Approaching the Capacity of Multiple Access Channels Using Interleaved Low-Rate Codes

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***Abstract*—We show that the capacity of a Gaussian multiple access channel can be approached by interleaved low-rate codes together with a simple chip-by-chip iterative decoding strategy. Based on a rate ≈ 1/69 code and with a total of 35 simultaneous users (the aggregate rate ≈ 1/2), performance of $\mathrm{BER} = 10^{-5}$ is observed at $E_b/N_0 \approx 1.4$ dB, which is close to the corresponding capacity limit ($E_b/N_0 \approx 0$ dB).**

***Index Terms*—Channel capacity, multiple access channels.**

I. INTRODUCTION

ENCOURAGED by the success of turbo codes [1] in additive white Gaussian noise (AWGN) channels, turbo-type iterative multiuser detection in spread-spectrum multiple access channels (MACs) has been extensively studied and significant progress has been made [2]–[6].

A conventional random waveform code-division multiple-access (CDMA) system (such as IS-95) involves separated coding and spreading operations. The optimal tradeoff of coding-spreading has been investigated [7]. It is known that the achievable system capacity reduces as the spreading factor increases [7]. To approach the optimal MAC capacity (which is the same as that of a single-user AWGN channel [7, Fig. 1]), the entire bandwidth expansion should be devoted to coding. This implies that coding and spreading should be combined using low-rate codes with maximized coding gain. Such a strategy has been previously considered [8], [9]. Several issues arise when applying this principle: 1) optimal design of the low-rate codes; 2) choice of multiple access methods; and 3) efficient detection techniques.

There are several approaches to issue 1) above [8]–[11]. This letter reports experimental work addressing issues 2) and 3) based on the low-rate codes devised in [11]. User-specific interleavers [9], [12]–[16] are employed for multiple access purposes. Detection is based on a low-cost iterative chip-by-chip technique. We show that, with a rate ≈ 1/69 code for each user and with 35 simultaneous users in the system (i.e., the aggregate rate ≈ 1/2), performance of $\mathrm{BER} = 10^{-5}$ is observed at $E_b/N_0 \approx 1.4$ dB, which is close to the capacity limit of a Gaussian MAC ($E_b/N_0 \approx 0$ dB) [7].

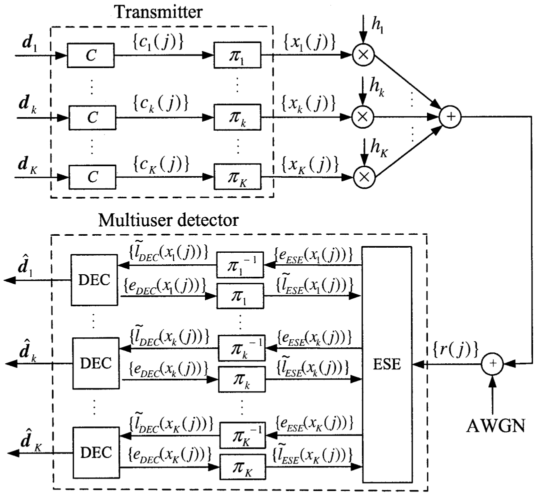

Fig. 1. Transmitter and receiver structures of the proposed scheme, where π_k is the interleaver of user-k.

II. TRANSMITTER AND RECEIVER PRINCIPLES

A. Transmitter Structure

The upper part of Fig. 1 shows the transmitter structure of the proposed scheme with K simultaneous users [15], [16]. The input data sequence $\boldsymbol{d}_k$ of user-k is encoded by a low-rate code C into $\boldsymbol{c}_k = \{c_k(j), j = 1, \ldots, J\}$, where J is the frame length. A chip level interleaver π_k is then used, producing $\boldsymbol{x}_k = \{x_k(j), j = 1, \ldots, J\}$. (Note: We follow the convention of CDMA and call the elements in $\boldsymbol{c}_k$ and $\boldsymbol{x}_k$ “chips”.) The key principle of the scheme is that the interleavers $\{\pi_k\}$ should be different for different users. We assume that the interleavers are generated independently and randomly.

Assuming a quasistatic MAC, after sampling at the chip rate, the received signal from K users can be written as

$$r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j), \qquad j = 1, 2, \ldots, J \tag{1}$$

where $x_k(j)$ is the jth chip transmitted by user-k, h_k is the channel coefficient for user-k, and $\{n(j)\}$ are samples of a zero-mean AWGN with variance $\sigma^2 = N_0/2$. We assume that the channel coefficients $\{h_k\}$ are known *a priori* at the receiver side.

