Discrete-Time Systems

EE4015 Digital Signal Processing

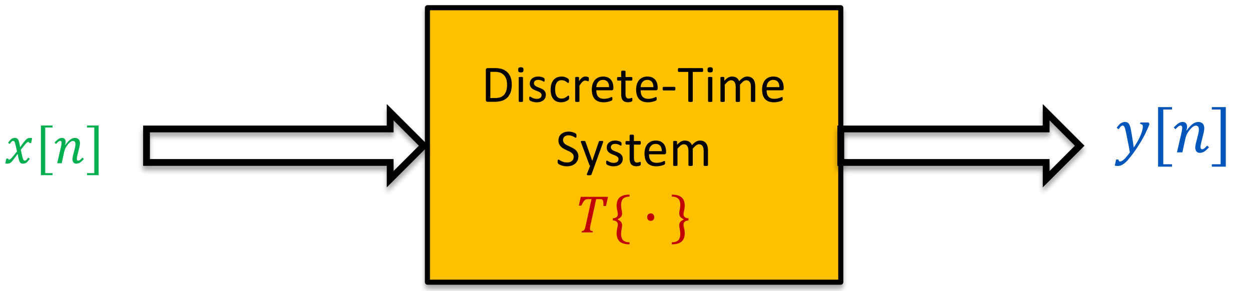
Dr. Lai-Man Po
Department of Electrical Engineering
City University of Hong Kong

Content

- Definition of Discrete-Time (DT) Systems
- Classification of DT Systems
  - Memory, Time-Invariant, Linear, Causal and Stable
- Linear Time-Invariant (LTI) System
  - Unit sample Response (Impulse Response)
  - Convolution
- Block Diagram Representation
- Difference Equations
- Finite Impulse Response (FIR) System
- Infinite Impulse Response (IIR) System
- Stability of LTI System

Discrete-Time Systems

- A discrete-time system is a device or algorithm that operates on a discrete-time signal (or sequence) $x[n]$ called the input to produce another discrete-time signal called the output or response $y[n]$.



$$y[n] = T\{x[n]\}$$

- where the symbol T denotes the transformation or processing performed by the system on $x[n]$ to produce $y[n]$

Classification of Discrete-Time Systems

- In the system analysis, it is desirable to classify the systems according to their general properties.
- General Categories of DT Systems are:
  - **Memoryless Systems**
  - **Time-Invariant Systems**
  - **Linear Systems**
  - **Causal Systems**
  - **Stable Systems**

Memoryless Systems

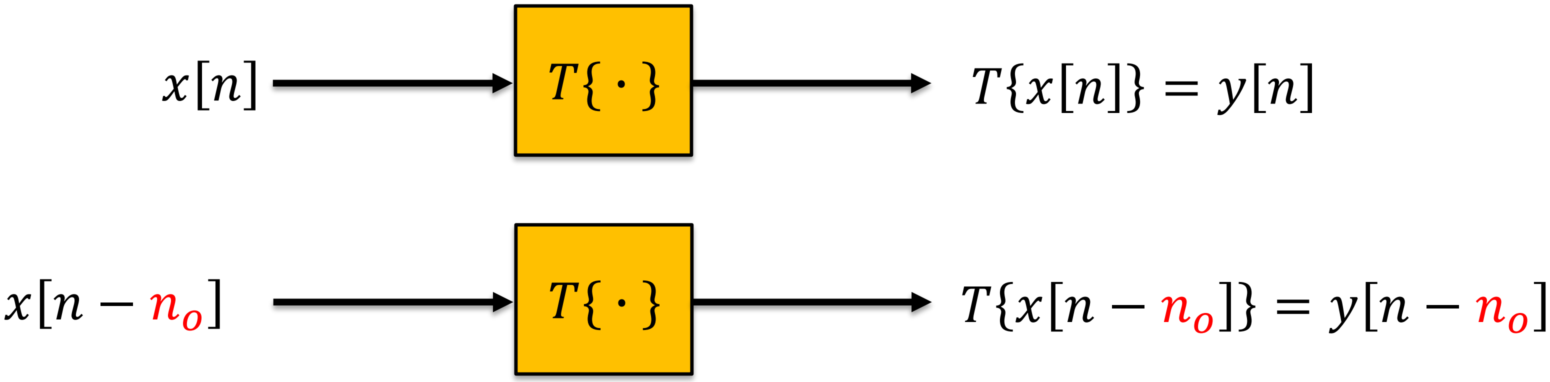
- A discrete-time system is called memoryless if its output at any instant n depends at most on the input sample at the same time, but not on past or future samples of the input.
- Example memoryless systems:
  - $y[n] = a\, x[n]$
  - $y[n] = n\, x[n] + b(x[n])^2$
- The output of these systems $y[n]$ are only depends on $x[n]$
- They are all memoryless systems

Memory Systems

- On the other hand, the systems described by the following input/output relations, such as
  - $y[n] = 2x[n] + 3x[n-1]$
  - $y[n] = \sum_{k=0}^{N} x[n-k]$
- These are systems with memory as their outputs depend on previous input samples.

Time-Invariant Systems

- A time-invariant system is defined as follows:



- Specifically, a system is time invariant if a time shift in the input signal results in an identical time shift in the output signal.

