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## DOWNLINK PERFORMANCE OF RANDOMIZED OFDM ACCESS

**Abstract.** This paper is concerned with the downlink design of mobile communication systems using R-OFDM. A noticeable feature of R-OFDM is that the intra-cell interference is eliminated and the inter-cell interference can be approximately treated as Gaussian noise. Simulation results show that R-OFDM combined with space-time coded transmit diversity can achieve significant downlink capacity gain.

### 1. INTRODUCTION

In [1][2], a randomized orthogonal frequency division multiplexing (R-OFDM) technique is proposed to alleviate the multiple access interference (MAI) problem in multi-cell wireless communication systems employing OFDM technique. In R-OFDM, a pair of pseudo-random scrambling and de-scrambling operations is inserted at the transmitter and receiver, respectively. They are designed to mitigate the inter-cell interference while maintaining the intra-cell orthogonality among OFDM sub-carriers. Such mitigation results from the randomization effect as the scrambling and de-scrambling of different cells do not cancel each other but, rather, lead to Gaussian-noise-like inter-cell interference behavior. The scrambling and de-scrambling are realized as convolutional operations that are commutative with the multipath channel. This ensures that the above-mentioned properties of R-OFDM can be preserved in a fading environment.

The work in [1][2] is mainly concerned with uplink. This paper presents the downlink design for cellular R-OFDM systems. The downlink OFDM schemes have been extensively studied by many authors. Recently, it has been shown that space-time coded transmit diversity [3] provides an efficient technique to combat the fading effect in downlink. In this paper, we will show that with appropriate error control codes and space-time codes, R-OFDM can achieve significantly improved spectral efficiency, compared with both ordinary OFDM and random waveform CDMA systems.

### 2. INTERFERENCE CHARACTERIZATION OF R-OFDM

#### 2.1. The Transmission Principle of R-OFDM

The R-OFDM transmission principle [2] is illustrated in Figure 1 and Figure 2.

The input is delivered in a framed structure  $\{\mathbf{u}^{(j)}, j = 0, 1, \dots, J-1\}$ . Each  $\mathbf{u}^{(j)}$  is a sequence of  $M$  input symbols. An inverse *DFT* (Discrete Fourier Transform), denoted by  $F^{-1}$ , is applied to each  $\mathbf{u}^{(j)}$ , producing

*T.H.E. K. Fazel and S. Kaiser (ed.), Multi-Carrier Spread-Spectrum & Related Topics, 3<sup>rd</sup> ed.*  
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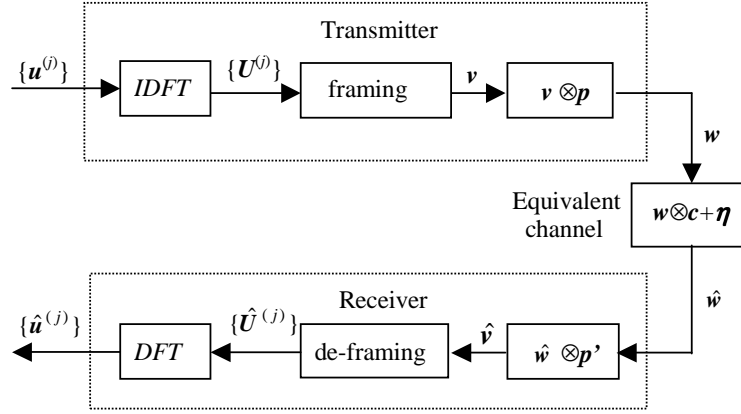


Figure 1. An illustration of the transmission principle of R-OFDM.  
 “ $\otimes$ ” denote convolution operations.

$$\mathbf{U}^{(j)} = F^{-1}(\mathbf{u}^{(j)}) \quad (1)$$

Each  $\mathbf{U}^{(j)}$  is referred to as a  $\mathbf{U}$ -frame and a cyclic prefix of length  $D$  is padded to every  $\mathbf{U}$ -frame. A total of  $J$  such  $\mathbf{U}$ -frames are collected to form a  $\mathbf{v}$ -frame, together with a pilot frame  $\mathbf{P}$ , Figure 2. The pilot frame is used to provide phase reference for the detection of remaining  $\mathbf{U}$ -frames multiplexed in  $\mathbf{v}$ . Thus a  $\mathbf{v}$ -frame is formed by

$$\mathbf{v} = \{\mathbf{P}, \mathbf{U}^{(0)}, \mathbf{U}^{(1)}, \dots, \mathbf{U}^{(J-1)}\} \quad (2)$$