B. Receiver Structure

A sub-optimal turbo-type receiver structure is illustrated in the lower part of Fig. 1, consisting of an elementary signal estimator (ESE) and K single-user *a posteriori* probability (APP)

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decoders (DECs). Overall, it follows the turbo multiuser detection principle developed in [2]–[6], [14]. Since traditional signature-sequence based spreading is not necessarily a system function, the correlator based receiver structure [2]–[5] is not suitable here. Instead, we adopt a chip-by-chip estimation technique [14]–[16].

Denote the *a priori* logarithm likelihood ratios (LLRs) about $\{x_k(j), \forall k, j\}$ by

$$\tilde{l}(x_k(j)) \equiv \log\left(\frac{\Pr(x_k(j)=+1)}{\Pr(x_k(j)=-1)}\right). \tag{2}$$

These *a priori* LLRs are further distinguished by subscripts, i.e., $\tilde{l}_{\mathrm{ESE}}(x_k(j))$ and $\tilde{l}_{\mathrm{DEC}}(x_k(j))$, depending on whether they are used in the ESE or DECs.

The ESE uses $\{r(j)\}$ and $\{\tilde{l}_{\mathrm{ESE}}(x_k(j))\}$ as its inputs and only the multiple access channel constraint is considered here. Let $\boldsymbol{h} \equiv \{h_k, \forall k\}$, $\tilde{\boldsymbol{L}}_{\mathrm{ESE}} \equiv \{\tilde{l}_{\mathrm{ESE}}(x_k(j)), \forall k, j\}$. The outputs of the ESE are the extrinsic information about $\{x_k(j), \forall k, j\}$ based on the channel observation and the *a priori* information of the other chips (excluding $x_k(j)$), as detailed in II-C.

$$\begin{aligned} &e_{\mathrm{ESE}}(x_k(j)) \\ &= \log\left(\frac{\Pr\left(x_k(j)=+1|r(j), \tilde{\boldsymbol{L}}_{\mathrm{ESE}}, \boldsymbol{h}\right)}{\Pr\left(x_k(j)=-1|r(j), \tilde{\boldsymbol{L}}_{\mathrm{ESE}}, \boldsymbol{h}\right)}\right) - \tilde{l}_{\mathrm{ESE}}(x_k(j)). \end{aligned} \tag{3}$$

Similarly, let $(\tilde{\boldsymbol{L}}_{\mathrm{DEC}})_k \equiv \{\tilde{l}_{\mathrm{DEC}}(x_k(j)), \forall j\}$ be the inputs to the DEC for user-k. The outputs of the DEC are the extrinsic information about $\{x_k(j), \forall k, j\}$ based on the code constraint of C.

$$\begin{aligned} &e_{\mathrm{DEC}}(x_k(j)) \\ &= \log\left(\frac{\Pr\left(x_k(j)=+1|C, (\tilde{\boldsymbol{L}}_{\mathrm{DEC}})_k\right)}{\Pr\left(x_k(j)=-1|C, (\tilde{\boldsymbol{L}}_{\mathrm{DEC}})_k\right)}\right) - \tilde{l}_{\mathrm{DEC}}(x_k(j)). \end{aligned}$$

During the turbo-type iterative process, the extrinsic information generated by the ESE/DEC is used (after appropriate deinterleaving/interleaving) as the *a priori* information in the DEC/ESE, i.e., the updates $\{e_{\mathrm{ESE}}(x_k(j))\} \Rightarrow \{\tilde{l}_{\mathrm{DEC}}(x_k(j))\}$ and $\{e_{\mathrm{DEC}}(x_k(j))\} \Rightarrow \{\tilde{l}_{\mathrm{ESE}}(x_k(j))\}$ are performed at the ESE/DEC interface, see Fig. 1. Suppose that we start from the ESE. All $\{\tilde{l}_{\mathrm{ESE}}(x_k(j))\}$ are initialized to zero, assuming that there is no initial *a priori* information. The DECs also produce hard decisions $\{\hat{\boldsymbol{d}}_k\}$ on information bits in the final iteration.

The APP decoding in the DECs is a standard function [1], [11], so we will not discuss it in detail. In the following, we will focus on the ESE.

C. Detailed Descriptions of the ESE

The ESE generates coarse estimates of $\{x_k(j), \forall k, j\}$ ignoring the constraint of C. The output of the ESE defined by (3) can be obtained with maximum *a posteriori* (MAP) based methods [2], [3], but it is difficult to process a large number of users due to the exponentially increasing complexity. We will adopt a low-complexity chip-by-chip detection technique [15] that is similar to the MMSE filtering methods in [4], [14].