Time-Invariant System Example 1

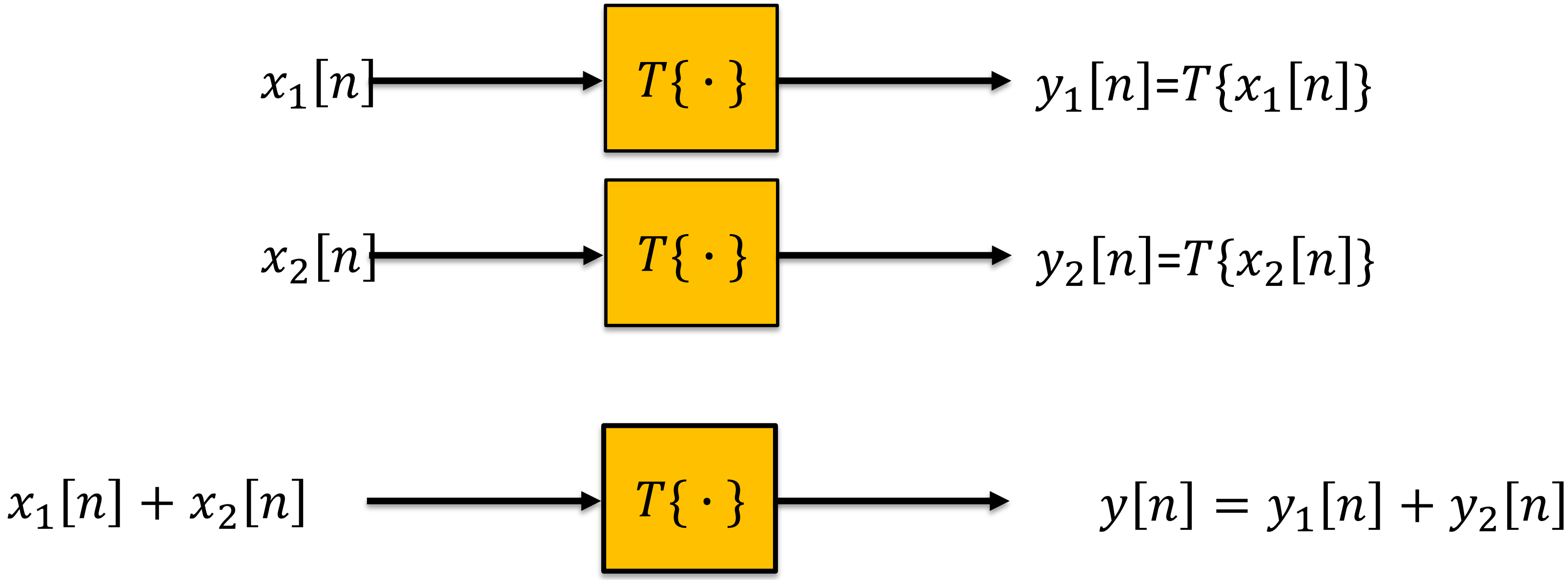
- Determine if the system is time variant or time invariant.
  - $y[n] = T\{x[n]\} = |x[n]|$
- The response of this system to $x[n-k]$ is
  - $T\{x[n-k]\} = |x[n-k]|$
- Now if we delay $y[n]$ by k units in time, we obtain
  - $y[n-k] = |x[n-k]|$
- This system is **time-invariant**, since
  - $T\{x[n-k]\} = |x[n-k]| = y[n-k]$

Time-Invariant System Example 2

- Determine if the system is time variant or time invariant.
  - $y[n] = T\{x[n]\} = nx[n]$
- The response of this system to $x[n-k]$ is
  - $T\{x[n-k]\} = nx[n-k]$
- Now if we delay $y[n]$ by k units in time, we obtain
  - $y[n-k] = (n-k)x[n-k]$
- This system is **time-variant**, since
  - $T\{x[n-k]\} = nx[n-k] \neq y[n-k] = (n-k)x[n-k]$

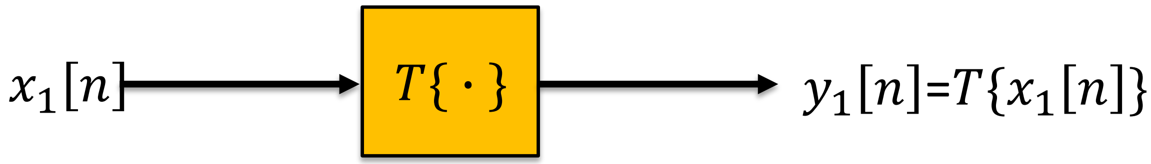
Superposition Condition

- A superposition system is defined as follows:

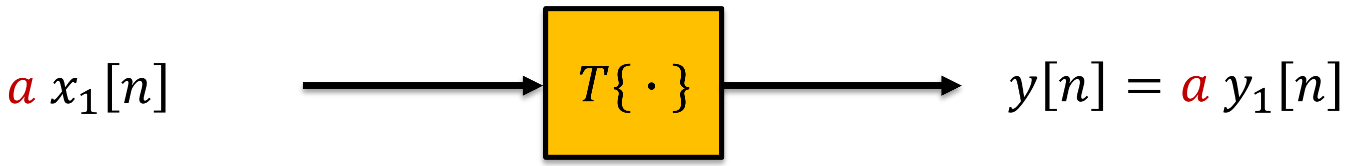


Homogeneity Condition

- A homogeneity system is defined as follows:

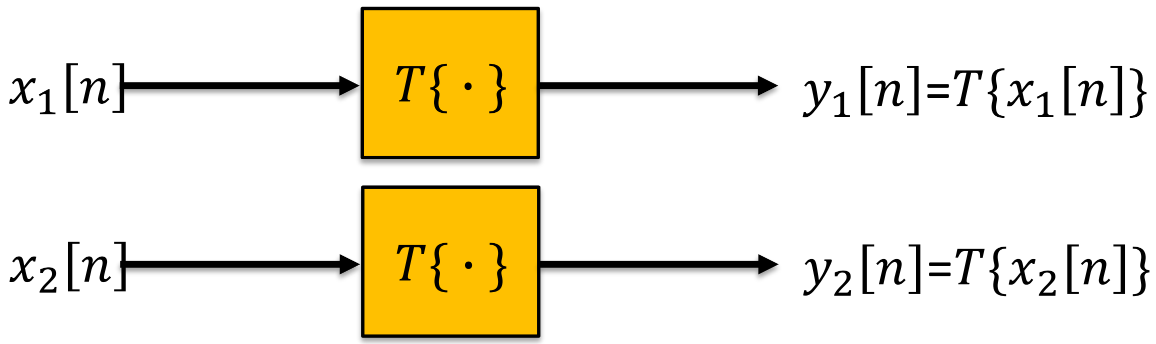


- For arbitrary constants a:

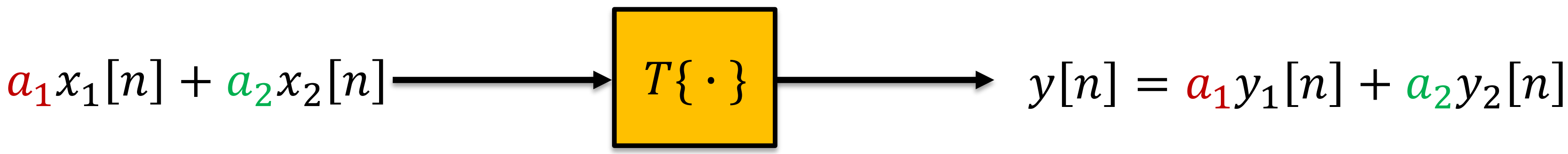


Linear Systems : Superposition and Homogeneity

- Linear systems satisfy both superposition and homogeneous conditions:



- For arbitrary constants a_1 and a_2 :



Linear System Example 1

- **Determine the 3-sample average system is linear or Not.**
  - $y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2]) = T\{x[n]\}$
- The response of this system to $\{a_1x_1[n] + a_2x_2[n]\}$ is $T\{a_1x_1[n] + a_2x_2[n]\}$

$$= \frac{1}{3}(a_1x_1[n] + a_2x_2[n] + a_1x_1[n-1] + a_2x_2[n-1] + a_1x_1[n-2] + a_2x_2[n-2])$$

$$= \frac{1}{3}(a_1x_1[n] + a_1x_1[n-1] + a_1x_1[n-2]) + \frac{1}{3}(a_2x_2[n] + a_2x_2[n-1] + a_2x_2[n-2])$$

$$= a_1y_1[n] + a_2y_2[n]$$

- The 3-sample average is a linear system

Linear System Example 2

- **Determine the squared input system is linear or Not.**
  - $y[n] = T\{x[n]\} = (x[n])^2$
- Let $y_1[n]$=$T\{x_1[n]\} = (x_1[n])^2$ and $y_2[n]$=$T\{x_2[n]\} = (x_2[n])^2$
- The response of this system to $\{a_1x_1[n] + a_2x_2[n]\}$ is $T\{a_1x_1[n] + a_2x_2[n]\}$

$$= (a_1x_1[n] + a_2x_2[n])^2 = (a_1x_1[n])^2 + (a_2x_2[n])^2 + 2a_1a_2x_1[n]\,x_2[n]$$

- This is NOT equal to $a_1y_1[n] + a_2y_2[n]$ = $a_1(x_1[n])^2 + a_2(x_2[n])^2$
- This system is non-linear.

Linear System Example 3

- **Determine this system is linear or Not.**
  - $y[n] = T\{x[n]\} = n\, x[n]$
- Let $y_1[n]$=$T\{x_1[n]\} = nx_1[n]$ and $y_2[n]$=$T\{x_2[n]\} = nx_2[n]$
- The response of this system to $\{a_1x_1[n] + a_2x_2[n]\}$ is

$$H\{a_1x_1[n] + a_2x_2[n]\} = n\,(a_1x_1[n] + a_2x_2[n]) = a_1nx_1[n] + a_2nx_2[n]$$

- This is equal to $a_1y_1[n] + a_2y_2[n]$
- This system is linear.

Causal Systems

- A system is said to be causal if the output of the system at any time '*n*' depends only on present and past inputs but does not depend on future inputs.
- If a system does not satisfy this definition, it is called noncausal.
  - The noncausal systems have outputs that depend not only on present and past inputs but also on future inputs.

Causal and Noncausal System Examples

- **Causal System Examples**
  - $y[n] = x[n] + 3x[n-1]$
  - $y[n] = 2x[n]$

- **Noncausal System Examples**
  - $y[n] = x[n] + 3x[n+2]$
  - $y[n] = x[-n]$
    - Let n = -1 ⇒ y[-1]= x [1], the output at n = -1 depends on the input at n = 1.

Causal System Exercise

- **Determine whether the following systems are causal or not.**

  1. $y[n] = 0.5x[n] + 2.5x[n-2], \quad for\ n \geq 0$

  2. $y[n] = 0.25x[n-1] + 2.5x[n+2] - 0.4y[n-1], \quad for\ n \geq 0$

Causal System Exercise

- **Determine whether the following systems are causal or not.**
  1. $y[n] = 0.5x[n] + 2.5x[n-2], \quad for\ n \geq 0$
  2. $y[n] = 0.25x[n-1] + 2.5x[n+2] - 0.4y[n-1], \quad for\ n \geq 0$

Solution

1) Causal
2) Non-causal

Stable Systems : BIBO Stable

- A discrete signal $x[n]$ is bounded if there exists a finite B_x such that $|x[n]| < B_x$ for all n.

