Approximate MMSE-APP Estimation for Linear Systems with Binary Inputs

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***Abstract*— This letter presents an equivalent form of the Wang-Poor algorithm. A single matrix equation is used to generate all of the estimates simultaneously, while in the Wang-Poor algorithm a recursive technique is used to generate these estimates one by one. The new method is mathematically more compact and computationally more efficient.**

***Index Terms*— Iterative detection, MMSE estimation, multi-user detection, turbo detection.**

I. Introduction

THIS letter is concerned with the linear real matrix system,

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{b} + \boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{r}$ is an observation vector, $\boldsymbol{H}$ a known matrix, $\boldsymbol{b}$ an length-K input vector with entries $\{b[k] \in \{+1, -1\}\}$, and $\boldsymbol{\eta}$ an additive noise vector. The discussion can be extended to complex matrix systems since any complex matrix equation has an equivalent real representation obtained by equating the real and imaginary parts separately.

It is well-known that the optimal *a posteriori* minimum mean square error (MMSE) estimator for $\boldsymbol{b}$ based on the observation $\boldsymbol{r}$ is the conditional mean $\mathrm{E}(\boldsymbol{b}|\boldsymbol{r})$ [1]. If the distributions of $\boldsymbol{b}$ and $\boldsymbol{\eta}$ are both Gaussian, $\mathrm{E}(\boldsymbol{b}|\boldsymbol{r})$ is the same as the linear MMSE (LMMSE) estimator [1]. However, in many communication applications, the entries of $\boldsymbol{b}$ are binary and then the LMMSE estimator is not optimal. In this case, we can first estimate the log likelihood ratios (LLRs) for the elements of $\boldsymbol{b}$ and then use these to compute $\mathrm{E}(\boldsymbol{b}|\boldsymbol{r})$ (see below). The BCJR algorithm [2] can be applied for the LLR evaluation if the system in (1) can be described by a trellis diagram, but the complexity involved is usually very high. The Wang-Poor algorithm [3] is a well-known approximate solution with a complexity much lower than the BCJR algorithm. In the following, we develop an equivalent form of the Wang-Poor algorithm. The new method is mathematically more compact and computationally more efficient.

II. Approximate Minimum Mean Square Error (MMSE) Estimation

A. MMSE and LLR Estimators for a Scalar System

Consider, initially, the trivial case where (1) reduces to the scalar equation: $r = hb + \eta$, with $b \in \{+1, -1\}$. We can evaluate $\mathrm{E}(b|r)$ by first computing the LLR for b based on Bayes Theorem,

$$\mathrm{L}(b|r) \equiv \log\frac{\Pr(b=+1|r)}{\Pr(b=-1|r)} = \log\frac{\Pr(b=+1)_{apriori}}{\Pr(b=-1)_{apriori}} + \lambda \tag{2}$$

where $\Pr(b = \pm 1)_{apriori}$ are *a priori* probabilities and are assumed to be known. In (2), λ is referred to as an extrinsic LLR. When η is a Gaussian random variable independent to b,

$$\lambda \equiv \log\frac{p(r|b=+1)}{p(r|b=-1)} = \frac{2h}{\mathrm{Var}(\eta)}(r - \mathrm{E}(\eta)). \tag{3}$$

Using $\mathrm{L}(b|r)$ we can then compute $\mathrm{E}(b|r)$ as

$$\mathrm{E}(b|r) = \Pr(b=+1|r) - \Pr(b=-1|r) = \frac{e^{\mathrm{L}(b|r)} - 1}{e^{\mathrm{L}(b|r)} + 1}. \tag{4}$$