Consider the jth chip of user-k with $x_k(j) \in \{+1, -1\}$ under BPSK modulation. We treat $x_k(j)$ as a random variable and use $e_{\mathrm{DEC}}(x_k(j))$ to approximate the *a priori* LLR $\tilde{l}_{\mathrm{ESE}}(x_k(j))$ about $x_k(j)$. Then from (2), we have [4]

$$\begin{aligned} \mathrm{E}(x_k(j)) &= (+1)\cdot\Pr(x_k(j)=+1) + (-1)\Pr(x_k(j)=-1) \\ &= \frac{\exp\left(\tilde{l}_{\mathrm{ESE}}(x_k(j))\right) - 1}{\exp\left(\tilde{l}_{\mathrm{ESE}}(x_k(j))\right) + 1} \\ &= \tanh\left(\frac{\tilde{l}_{\mathrm{ESE}}(x_k(j))}{2}\right) \\ \mathrm{var}(x_k(j)) &= 1 - (\mathrm{E}(x_k(j)))^2 \end{aligned}$$

where $\mathrm{E}(x)$ and $\mathrm{var}(x)$ denote the mean and variance of x, respectively.

For user-k, denoting $\zeta_k(j) = \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j)$, we can rewrite (1) as

$$r(j) = h_k x_k(j) + \zeta_k(j). \tag{4}$$

Assume that $\{x_k(j), \forall k\}$ are independent and identically distributed (i.i.d.) random variables. Applying the central limit theorem, $\zeta_k(j)$ in (4) can be approximated by a Gaussian random variable with mean and variance $\mathrm{E}(\zeta_k(j)) = \sum_{k' \neq k} h_{k'} \mathrm{E}(x_{k'}(j))$ and $\mathrm{var}(\zeta_k(j)) = \sum_{k' \neq k} |h_{k'}|^2 \mathrm{var}(x_{k'}(j)) + \sigma^2$. Applying this approximation to (4), $e_{\mathrm{ESE}}(x_k(j))$ in (3) can be calculated as [15]

$$\begin{aligned} e_{\mathrm{ESE}}(x_k(j)) &= \log\left(\frac{p\left(r(j)|x_k(j)=+1, \tilde{\boldsymbol{L}}_{\mathrm{ESE}}, \boldsymbol{h}\right)}{p\left(r(j)|x_k(j)=-1, \tilde{\boldsymbol{L}}_{\mathrm{ESE}}, \boldsymbol{h}\right)}\right) \\ &= \log\left(\frac{\exp\left(-\frac{(r(j)-\mathrm{E}(\zeta_k(j))-h_k)^2}{2\mathrm{var}(\zeta_k(j))}\right)}{\exp\left(-\frac{(r(j)-\mathrm{E}(\zeta_k(j))+h_k)^2}{2\mathrm{var}(\zeta_k(j))}\right)}\right) \\ &= 2h_k \cdot \frac{r(j) - \mathrm{E}(r(j)) + h_k \mathrm{E}(x(j))}{\mathrm{var}(r(j)) - |h_k|^2 \mathrm{var}(x(j))} \end{aligned} \tag{5}$$

where (based on (1))

$$\begin{aligned} \mathrm{E}(r(j)) &= \sum_{k'=1}^{K} h_{k'} \mathrm{E}(x_{k'}(j)) \\ \mathrm{var}(r(j)) &= \sum_{k'=1}^{K} |h_{k'}|^2 \mathrm{var}(x_{k'}(j)) + \sigma^2. \end{aligned}$$

The result in (5) is an extremely reduced form of the method derived in [4] (i.e., when spreading sequences are all of length-1).

D. A Summary of the Chip-by-Chip Detection Algorithm

For clarity, we summarize the chip-by-chip detection algorithm as follows (initialized with $\tilde{l}_{\mathrm{ESE}}(x_k(j)) = 0, \forall k, j$).

