Exercises for Week 1

1. Determine the values of the following functions:

(a) $\cos\left(\frac{\pi}{6}\right)$ (b) $\sin\left(\frac{\pi}{6}\right)$

(c) $\cos\left(\frac{3\pi}{2}\right)$ (d) $\sin\left(\frac{5\pi}{4}\right)$

(e) $e^{j\frac{\pi}{3}}$ (f) $e^{j\pi}$

2. Determine the values of the following geometric series:

(a) $\sum_{k=0}^{9} 0.2^k$ (b) $\sum_{k=0}^{\infty} 0.2^k$

(c) $\sum_{k=5}^{10} 2^k$ (d) $\sum_{k=0}^{\infty} 2^k$

(e) $\sum_{k=0}^{4} 5 \cdot (-0.3)^k$ (f) $\sum_{k=0}^{\infty} (-0.9)^k$

3. Solve the following Integrations.

(a) $\int e^{-jk\Omega_o t} dt$

(b) $\int_{-T_o}^{T_o} e^{-jk\Omega_o t} dt$

Solution of Exercise 1

Question 1

(a)
$$\cos\left(\frac{\pi}{6}\right) = \cos(30^o\ ) = \frac{\sqrt{3}}{2}$$

(b)
$$\sin\left(\frac{\pi}{6}\right) = \sin(30^o) = \frac{1}{2}$$

(c)
$$\cos\left(\frac{3\pi}{2}\right) = \cos(270^o) = 0$$

(d)
$$\sin\left(\frac{5\pi}{4}\right) = \sin\left(\pi + \frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\sin(45^o) = -\frac{\sqrt{2}}{2}$$

(e)
$$e^{j\frac{\pi}{3}} = \cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right) = \cos(60^o) + j\sin(60^o) = \frac{1}{2} + j\frac{\sqrt{3}}{2}$$

(f)
$$e^{j\pi} = \cos(\pi) + j\sin(\pi\ ) = -1$$

Question 2

$$a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{N-1} = \sum_{k=0}^{N-1} ar^k = a\left(\frac{1-r^N}{1-r}\right)$$

(a)
$$\sum_{k=0}^{9} 0.2^k = \frac{1-0.2^{10}}{1-0.2} \cong 1.25$$

(b)
$$\sum_{k=0}^{\infty} 0.2^k = \frac{1}{1-0.2} = \frac{1}{0.8} = 1.25$$

(c)
$$\sum_{k=5}^{10} 2^k = \sum_{k=0}^{10} 2^k - \sum_{k=0}^{4} 2^k = \frac{1-2^{11}}{1-2} - \frac{1-2^5}{1-2} = 2^{11} - 2^5 = 2016$$

(d) $$\sum_{k=0}^{\infty} 2^k = \infty$$

(e) $$\sum_{k=0}^{4} 5 \cdot (-0.3)^k = 5\left(\frac{1-(-0.3)^5}{1-(-0.3)}\right) = 3.855$$

(f) $$\sum_{k=0}^{\infty} (-0.9)^k = \frac{1}{1-(-0.9)} = \frac{1}{1.9} = 0.526$$

Question 3

(a) $$\int e^{-jk\Omega_o t} dt = -\frac{1}{jk\Omega_o} e^{-jk\Omega_o t} + C$$

(b) $$\int_{-T_o}^{T_o} e^{-jk\Omega_o t} dt = \left[-\frac{1}{jk\Omega_o} e^{-jk\Omega_o t}\right]_{-T_o}^{T_o} = -\frac{1}{jk\Omega_o}\left[e^{-jk\Omega_o T_o} - e^{jk\Omega_o T_o}\right]$$

$$= \frac{2}{k\Omega_o} \cdot \frac{1}{2j}\left[e^{jk\Omega_o T_o} - e^{-jk\Omega_o T_o}\right] = \frac{2\sin(k\Omega_o T_o)}{k\Omega_o}$$