B. Approximate a Posteriori LLR Evaluation

Consider now the general situation described by (1). The LMMSE estimator [1] for $\boldsymbol{b}$ based on $\boldsymbol{r}$ is defined as

$$\hat{\boldsymbol{b}} = \tilde{\boldsymbol{b}} + \boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1}(\boldsymbol{r} - \boldsymbol{H}\tilde{\boldsymbol{b}} - \tilde{\boldsymbol{\eta}}) \tag{5}$$

where $\tilde{\boldsymbol{b}}$ is the *a priori* mean of $\boldsymbol{b}$, $\tilde{\boldsymbol{\eta}}$ the *a priori* mean of $\boldsymbol{\eta}$, $\boldsymbol{C}_{br} \equiv \mathrm{Cov}(\boldsymbol{b}, \boldsymbol{r})$ and $\boldsymbol{C}_{rr} \equiv \mathrm{Cov}(\boldsymbol{r}, \boldsymbol{r})$. We assume that $\tilde{\boldsymbol{b}}$, $\boldsymbol{C}_{br}$ and $\boldsymbol{C}_{rr}$ are all known (see Appendix I), so that (5) can be readily evaluated. Since the entries of $\boldsymbol{b}$ are not Gaussian, $\hat{\boldsymbol{b}}$ is not the optimal MMSE solution. However, following the principle in [3], we will consider using $\hat{\boldsymbol{b}}$ to generate an approximate solution.

Rewrite (5) in signal plus interference-noise form as

$$\hat{\boldsymbol{b}} = \boldsymbol{\Phi}\boldsymbol{b} + \boldsymbol{\xi} \tag{6}$$

where $\boldsymbol{\Phi}$ is a diagonal matrix. The expressions for $\boldsymbol{\Phi}$ and $\boldsymbol{\xi}$ are given in Appendix I. The row-by-row form of (6) is

$$\hat{b}[k] = \Phi[k,k]b[k] + \xi[k], \qquad k = 1, 2, \cdots, K \tag{7}$$

where $\Phi[k,k]$ is the (k,k)th entry of $\boldsymbol{\Phi}$. In (7), $\xi[k]$ can be regarded as a noise-plus-interference term with respect to $b[k]$. We assume that $b[k]$ and $\xi[k]$ are independent to each other and that the elements in $\boldsymbol{b}$ are mutually independent. Similar to [3], we use $\hat{b}[k]$ to estimate $b[k]$. Applying (2) and (3) to (7), we have

Manuscript received May 7, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. Paul Cotae. This work was fully supported by a grant from the Research Grant Council of the Hong Kong SAR, China (Project No. CityU 1010/01E).

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Digital Object Identifier 10.1109/LCOMM.2005.02021.

$$\mathrm{L}(b[k]|\hat{b}[k]) \equiv \log\frac{\Pr(b[k]=+1)_{apriori}}{\Pr(b[k]=-1)_{apriori}} + \lambda[k], \tag{8a}$$

$$\lambda[k] \equiv \frac{2\Phi[k,k]}{\mathrm{Var}(\xi[k])}(\hat{b}[k] - \mathrm{E}(\xi[k])). \tag{8b}$$

Equation (4) can then be computed. (In some applications, such as multi-user detection [3], the $\{\lambda[k]\}$ are used directly in the iterative process.)

Denote $\boldsymbol{\lambda} = [\lambda[1], \lambda[2], \cdots, \lambda[K]]^T$. Equation (8b) can be rewritten as

$$\boldsymbol{\lambda} = 2(\mathrm{Cov}(\boldsymbol{\xi},\boldsymbol{\xi})_{diag})^{-1}\boldsymbol{\Phi}(\hat{\boldsymbol{b}} - \mathrm{E}(\boldsymbol{\xi})) \tag{9}$$

where $\mathrm{Cov}(\boldsymbol{\xi},\boldsymbol{\xi})_{diag}$ is the matrix obtained by setting all the entries of $\mathrm{Cov}(\boldsymbol{\xi},\boldsymbol{\xi})$ to zero except those on the diagonal line. The following is the main result of this letter. Its proof is given in Appendix I.