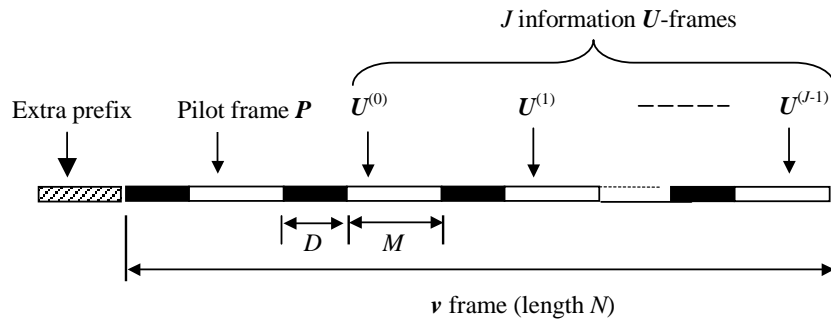


Figure 2. Structure of a  $\mathbf{v}$ -frame in R-OFDM. The black segments indicate cyclic prefixes.

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A randomization operation is applied to  $\mathbf{v}$ , realized in a convolution form as

$$\mathbf{w} = \mathbf{v} \otimes \mathbf{p} \quad (3)$$

where symbol “ $\otimes$ ” denotes a cyclic convolution. An extra cyclic prefix is padded to each  $\mathbf{w}$ -frame before transmission. The use of an extra prefix mentioned above is to transform the sliding convolution of the multipath channel into a form of cyclic convolution. As a result [1], at the receiver, the received signal, after stripping off the extra prefix, can be modeled as

$$\hat{\mathbf{w}} = \mathbf{w} \otimes \mathbf{c} + \boldsymbol{\eta} \quad (4)$$

where  $\mathbf{c}$  is a multipath reflection coefficient vector and  $\boldsymbol{\eta}$  contains the additive Gaussian noise samples.

### 2.2. Intra-cell Transmission Characteristics for R-OFDM

A de-randomization operation is then performed as follows

$$\hat{\mathbf{v}} = \hat{\mathbf{w}} \otimes \mathbf{p}' \quad (5)$$

where  $\mathbf{p}'$  is a random sequence. For the same-cell users, a crucial constraint is imposed on  $\mathbf{p}$  and  $\mathbf{p}'$  as <sup>1</sup>,

$$\mathbf{p} \otimes \mathbf{p}' = \boldsymbol{\delta} = \{1, 0, 0, \dots, 0\} \quad (6)$$

Based on this, since the convolutions involving  $\mathbf{c}$ ,  $\mathbf{p}$ , and  $\mathbf{p}'$  are commutative, the effects of the two convolutions involving  $\mathbf{p}$  and  $\mathbf{p}'$  cancel each other. The resultant  $\hat{\mathbf{v}}$  can be decomposed corresponding to (2) as

$$\hat{\mathbf{v}} = \{\hat{\mathbf{P}}, \hat{\mathbf{U}}^{(0)}, \hat{\mathbf{U}}^{(1)}, \dots, \hat{\mathbf{U}}^{(J-1)}\} \quad (7)$$

Due to the use of prefixes <sup>2</sup>, it can be verified that each  $\hat{\mathbf{U}}^{(j)}$  can be expressed as

$$\hat{\mathbf{U}}^{(j)} = \mathbf{U}^{(j)} \otimes \mathbf{c} + \boldsymbol{\xi} \quad (8)$$

where  $\boldsymbol{\xi}$  is a noise vector. Finally, a *DFT* is applied to each  $\hat{\mathbf{U}}^{(j)}$  to obtain

$$\hat{\mathbf{u}}^{(j)} = F(\hat{\mathbf{U}}^{(j)}) \quad (9)$$

where  $F(\cdot)$  represents the *DFT*. Substituting (1) and (8) into (9), we arrive at the following end-to-end relationship,