- A discrete-time system in Bounded Input-Bounded Output (**BIBO**) stable if every bounded input sequence $x[n]$ produced a bounded output sequence.
  - If $\underbrace{x[n]_{\max} \leq B_x}_{\text{Bounded Input}}$, then $\underbrace{y[n]_{\max} \leq B_y}_{\text{Bounded Output}}$

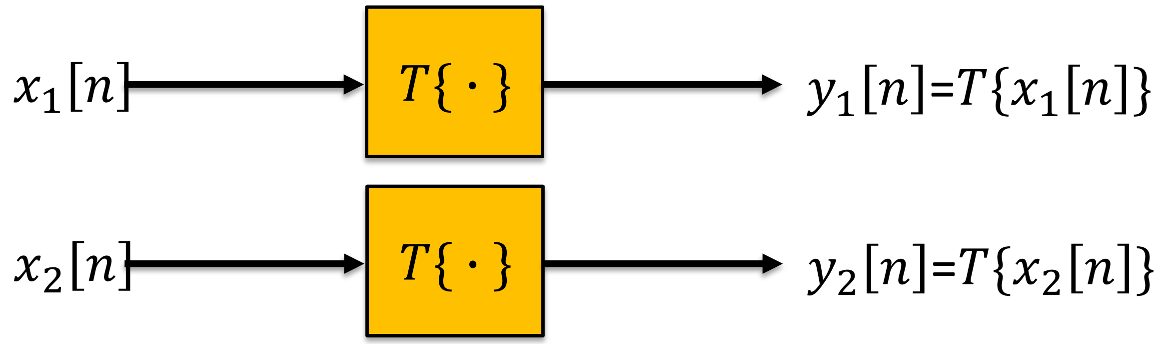
Stable System Example

- A discrete-time system with difference equation of
  - $y[n] = ny[n-1] + x[n], \quad n > 0$
  - The system at rest (i.e. $y[-1] = 0$)
- Check if the system is BIBO stable?
- If $x[n] = u[n]$, then $|x[n]| \leq 1$. But for this bounded input, the output is
  - $n = 0 \Rightarrow y[0] = x[0] = 1$
  - $n = 1 \Rightarrow y[1] = 1y[0] + x[1] = 2$
  - $n = 2 \Rightarrow y[2] = 2y[1] + x[2] = 5$
  - ... => ∞
- The input of unit step sequence is bounded, but the output is unbounded. Hence the system is BIBO unstable.

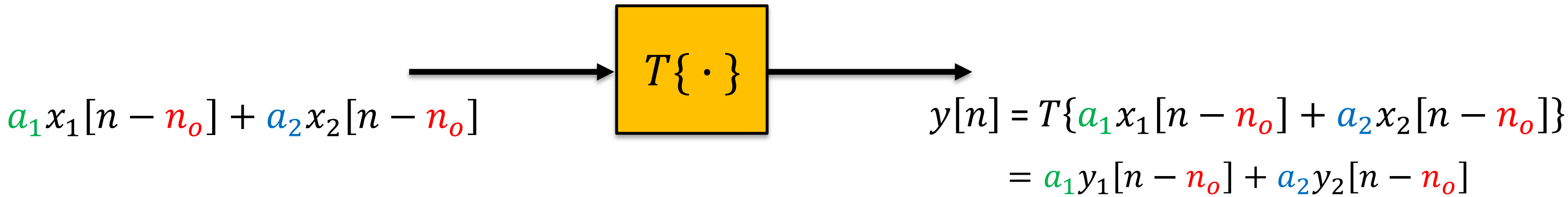
LTI Systems

Linear Time-Invariant (LTI) Discrete-Time Systems

- LTI discrete-time systems satisfy both Linear and Time-Invariant properties.



- For an integer n_o and arbitrary constants a_1 and a_2, LTI system property is



LTI System Examples

- Absolute Magnitude System: $y[n] = |x[n]|$
  - It is Time-Invariant but not Linear
- Time scaling System : $y[n] = n\,x[n]$
  - It is Linear but not Time-Invariant
- 3-sample average System : $y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$
  - It satisfies both Linear and Time-Invariant properties
  - **It is an LTI system**

LTI System Exercise 1

Determine whether the linear system $y[n] = 2x\,[n-5]$ is time invariant.

<u>**Solution**</u>

- Let the input and output be $x_1[n]$ and $y_1[n]$, respectively, then the system output is
  - $y_1[n] = 2x_1[n-5]$
- Again, let $x_2[n] = x_1[n-n_0]$ be the shifted input and $y_2[n]$ be the output using the shifted input can be described as
  - $y_2[n] = 2x_2[n-5] = x_1[n-n_0-5] = y_1[n-n_0]$
- As $y_2[n] = y_1[n-n_0]$, then the system is **time-invariant**.