Method 1: $\boldsymbol{\lambda} = ((\boldsymbol{Q}^{-1})_{diag})^{-1}\boldsymbol{Q}^{-1}(\boldsymbol{\alpha}+\boldsymbol{\beta})-\boldsymbol{\beta}$ (10)

with

$$\boldsymbol{\alpha} \equiv 2\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}(\boldsymbol{r} - \tilde{\boldsymbol{\eta}}),$$
$$\boldsymbol{\beta} \equiv 2\boldsymbol{C}_{bb}^{-1}\tilde{\boldsymbol{b}},$$
$$\boldsymbol{Q} \equiv \boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H} + \boldsymbol{C}_{bb}^{-1},$$
$$\boldsymbol{C}_{bb} \equiv \mathrm{Cov}(\boldsymbol{b},\boldsymbol{b}),$$
$$\boldsymbol{C}_{\eta\eta} \equiv \mathrm{Cov}(\boldsymbol{\eta},\boldsymbol{\eta}).$$

C. Equivalence to the Wang-Poor Algorithm [3]

Let $\boldsymbol{C}_{\eta\eta} = \sigma^2\boldsymbol{I}$ and $\tilde{\boldsymbol{\eta}} = \boldsymbol{0}$ and use the following notations as in [3].

- Let $\boldsymbol{V}_k$ be obtained from $\boldsymbol{C}_{bb}$ by replacing its (k,k)th entry $C_{bb}[k,k]$ by 1.
- Let $\tilde{\boldsymbol{b}}_k$ be obtained from $\tilde{\boldsymbol{b}}$ by replacing its kth entry $\tilde{b}[k]$ by 0.
- Let $\boldsymbol{Q}_k = \boldsymbol{R}\sigma^{-2} + \boldsymbol{V}_k^{-1}$, where $\boldsymbol{R} \equiv \boldsymbol{H}^T\boldsymbol{H}$.

An alternative expression for $\boldsymbol{\lambda} = \{\lambda[k]\}$ is given in (11) below. The derivation is given in Appendix II. It can be shown that (11) is equivalent to the Wang-Poor Algorithm given by (50) in [3].

Method 2 (Wang-Poor Algorithm):

$$\lambda[k] = 2\cdot\frac{z[k]}{1-\mu[k]}, \qquad \forall k \tag{11}$$

where

$$z[k] \equiv \boldsymbol{w}_k^T(\boldsymbol{y} - \boldsymbol{R}\tilde{\boldsymbol{b}}_k),$$
$$\mu[k] \equiv \boldsymbol{w}_k^T\boldsymbol{R}\boldsymbol{e}_k,$$
$$\boldsymbol{y} \equiv \boldsymbol{H}^T\boldsymbol{r},$$
$$\boldsymbol{w}_k^T \equiv \boldsymbol{e}_k^T\boldsymbol{Q}_k^{-1}\sigma^{-2}$$

and $\boldsymbol{e}_k$ is the kth column of the unit matrix.

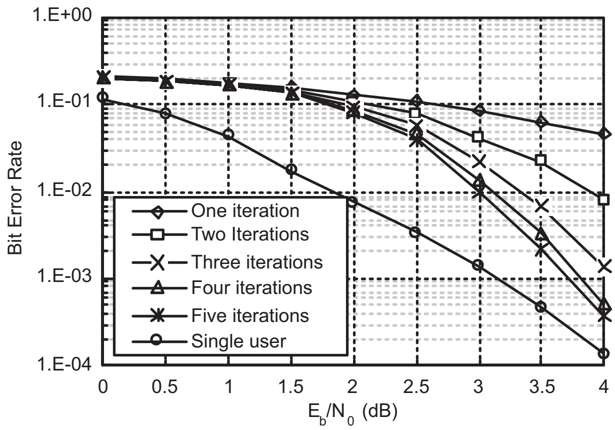

Fig. 1. Performance of a turbo multiuser receiver employing the proposed algorithm.

III. Discussions and Conclusions

For the Wang-Poor algorithm in (11), a different set of matrix operations and vectors are required to generate each $\lambda[k]$. In particular, $\boldsymbol{Q}_k$ is different for different k and so K matrix inverses $\{\boldsymbol{Q}_k^{-1}\}$ are required. An efficient technique is derived in [3] to generate the inverses recursively. The related cost is approximately $2K^3$ FLOPs, where a FLOP involves an addition and a multiplication. (Note that for a multiuser detection application [3], the cost here is for all K users.) For the proposed method using (10), the complexity is dominated by the matrix inversion of $\boldsymbol{Q}$, costing K^3 FLOPs [4]. This is half of the complexity using (11). Equation (10) is also more compact than (11) from a programming point of view.