$$\hat{\mathbf{u}}^{(j)} = F(\mathbf{c}) \circ \mathbf{u}^{(j)} + F(\boldsymbol{\xi}) \quad (10)$$

where operation “ $\circ$ ” denotes symbol-by-symbol multiplication. It can be seen that equation (10) is equivalent to a conventional OFDM system, which implies orthogonality among the elements of  $\hat{\mathbf{u}}^{(j)}$ . Clearly,  $\hat{\mathbf{u}}^{(j)}$  can be used to estimate  $\mathbf{u}^{(j)}$  provided that  $\mathbf{c}$  is known.

### 2.3. Inter-cell Transmission Characteristics for R-OFDM

In a multiple cell system, the signal from a cell site is the interference for users within other cells. In this case, we assume that  $\mathbf{p}' \neq \mathbf{p}^H$  (see footnote 1) in Figure 1 and, in general, they are approximately random to each other. It can be shown that this leads to Gaussian-type characteristics of inter-cell interference for R-OFDM (see the histogram in [2]).

The Gaussian-like behavior of inter-cell interference implies that the interference signals from other cells have a benign effect on the desired signal. This property gives R-OFDM a significant advantage over ordinary OFDM in a multi-cell environment, as will be shown by the capacity analysis later.

## 3. CHANNEL CODING SCHEME FOR DOWNLINK

Since we are considering downlink, the space-time coding and transmit antenna diversity techniques can be employed to treat the fading problem [3]. Besides, a high layer coding can be used for error protection.

We employ the concatenated tree (CT) code introduced in [4] due to its low complexity and good performance. Figure 3 shows the overall coding scheme, where a bit-interleaved CT-coded modulation scheme is used in conjunction with a two-antenna space-time block code [5]. The user data is encoded by a rate-1/2 CT code, interleaved, then mapped to a QPSK constellation using Gray codes. After space-time coding, the resultant signals are fed into the system in Figure 1.

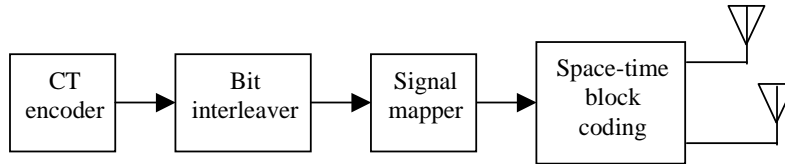


Figure 3. A concatenated space-time coding scheme for downlink

The performance of this coding scheme is shown in Figure 4, where the CT code

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consists of four component codes with pseudo-random interleavers of size 512bits. For correlated Rayleigh fading channel,  $f_D$  is the Doppler bandwidth and  $T_s$  is the signal duration.

It can be seen from Figure 4 that an adequate performance of  $\text{BER} < 10^{-3}$  is achievable over correlated Rayleigh channels with  $E_b / N_0 \geq 3.4$  (5.3dB). (For independent fading channel, this value can be lowered.) For code rate  $R_c=1/2$ , it is equivalent to the requirement of

$$E_c / N_0 \geq 1.7 \text{ (2.3dB)} \quad (11)$$

where  $E_c$  is the average received energy per coded bit at the output of the demodulator (in front of the decoder). Equation (11) will be used as a criterion to assess the system capacity below.

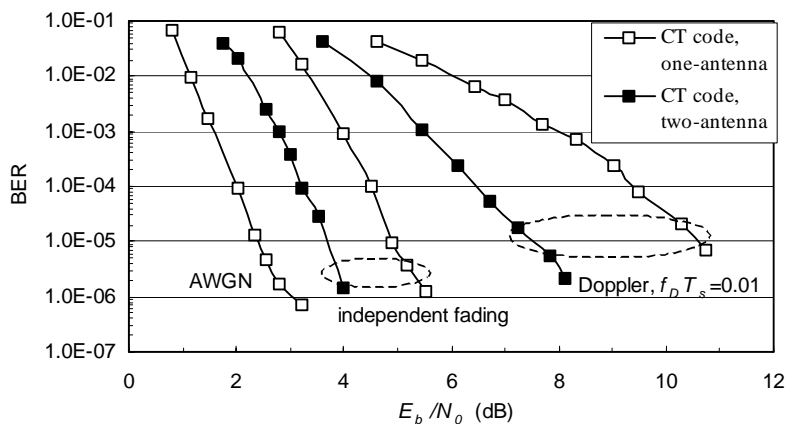


Figure 4. Performance of rate-1/2 CT-space-time codes.

