Exercises for Week 8

1. Let the impulse response of a digital system be

$$h[n] = \{\mathbf{1}, 1, 1\}$$

The first sample is at index $n = 0$. Is it a linear-phase system? Determine its magnitude and amplitude response. Find also the corresponding phase responses.

2. Let the impulse response of a digital system be

$$h[n] = \left\{\frac{\mathbf{1}}{\mathbf{5}}, -\frac{1}{4}, \frac{1}{3}, -\frac{1}{2}, 1, 0, -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}\right\}$$

The first sample is at index $n = 0$. Is it a linear-phase system? Why?

3. Compute the energy for $h_d[n]$:

$$h_d[n] = \frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c n}{\pi}\right), \qquad \omega_c = 0.1\pi$$

which is the impulse response of an ideal lowpass filter with cutoff frequency 0.1π.

Solution

Question 1

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j2\omega}$$

$$= e^{-j\omega}(e^{j\omega} + 1 + +e^{-j\omega})$$

$$= e^{-j\omega}[1 + 2\cos(\omega)]$$

The amplitude response is

$$H_r(e^{j\omega}) = 1 + 2\cos(\omega)$$

and the corresponding phase response is:

$$\angle H(e^{j\omega}) = -\omega$$

As $\angle H(e^{j\omega})$ is a linear function of ω, the system is of linear-phase.

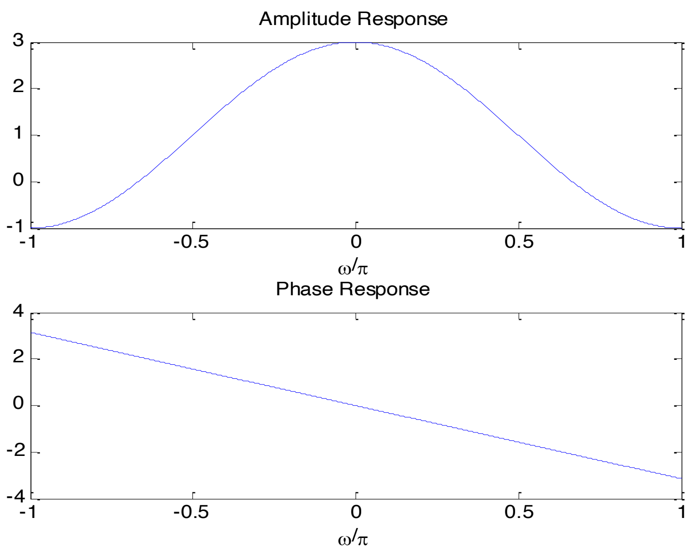
On the other hand, the magnitude response is:

$$|H(e^{j\omega})| = 1 + 2\cos(\omega)$$

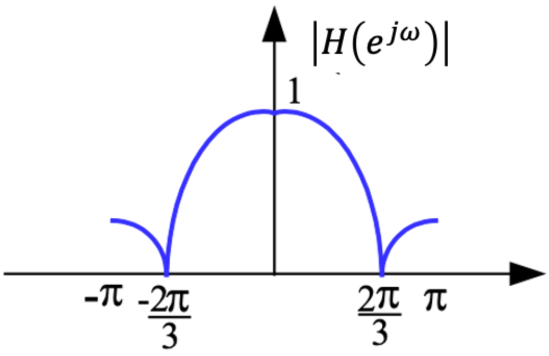
and the corresponding phase response is:

$$\angle H(e^{j\omega}) = \begin{cases} -\omega & -\frac{2\pi}{3} < \omega < \frac{2\pi}{3} \\ -\omega \pm \pi & -\pi \le \omega \le -\frac{2\pi}{3} \; and \; \frac{2\pi}{3} < \omega < \pi \end{cases}$$

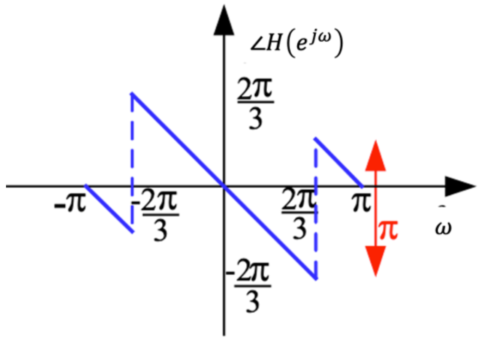
In$(-\pi, \pi)$, $(1 + 2\cos(\omega)) \ge 0$ for $|\omega| \le 2\pi/3$ and it is equal to zero otherwise.



Magnitude Response



Phase Response



Question 2

No. It is not linear-phase because it does not fulfil the symmetric or anti-symmetric condition.

Question 3

Using Parseval's relation for DTFT

$$\text{Signal Energy} = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$$

Based on the frequency domain formula, the energy is

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left|H_d\left(e^{j\omega}\right)\right|^2 d\omega = \frac{1}{2\pi}\int_{-0.1\pi}^{0.1\pi} d\omega = \frac{0.2\pi}{2\pi} = 0.1$$

On the other hand, it is difficult to compute the energy in the time domain.