$$\mathrm{E}(x_k(j)) \Leftarrow \tanh\left(\frac{\tilde{l}_{\mathrm{ESE}}(x_k(j))}{2}\right) \quad \forall k, j. \tag{6a}$$

$$\mathrm{var}(x_k(j)) \Leftarrow 1 - (\mathrm{E}(x_k(j)))^2 \quad \forall k, j. \tag{6b}$$

$$\mathrm{E}\left(r(j)\right) \Leftarrow \sum_{k'=1}^{K} h_{k'}\mathrm{E}\left(x_{k'}(j)\right) \quad \forall j. \tag{7a}$$

$$\mathrm{var}\left(r(j)\right) \Leftarrow \sum_{k'=1}^{K} |h_{k'}|^2\mathrm{var}\left(x_{k'}(j)\right) + \sigma^2 \quad \forall j. \tag{7b}$$

$$e_{\mathrm{ESE}}\left(x_k(j)\right) \Leftarrow 2h_k \cdot \frac{r(j) - \mathrm{E}\left(r(j)\right) + h_k\mathrm{E}\left(x_k(j)\right)}{\mathrm{var}\left(r(j)\right) - |h_k|^2\mathrm{var}\left(x_k(j)\right)} \quad \forall k, j. \tag{8}$$

The APP decoding in the DECs is performed after (8) to generate $\{e_{\mathrm{DEC}}(x_k(j))\}$. Update $\{e_{\mathrm{DEC}}(x_k(j))\} \Rightarrow \{\tilde{l}_{\mathrm{ESE}}(x_k(j))\}$, then go back to (6) for the next iteration.

Note that the operations in (6) and (8) only involve several additions and multiplications per chip per user. The results in (7), i.e., $\mathrm{E}(r(j))$ and $\mathrm{var}(r(j))$, are shared by all the users, costing only three multiplications and two additions per chip per user. Clearly, the complexity involved in (6)–(8) is very modest. The complexity per user is independent of the number of users K.

III. SIMULATION RESULTS

Good low-rate codes with high coding gain are required in the proposed scheme. We adopt turbo-Hadamard codes [11] for this purpose. A performance comparison between systems based on turbo-Hadamard codes and other codes can be found in [16].

We assume that all the users have equal power and equal rate. The simulation work is based on a serial schedule [15]. Fig. 2 shows the performance of the system in Fig. 1 based on a low-rate ($\approx$ 1/17.2) turbo-Hadamard code[1] [11] with an information length of 4095 bits per user. An extra spreading operation is applied using a length-4 spreading sequence $[+1, -1, +1, -1]$ for further bandwidth expansion, reducing the overall rate R_C to approximately 1/69. (Note: Alternatively, we can directly use a rate-1/69 turbo-Hadamard code. However, the related complexity is much higher and the performance gain is marginal.) The same spreading sequence is used for all users. The turbo-Hadamard code and the spreading operation together form C in Fig. 1. From Fig. 2, performance of $\mathrm{BER} = 10^{-5}$ is observed at $E_b/N_0 \approx 1.4$ dB with $K = 35$. This corresponds to a total system rate of $K \times R_C = 35/69 = 0.5084$, which is the overall measurement of the system bandwidth efficiency. This result is only 1.4 dB away from the corresponding optimal MAC capacity, which is $E_b/N_0 = 0$ dB for $K \times R_C = 0.5$ [7], the same as that for a single-user channel. This compares favorably with conventional CDMA multiuser detection schemes. For example, [5] reported throughput of $K \times R_C = 1/3$ at $E_b/N_0 = 2.8$ dB (with $K = 15$, FEC coding rate $= 1/3$, spreading sequence length $= 15$ and $R_C = 1/45$), which is about 3.35 dB away from the corresponding optimal MAC capacity limit of $E_b/N_0 = -0.55$ dB.

IV. CONCLUSIONS

We have studied a multiple access scheme exploiting the high coding gain offered by low-rate turbo-type codes. Interleavers are employed as the only means to separate the signals from different users, which facilitates a low-cost chip-by-chip multiuser detection technique. Performance about 1.4 dB away from the optimal MAC capacity has been observed for a system with 35 users.

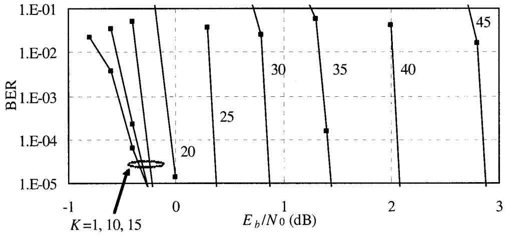

Fig. 2. Performance of the system in Fig. 1 based on interleaved turbo-Hadamard codes [11] over AWGN channels. All channel coefficients $\{h_k\}$ are set to unity. The number of users K is marked in the figure. $\mathrm{Iteration\ number} = 30$.

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[1] The rate-1/17.2 turbo-Hadamard code is constructed by concatenating three convolutional-Hadamard codes, each of which is generated from a length-32 Hadamard code and a convolutional code with polynomial $G(x) = 1/(1+x)$. The information bits in all component codes except one are punctured. The related decoding complexity is quite low, as analyzed in [11].