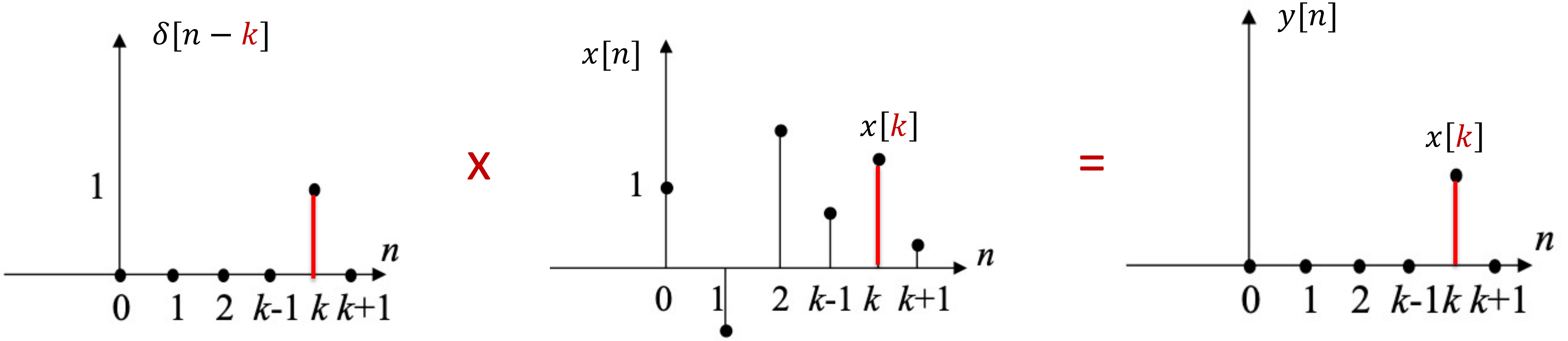
Impulse Response and Convolution

Representation of Discrete-Time Signals by Sum of Scaled and Shifted Unit Impulse Signals (1)

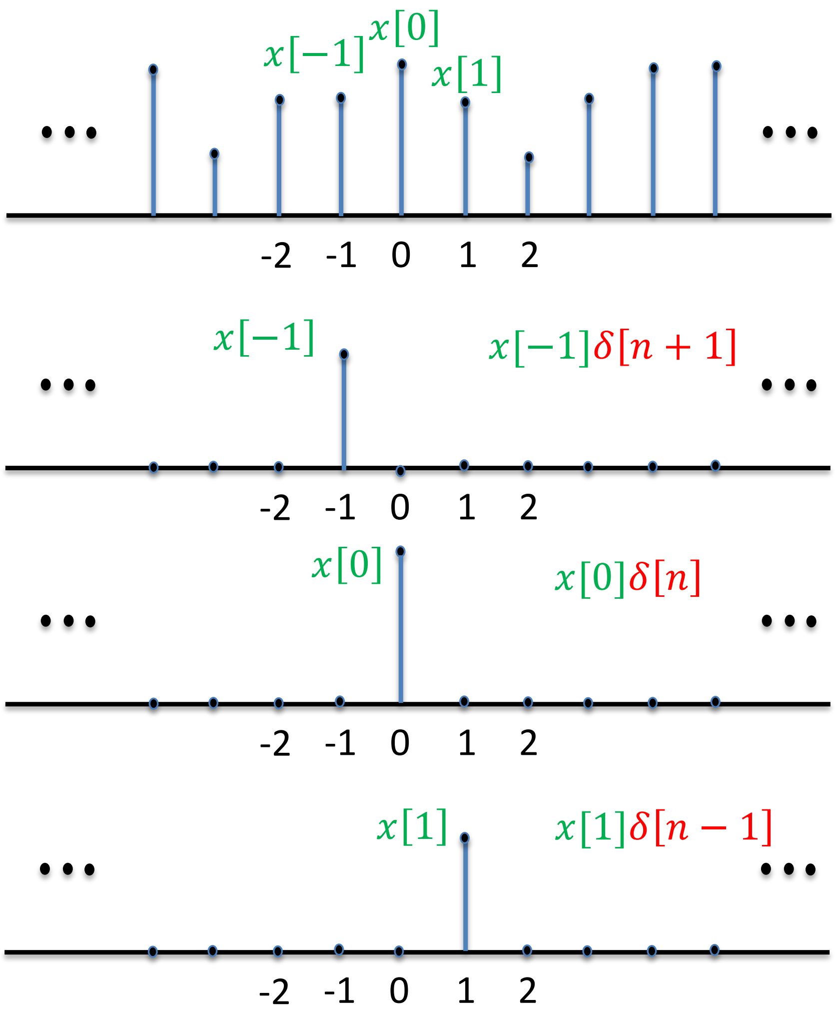
- A discrete-time signal, $x[n]$ may be shifted in time (delayed or advanced) by replacing the variables n with $(n-k)$ where $k>0$ is an integer
  - $x[n-k] \Rightarrow x[n]$ delayed by k samples
  - $x[n+k] \Rightarrow x[n]$ advanced by k samples
- For example, consider a shifted version of the unit impulse function. If we multiply an arbitrary signal $x[n]$ by this function, we obtain a signal that is zero everywhere, except at $n=k$.
  - $y[n] = x[n] \cdot \delta[n-k] = x[k] \cdot \delta[n-k]$