To verify the proposed method, we consider a multi-user detection system with K users for which the kth column of matrix $\boldsymbol{H}$ in (1) is the signature sequence for user-k. Let $\boldsymbol{r}$ in (1) and $\boldsymbol{y} = \boldsymbol{H}^T\boldsymbol{r}$ represent the signals before and after the matched filtering, respectively. The system equation (1) can be alternatively written as

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{b} + \boldsymbol{n} \tag{12}$$

where $\boldsymbol{R} \equiv \boldsymbol{H}^T\boldsymbol{H}$ and $\boldsymbol{n} \equiv \boldsymbol{H}^T\boldsymbol{\eta}$. The above is equivalent to (21) in [3] (after setting the gain matrix $\boldsymbol{A}$ in [3] to unit). We adopt the same settings as those for Fig. 3 in [3] with equal cross-correlation coefficients (= 0.7). The simulation results given in Fig. 1 using (10) agree well with Fig. 3 in [3] obtained using the Wang-Poor algorithm, which is as expected, since the two methods are mathematically equivalent.

In conclusion, we have derived an alternative approach to evaluate the approximate extrinsic LLRs for linear systems with binary input. The new method is computationally more efficient than the well-known Wang-Poor algorithm. It can be applied to multi-user detection [3] and other applications [5].

Appendix I
Proof of Method 1

The covariance matrices in (5) can be obtained from (1) as

$$\boldsymbol{C}_{br} = \boldsymbol{C}_{bb}\boldsymbol{H}^T, \tag{13}$$

$$\boldsymbol{C}_{rr} = \boldsymbol{H}\boldsymbol{C}_{bb}\boldsymbol{H}^T + \boldsymbol{C}_{\eta\eta}. \tag{14}$$

We assume that $\boldsymbol{C}_{bb}$, $\boldsymbol{C}_{\eta\eta}$ and $\tilde{\boldsymbol{b}}$ are known and $\boldsymbol{C}_{\eta\eta}^{-1}$ and $\boldsymbol{C}_{bb}^{-1}$ exist. It can be verified that

$$\boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1} = \boldsymbol{Q}^{-1}\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}, \tag{15a}$$

$$\boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1}\boldsymbol{H} = \boldsymbol{Q}^{-1}\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H} = \boldsymbol{I} - \boldsymbol{Q}^{-1}\boldsymbol{C}_{bb}^{-1}, \tag{15b}$$

$$\text{where} \quad \boldsymbol{Q} \equiv \boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H} + \boldsymbol{C}_{bb}^{-1}. \tag{15c}$$

We can then rewrite (5) as

$$\hat{\boldsymbol{b}} = \boldsymbol{Q}^{-1}(\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}(\boldsymbol{r} - \tilde{\boldsymbol{\eta}}) + \boldsymbol{C}_{bb}^{-1}\tilde{\boldsymbol{b}}). \tag{16}$$

Notice that (16) can be expanded using (1) as:

$$\hat{\boldsymbol{b}} = \boldsymbol{Q}^{-1}(\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H}\boldsymbol{b} + \boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + \boldsymbol{C}_{bb}^{-1}\tilde{\boldsymbol{b}}). \tag{17}$$

Define $\boldsymbol{\Phi} \equiv (\boldsymbol{Q}^{-1}\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H})_{diag}$ and $\boldsymbol{\xi} \equiv \boldsymbol{Q}^{-1}(\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H}(\boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}) + \boldsymbol{C}_{bb}^{-1}\tilde{\boldsymbol{b}})$. We can rewrite (17) in the form of (6): $\hat{\boldsymbol{b}} = \boldsymbol{\Phi}\boldsymbol{b} + \boldsymbol{\xi}$. It is easy to show [1] that $\mathrm{E}(\hat{\boldsymbol{b}}) = \tilde{\boldsymbol{b}}$ and so

$$\mathrm{E}(\boldsymbol{\xi}) = \mathrm{E}(\hat{\boldsymbol{b}} - \boldsymbol{\Phi}\boldsymbol{b}) = (\boldsymbol{I} - \boldsymbol{\Phi})\tilde{\boldsymbol{b}}. \tag{18}$$

