the same way as DEC-k.

We rewrite (1) as

$$y_j = \alpha^{(n)} \sqrt{p_k} x_{k,j}^{(n)} + \zeta_{k,j}^{(n)}, \tag{2}$$

where

$$\zeta_{k,j}^{(n)} = \sum_{(n',k')\neq(n,k)} \alpha^{(n')}\sqrt{p_{k'}}x_{k',j}^{(n')} + n_j, \quad (3)$$

is the distortion in y_j with respect to $x_{k,j}^{(n)}$. Treat $\{x_{k,j}^{(n)}\}$ as independent and identically distributed random variables. Using the central limit theorem, $\zeta_{k,j}^{(n)}$ can be approximated by a Gaussian random variable, and $x_{k,j}^{(n)}$ can be estimated from (2) provided that the mean and variance of $\zeta_{k,j}^{(n)}$ are available. Denote by E(•) and Var(•) the mean and variance functions, respectively. The main decoding operations are listed as follows for BPSK signaling and real $\{\alpha^{(n)}\}$ [7]. (Refer to Fig. 2 for the notations involved.)

(a) Initialization: Set $\tilde{L}(x_{k,j}^{(n)}) = 0, \quad \forall k, n, j.$

(b) Main iteration:

$$\mathrm{E}(x_{k,j}^{(n)}) = \tanh(\tilde{L}(x_{k,j}^{(n)})/2), \quad \forall k, n, j. \quad (4a)$$

$$\mathrm{Var}(x_{k,j}^{(n)}) = 1 - (\mathrm{E}(x_{k,j}^{(n)}))^2, \quad \forall k, n, j. \quad (4b)$$

$$\mathrm{E}(y_j) = \sum_{n,k} \alpha^{(n)}\sqrt{p_k}\mathrm{E}(x_{k,j}^{(n)}), \quad \forall j. \quad (5a)$$

$$\mathrm{Var}(y_j) = \sum_{n,k} \left|\alpha^{(n)}\right|^2 p_k \mathrm{Var}(x_{k,j}^{(n)}) + \sigma^2, \quad \forall j. \quad (5b)$$

$$Ext(x_{k,j}^{(n)}) = \frac{2\alpha^{(n)}\sqrt{p_k}(y_j - \mathrm{E}(y_j) + \alpha^{(n)}\sqrt{p_k}\mathrm{E}(x_{k,j}^{(n)}))}{\mathrm{Var}(y_j) - \left|\alpha^{(n)}\right|^2 p_k \mathrm{Var}(x_{k,j}^{(n)})}, \quad \forall k,n,j. \quad (6)$$

$$\tilde{L}(c_{k,i}) = \sum_{(n,j)\in S(c_{k,i})} Ext(x_{k,j}^{(n)}), \quad \forall k, i. \quad (7)$$

where $S(c_{k,i})$ is the index set of N replicas in $\{x_{k,j}^{(n)}, \forall n, j\}$ related to $c_{k,i}$ (for all (n, j) combinations). Each DEC-k carries out an APP decoding at this stage using $\{\tilde{L}(c_{k,i}), \forall i\}$ as the input to produce *a posteriori* log-likelihood ratios (LLRs) $\{L(c_{k,i}), \forall i\}$ for $\boldsymbol{c}_k$, which are used to update $\{\tilde{L}(x_{k,j}^{(n)}), \forall n, j\}$ as

$$\tilde{L}(x_{k,j}^{(n)}) = L(c_{k,i}) - Ext(x_{k,j}^{(n)}), \quad \forall k, n, j. \quad (8)$$

Then go back to (4) for the next iteration. The cost per coded bit involved in (4) ~ (8) is independent of K and grows only linearly with N [7].

C. Complex Signaling

To increase spectral efficiency, each $x_{k,j}^{(n)}$ can carry two coded bits in the real and quadrature parts, respectively, so the overall rate doubles. We introduce a phase rotation for each layer to ensure evenly distributed interference among layers. The signal transmitted from the nth antenna is then

$$\sum_{k=1}^{K} e^{j\frac{k\pi}{2K}}\sqrt{p_k}\boldsymbol{x}_k^{(n)} \quad (9)$$

The decoding principle for complex signaling is a generalization of the procedure discussed above. The detail has been considered in [7][8] and we omit it here due to the space limitation.

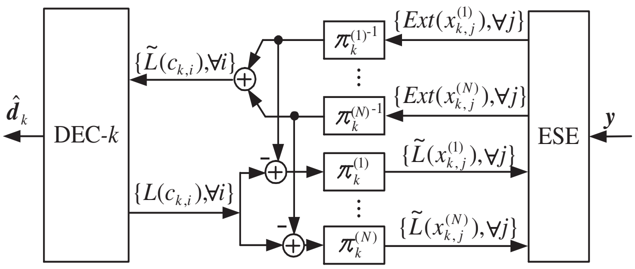

Fig. 2. A part of the receiver structure of the ML-IDM-ST code related to layer-k, where $\boldsymbol{y} = \{y_j\}$, $\pi_k^{(n)}$ and $\pi_k^{(n)^{-1}}$ are the interleaver and deinterleaver for layer-k on the nth transmit antenna. The DECs for other layers are connected to the ESE in the same way as DEC-k.

D. Power Allocation

Motivated by the work on power allocation for layered ST coding systems [9], we now consider introducing proper power allocation. We observed that when $P \equiv \sum_k p_k$ is fixed, the performance of the system in Fig. 1 is a function of the distribution of $\{p_k\}$. The intuition is that, with high probabilities, strong signals can be correctly detected first and their interference to weak signals can be correctly cancelled, which in turn facilitates the detection of weak signals. Thus an optimized set of $\{p_k\}$ can potentially improve the overall performance.