Representation of Discrete-Time Signals by Sum of Scaled and Shifted Unit Impulse Signals (2)

$$y[n] = x[k] \cdot \delta[n-k]$$



Unit Impulse based Composite Sequence Expression



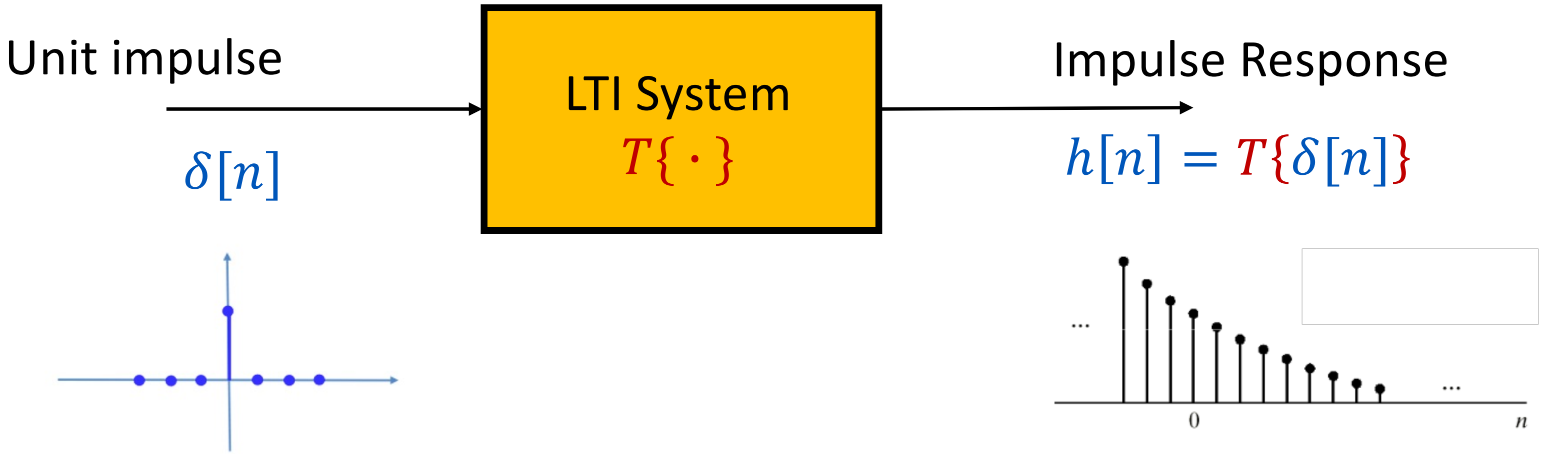
$$x[n] = \cdots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \cdots$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Any discrete-time signals can be expressed in terms of summation of scaled and shifted unit sample sequences $x[k]\delta[n-k]$.

Impulse Response

- If the input is unit impulse (unit sample) sequence $\delta[n]$, the corresponding output is called the impulse response $h[n]$ of the LTI system

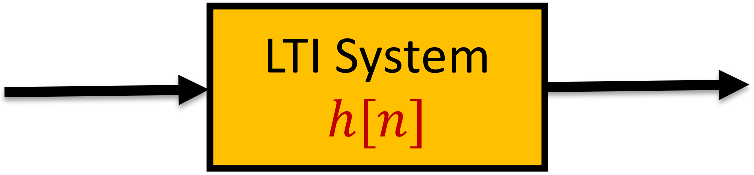


Convolution : Why Impulse Response is so important?

- The output of any LTI system is a convolution operation of the input signal with the unit impulse response:

Convolution operator

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



$$y[n] = x[n] * h[n]$$

$$y[n] = T\{x[n]\} = T\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\} = \sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

Linear Property　　　Time-Invariant Property

- Any Discrete-Time LTI system can be **completely characterized by its unit impulse response (*h*[*n*]).**

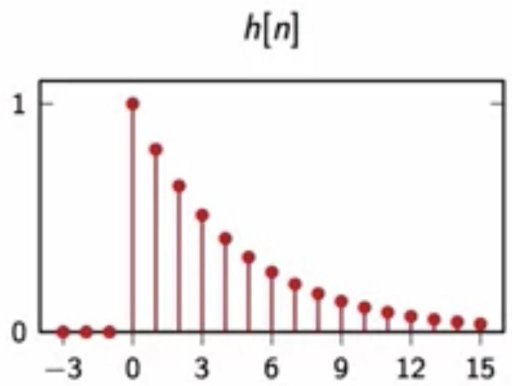
Performing the Convolution Algorithmically

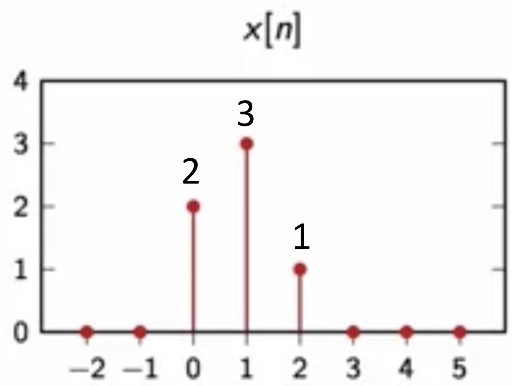
$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- **Ingredients**
  - A sequence $x[k]$
  - A second sequence $h[k]$
- **The recipe:**
  - Time-reverse $h[k]$
  - At each step n (from -∞ to ∞):
    - Center the time-reversed $h[k]$ in n (i.e. shift by $-n$)
    - Compute the inner product