Recall the assumption in II.B that $b[k]$ and $\xi[k]$ are independent to each other and take the covariance of both sides of (6). The diagonal entries of these covariance matrices give

$$\mathrm{Cov}(\hat{\boldsymbol{b}}, \hat{\boldsymbol{b}})_{diag} = \mathrm{Cov}(\boldsymbol{\Phi}\boldsymbol{b}, \boldsymbol{\Phi}\boldsymbol{b})_{diag} + \mathrm{Cov}(\boldsymbol{\xi}, \boldsymbol{\xi})_{diag}. \tag{19}$$

Using the expression of $\hat{\boldsymbol{b}}$ in (5), we have

$$\mathrm{Cov}(\hat{\boldsymbol{b}}, \hat{\boldsymbol{b}}) = \mathrm{E}(\boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1}(\boldsymbol{r} - \mathrm{E}(\boldsymbol{r}))(\boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1}(\boldsymbol{r} - \mathrm{E}(\boldsymbol{r})))^T)$$

$$= \boldsymbol{C}_{br}\boldsymbol{C}_{rr}^{-1}\boldsymbol{C}_{br}^T. \tag{20}$$

Substituting (13), (15b) and (15c) into (20),

$$\mathrm{Cov}(\hat{\boldsymbol{b}}, \hat{\boldsymbol{b}})_{diag} = (\boldsymbol{Q}^{-1}\boldsymbol{H}^T\boldsymbol{C}_{\eta\eta}^{-1}\boldsymbol{H}\boldsymbol{C}_{bb})_{diag} = \boldsymbol{\Phi}\boldsymbol{C}_{bb}. \tag{21}$$

Noting that $\mathrm{Cov}(\boldsymbol{\Phi}\boldsymbol{b}, \boldsymbol{\Phi}\boldsymbol{b})_{diag} = \boldsymbol{\Phi}^2\boldsymbol{C}_{bb}$, we can rewrite (19) as $\boldsymbol{\Phi}\boldsymbol{C}_{bb} = \boldsymbol{\Phi}^2\boldsymbol{C}_{bb} + \mathrm{Cov}(\boldsymbol{\xi}, \boldsymbol{\xi})_{diag}$ or

$$(\mathrm{Cov}(\boldsymbol{\xi}, \boldsymbol{\xi})_{diag})^{-1}\boldsymbol{\Phi} = (\boldsymbol{I} - \boldsymbol{\Phi})^{-1}\boldsymbol{C}_{bb}^{-1}. \tag{22}$$

Substituting (15b) into the definition of $\boldsymbol{\Phi}$, we have $\boldsymbol{\Phi} = (\boldsymbol{I} - \boldsymbol{Q}^{-1}\boldsymbol{C}_{bb}^{-1})_{diag}$, or equivalently,

$$\boldsymbol{I} - \boldsymbol{\Phi} = (\boldsymbol{Q}^{-1}\boldsymbol{C}_{bb}^{-1})_{diag}. \tag{23}$$

Combining (22) and (23), we have

$$(\mathrm{Cov}(\boldsymbol{\xi}, \boldsymbol{\xi})_{diag})^{-1}\boldsymbol{\Phi} = ((\boldsymbol{Q}^{-1})_{diag})^{-1}. \tag{24}$$

From our earlier assumption that the elements in $\boldsymbol{b}$ are mutually independent, $\boldsymbol{C}_{bb}$ is a diagonal matrix. Then (23) leads to

$$((\boldsymbol{Q}^{-1})_{diag})^{-1}(\boldsymbol{I} - \boldsymbol{\Phi}) = \boldsymbol{C}_{bb}^{-1}. \tag{25}$$

Equation (10) follows from substituting (16), (18), (24) and (25) into (9).

APPENDIX II
PROOF OF METHOD 2

We first prove a useful result.

Lemma: The kth entry of $\boldsymbol{\lambda}$ in (11) is not a function of $\beta[k]$ and $Q[k,k]$.