## 4. DOWNLINK CAPACITY OF R-OFDM

### 4.1. Cellular System Model

As mentioned above, the inter-cell interference of an R-OFDM system can be modeled as Gaussian-noise-like. This implies that the downlink capacity of R-OFDM can be analyzed using the technique presented in [6]. However, there is an essential difference between an R-OFDM and a random CDMA system: The same-cell interference is eliminated in R-OFDM, but not in CDMA. Therefore, the derivations in [6] should be modified for the capacity analysis of R-OFDM.

We followed the system model as used in [6], and carried out Monte Carlo

simulation to assess the downlink performances of different systems. It is assumed that all cell sites beyond the second ring around a cell contribute negligible received power, and  $N_s$  subscribers are uniformly distributed in a sector. The path loss between the subscriber and the cell site is proportional to  $10^{\xi/10} r^{-4}$ , where  $r$  is the distance from the subscriber to cell site and  $\xi$  is a Gaussian random variable with zero mean and standard deviation  $\sigma = 8$ . Perfect power control is assumed and the voice activity factor has a value of 3/8.

We assume that the total bandwidth of the system  $W=1.024\text{MHz}$  (i.e., chip duration  $\tau=0.98\mu\text{s}$ ), and the user data rate  $R=8\text{kbps}$ . The rare-1/2 coding scheme shown in figure 3 is used.

Besides the common conditions above, some parameters for individual systems are listed below.

- Refer to Figures 1 and 2. The only difference between OFDM and R-OFDM is that no  $p$ -sequences are used in the former. For both OFDM and R-OFDM, the number of sub-carriers  $M=32$  and cyclic prefix length  $D=10$ . Each sub-carrier is modulated by two coded bits in QPSK format. Each  $\nu$ -frame contains 12  $U$ -frames (i.e.,  $J=11$  in Figure 2), with one frame used as phase reference (pilot). For R-OFDM, the  $p$  sequence length is  $N=511$  (i.e., the length of a  $\nu$ -frame is 511).
- For CDMA, random binary spreading sequences of length 64 are used.

The selection of the  $\nu$ -frame length of 511 above is a compromise between the channel coherent time and the requirement of a sufficient number of  $U$ -frames contained in  $\nu$  (so that the randomization can be effective [2]).

#### 4.2. Simulation Results

With the assumption that the background can be ignored, the output signal-to-interference ratio (SIR) of the demodulator is

$$SIR = E_c / I_0 \quad (12)$$

where  $E_c$  is the average received energy per coded bit from the desired cell site and  $I_0$  is the interference power per hertz. Figure 5 shows the cumulative distribution functions of  $E_c / I_0$  for various systems with different user numbers.

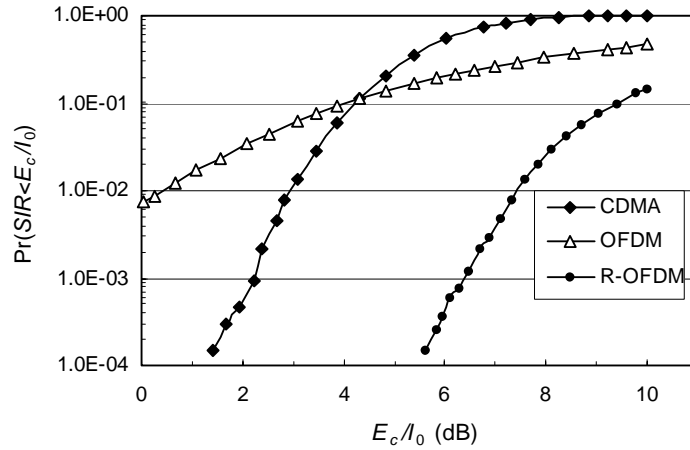
From (11), it is required that  $E_c / I_0 \geq 1.7$  to guarantee an acceptable performance of  $\text{BER} < 10^{-3}$ . Assume that 20% of each site's transmitted power in the sector is devoted to the pilot signal. Consequently, we define the common performance criterion on the outage probability as

$$P_{\text{out}} = \Pr(0.8 \times E_c / I_0 < 1.7) = \Pr(E_c / I_0 < 3.2\text{dB}) \leq 0.01 \quad (13)$$