Convolution Example 1

- Compute the $x[n] * h[n]$

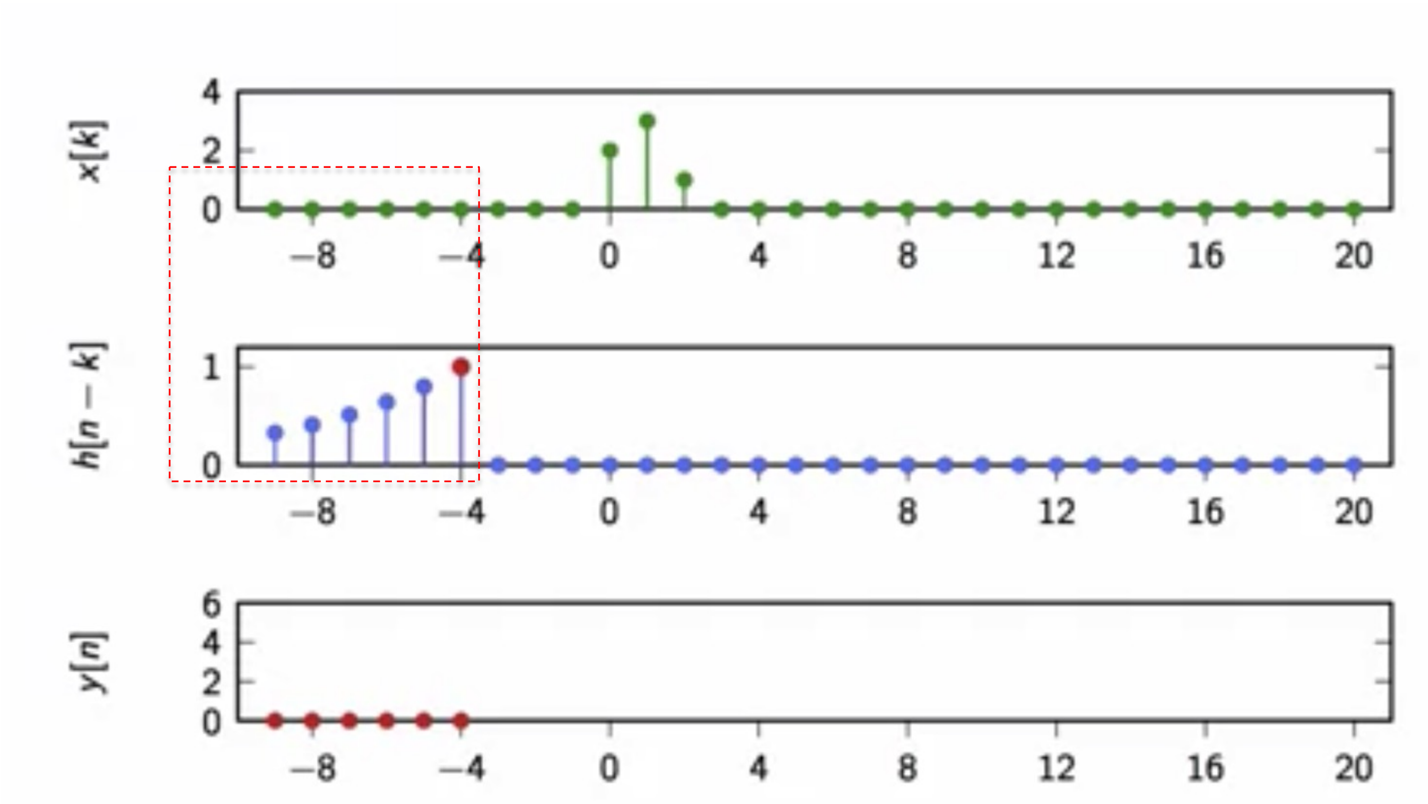




$$h[n] = \alpha^n u[n]$$

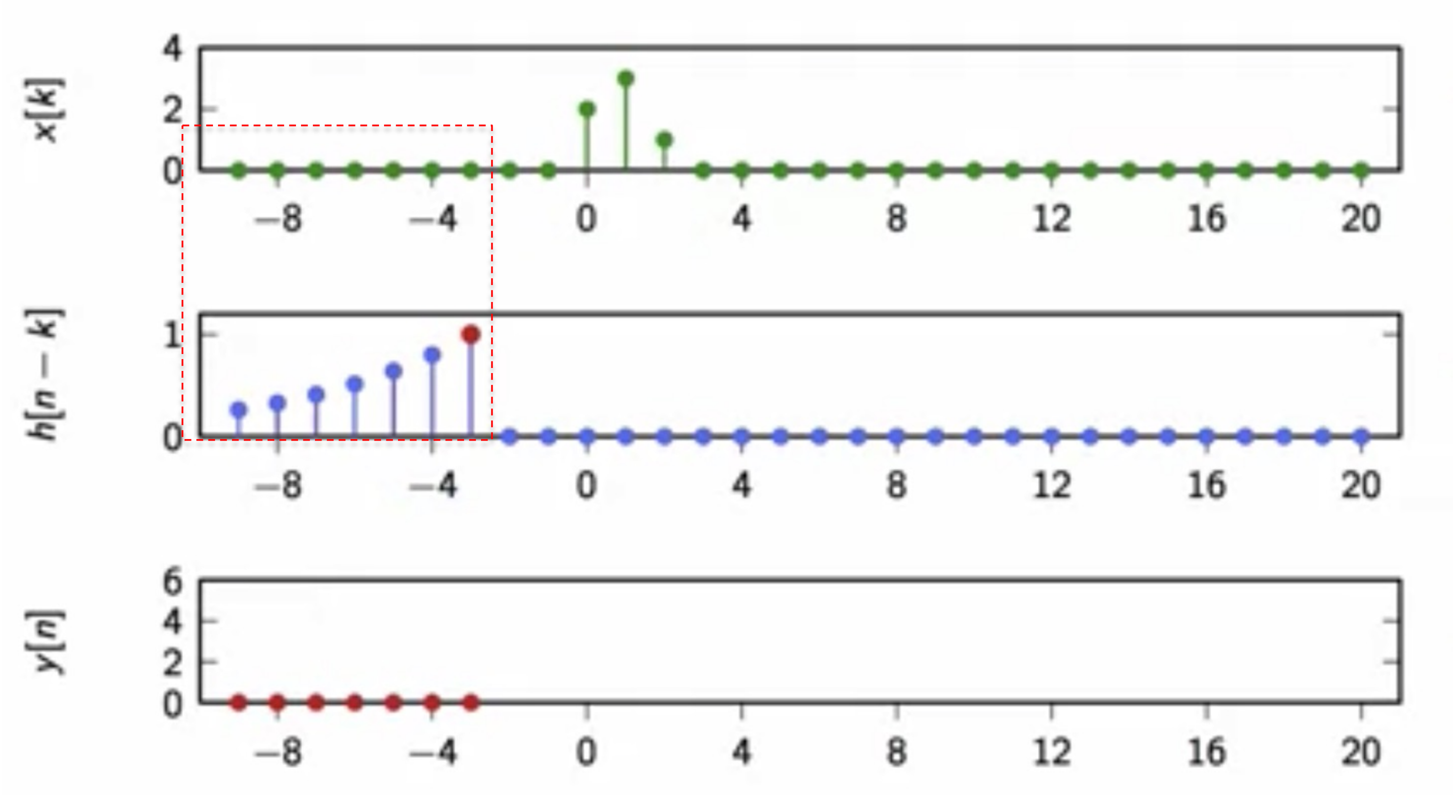
$$x[n] = \begin{cases} 2 & n = 0 \\ 3 & n = 1 \\ 1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