To reduce computational cost, we adopt the following simple (but sub-optimal) recursive search procedure. (More complicated power allocation techniques have been proposed in [10][11].) Suppose that we have obtained a sub-optimal solution for a (K-1)-layer system, denoted by $\{p_1^{(K-1)}, p_2^{(K-1)}, \ldots, p_{K-1}^{(K-1)}\}$. We then consider introducing an extra layer-K and finding $\{p_1^{(K)}, p_2^{(K)}, \ldots, p_K^{(K)}\}$ for the resultant K-layer system. We assume that the relative power ratios for the lower $K-1$ layers are preserved after layer-K is introduced, i.e., $p_k^{(K)} = \beta p_k^{(K-1)}$ for $k = 1, \ldots, K-1$. Then we only need to search for the best $(\beta, p_K^{(K)})$ pair for the K-layer system, which greatly reduces the searching cost. Some simulation-based searching results are provided below.

E. Examples

We call each $\boldsymbol{d}_k$ a frame and $\{\boldsymbol{d}_k, k = 1, \ldots, K\}$ a super-frame. The frame error rate (FER) and super-frame error rate (SFER) are defined accordingly. Clearly, SFER≥FER.

Consider a ML-IDM-ST coding system employing a rate-1/3 turbo code with generator $G(x)$ = (1+x+x^3)/(1+x^2+x^3) for all layers. Fig. 3 shows the FER and SFER performance of this system with complex signaling, R = 2 bits per channel use (corresponding to K = 3) and N = 2 and 4. The number of information bits per layer per frame = 4096, and the frame size = 4096/(2×R_C) = 6144. The power levels of different layers obtained through searching (by simulation) are listed in Table 1. Curves of outage probabilities [6][12] are included for reference, which are theoretical limits of SFER. In practice, FER can be a more useful performance measurement, as in case of error it is only necessary to discard the erroneous frames, instead of a complete super-frame. It is observed that the FER curves in Fig. 3 are quite close to the curves of the

TABLE I
POWER LEVELS ALLOCATED TO DIFFERENT LAYERS, WHERE P IS THE TOTAL POWER.

N	Layer		
	1	2	3
2	$0.538P$	$0.3003P$	$0.1617P$
4	$0.528P$	$0.298304P$	$0.173696P$

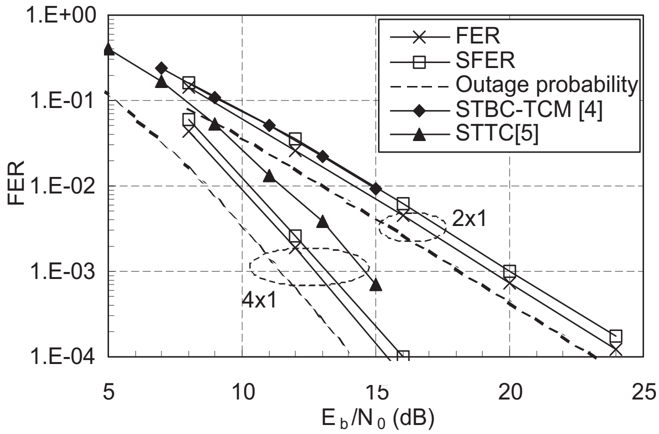

Fig. 3. FER and SFER performance of 2×1 and 4×1 ML-IDM-ST codes with complex signaling over a quasi-static complex Rayleigh fading channel. A rate-1/3 turbo code with generator $G(x) = (1 + x + x^3)/(1 + x^2 + x^3)$ is used for all layers. The number of information bits per layer per frame = 4096, iteration number = 30, K = 3 and R = 2 bits per channel use. The curves of outage probabilities [6][12] are included. The simulation results of a 16-state serially concatenated STBC-TCM code [4] and a 64-state STTC [5] are also included.

corresponding theoretical limits (within 1.1 dB). The SFER curves are about 0.5 dB worse than the corresponding FER curves. Statistically, we observed that the layers with lower power levels have higher frame error rates.

In Fig. 3, we have also included the performance curves of a 2×1 concatenated ST code [4] and a 4×1 64-state ST trellis code (STTC) [5]. The concatenated ST code [4] is constructed by serially concatenating an orthogonal ST block code (STBC) [2] and a 16-state rate-2/3 8PSK trellis-coded modulation (TCM) code [13]. The STTC [5] is designed based on a search to maximize the minimum square Euclidean distance between any pair of codewords. To the best knowledge of the authors, these two codes demonstrate the best performance for 2×1 and 4×1 systems with R = 2. It is seen that the SFER of the proposed code is similar to the performance of the STBC-TCM [4] with N = 2, and about 1.3 dB better than the performance of the STTC [5] with N = 4.

III. DISCUSSIONS

A key ingredient for IDM-ST codes is the use of random interleavers, which avoids the sophisticated methodology required by other alternative methods (based on, for example, algebraic or trellis structures). This approach has several features, such as flexibility regarding the rate and the number of transmit antennas, natural integration with turbo or LDPC coding [7] and simple treatment for multipath effects (see [14] for a detailed discussion).

In this letter, we have shown that the transmission rate of the IDM-ST codes in [7] can be enhanced with a multi-layer structure and a power allocation strategy. Our work is experimental and it demonstrates the efficiency of the random-interleaver-based approach. Some detailed analysis results are proposed in [10], which provides more solid understanding and more efficient power allocation strategies for the proposed scheme.

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