Proof: We write the kth entry of $\boldsymbol{\lambda}$ as

$$\lambda[k] = 2\boldsymbol{e}_k^T(((\boldsymbol{Q}^{-1})_{diag})^{-1}\boldsymbol{Q}^{-1}(\boldsymbol{\alpha} + \boldsymbol{\beta}) - \boldsymbol{\beta}), \tag{26}$$

where $\boldsymbol{e}_k$ is the kth column of the unit matrix. Since the (k,k)th entry of $((\boldsymbol{Q}^{-1})_{diag})^{-1}\boldsymbol{Q}^{-1}$ is 1, the contribution of $\beta[k]$ to $\lambda[k]$ is canceled in (26). From Cramer's rule, the (k,i)th entry of $\boldsymbol{Q}^{-1}$ is $(-1)^{i+k}\det(\boldsymbol{M}_{i,k})/\det(\boldsymbol{Q})$ where $\boldsymbol{M}_{i,k}$ is obtained from $\boldsymbol{Q}$ by deleting its ith row and kth column, so $\{\boldsymbol{M}_{i,k}, \forall i\}$ are not functions of $Q[k,k]$. The ith entry of $\boldsymbol{e}_k^T((\boldsymbol{Q}^{-1})_{diag})^{-1}\boldsymbol{Q}^{-1}$ is given by

$$(-1)^{k+k}\frac{\det(\boldsymbol{Q})}{\det(\boldsymbol{M}_{k,k})} \times (-1)^{i+k}\frac{\det(\boldsymbol{M}_{i,k})}{\det(\boldsymbol{Q})} = (-1)^{i+k}\frac{\det(\boldsymbol{M}_{i,k})}{\det(\boldsymbol{M}_{k,k})}, \quad \forall i$$

which is not a function of $Q[k,k]$ either. ∎

The above lemma indicates that we can freely alter the values of $\beta[k]$ and $Q[k,k]$ in (26) without affecting $\lambda[k]$. In particular, substituting $\boldsymbol{C}_{bb}^{-1}$ by $\boldsymbol{V}_k^{-1}$, $\boldsymbol{Q}$ by $\boldsymbol{Q}_k$, and $\tilde{\boldsymbol{b}}$ by $\tilde{\boldsymbol{b}}_k$ into (26) and noting that (see II.C) $\boldsymbol{C}_{\eta\eta} = \sigma^2\boldsymbol{I}$, $\tilde{\boldsymbol{\eta}} = \boldsymbol{0}$ and $\boldsymbol{e}_k^T\boldsymbol{V}_k^{-1}\tilde{\boldsymbol{b}}_k = \boldsymbol{e}_k^T\tilde{\boldsymbol{b}}_k$ leads to

$$\lambda[k] = 2 \cdot \frac{\boldsymbol{e}_k^T(\sigma^{-2}\boldsymbol{Q}_k^{-1}\boldsymbol{y} - (\boldsymbol{I} - \boldsymbol{Q}_k^{-1}\boldsymbol{V}_k^{-1})\tilde{\boldsymbol{b}}_k)}{\boldsymbol{e}_k^T\boldsymbol{Q}_k^{-1}\boldsymbol{e}_k}. \tag{27}$$

Using the definition of $\boldsymbol{Q}_k$ in II.C, we have $\boldsymbol{I} - \boldsymbol{Q}_k^{-1}\boldsymbol{V}_k^{-1} = \sigma^{-2}\boldsymbol{Q}_k^{-1}\boldsymbol{R}$ and (since $\boldsymbol{V}_k\boldsymbol{e}_k = \boldsymbol{e}_k$)

$$\boldsymbol{e}_k^T\boldsymbol{Q}_k^{-1}\boldsymbol{e}_k = \boldsymbol{e}_k^T(\boldsymbol{Q}_k^{-1}\boldsymbol{V}_k^{-1})\boldsymbol{e}_k = \boldsymbol{e}_k^T(\boldsymbol{I} - \sigma^{-2}\boldsymbol{Q}_k^{-1}\boldsymbol{R})\boldsymbol{e}_k = 1 - \mu[k].$$

Substituting these into (27) we arrive at (11):
$\lambda[k] = 2 \cdot z[k]/(1 - \mu[k])$.

ACKNOWLEDGMENT

The author wishes to thank Mr. Lihai Liu and Mr. Xiaojun Yuan for their invaluable help.

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