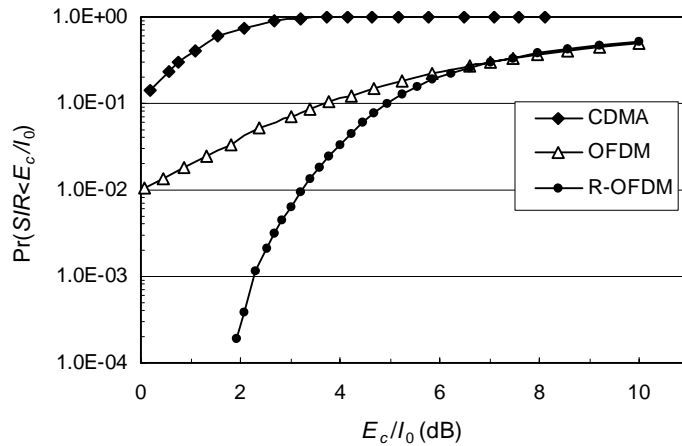
Using criterion (13), we can see from Figure 5 that the R-OFDM downlink can

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support 88 users/sector with  $\text{BER} < 10^{-3}$  at more than 99% of the time, as compared to about 30 users/sector for random CDMA. (If we consider the diversity provided by RAKE receivers, the required  $E_c/I_0$  can be lowered for CDMA in multipath channels. The capacity can be increased to about 40 users/sector.) The ordinary OFDM (not explicitly shown) with frequency hopping can support only about 15 users/sector. The advantage of R-OFDM is clearly seen here.



(a)  $N_s=30$



(b)  $N_s=88$

Figure 5. The cumulative distributions of  $E_c / I_0$  for different systems

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In summary, the simulation results show that a significant downlink capacity gain can be achieved using R-OFDM in conjunction with space-time codes. Compared with ordinary OFDM, the capacity improvement of R-OFDM is due to the randomization effect that mitigates inter-cell interference. Compared with random waveform CDMA, the capacity improvement of R-OFDM is due to the orthogonality among same-cell user signals.

## 5. CONCLUSIONS

In this paper, the performance of R-OFDM downlink has been discussed. Concatenated space-time codes are used to combat the channel impairments. Simulation results show that a significant downlink capacity gain can be achieved by eliminating intra-cell interference and by mitigating inter-cell interference, based on the R-OFDM techniques.

## ACKNOWLEDGMENT

This work was supported by CERG under grand 7001199.

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## NOTES

<sup>1</sup> In both [1] and [2], this constraint is stated as  $\mathbf{p} \otimes \mathbf{p} = \boldsymbol{\delta}$  (i.e., let  $\mathbf{p}' = \mathbf{p}$ ). However, it is more convenient to choose  $\mathbf{p}'$  as  $\mathbf{p}^H = (p_0, p_{N-1}, p_{N-2}, \dots, p_1)$ , i.e.,  $\mathbf{p}^H$  is the reversed sequence of  $\mathbf{p}$ . Let  $s$  be a length- $N$   $m$ -sequence over  $\{+1, -1\}$ . Let  $\alpha = 1/\sqrt{N+1}$ ,  $\beta = (1 \pm \sqrt{N+1})/N$  and  $\mathbf{e}$  be an all-1 sequence. Then it can be verified that  $\mathbf{p} \otimes \mathbf{p}^H = \boldsymbol{\delta}$  if  $\mathbf{p} = \alpha(s + \beta \mathbf{e})$ .

<sup>2</sup> These are the black signals in figure 2 and they should be distinguished from the extra prefix padded to  $\mathbf{v}$ .

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