$x[n] * h[n]$ when n = -4



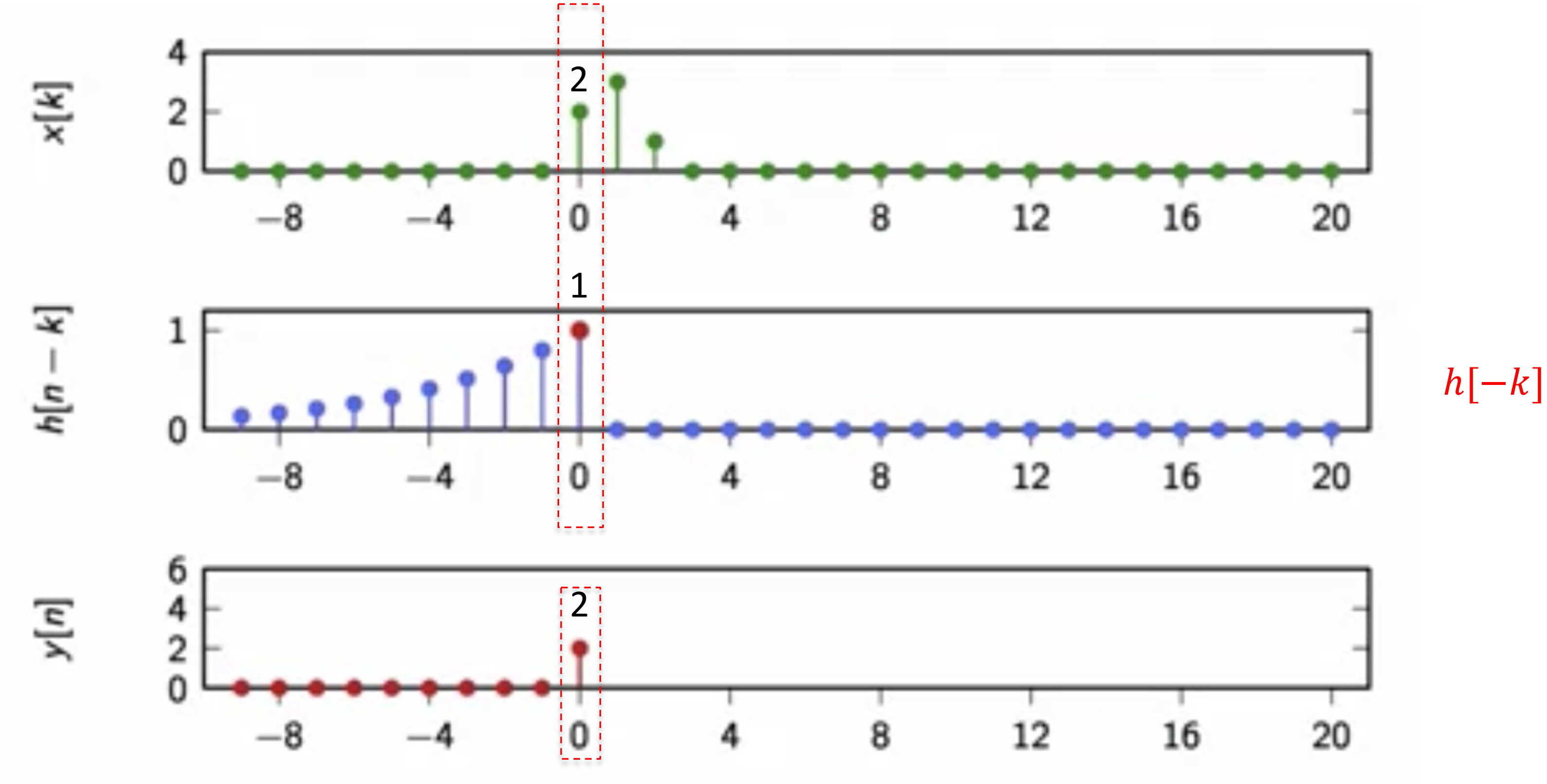
$h[-4-k]$

$x[n] * h[n]$ when n = -3

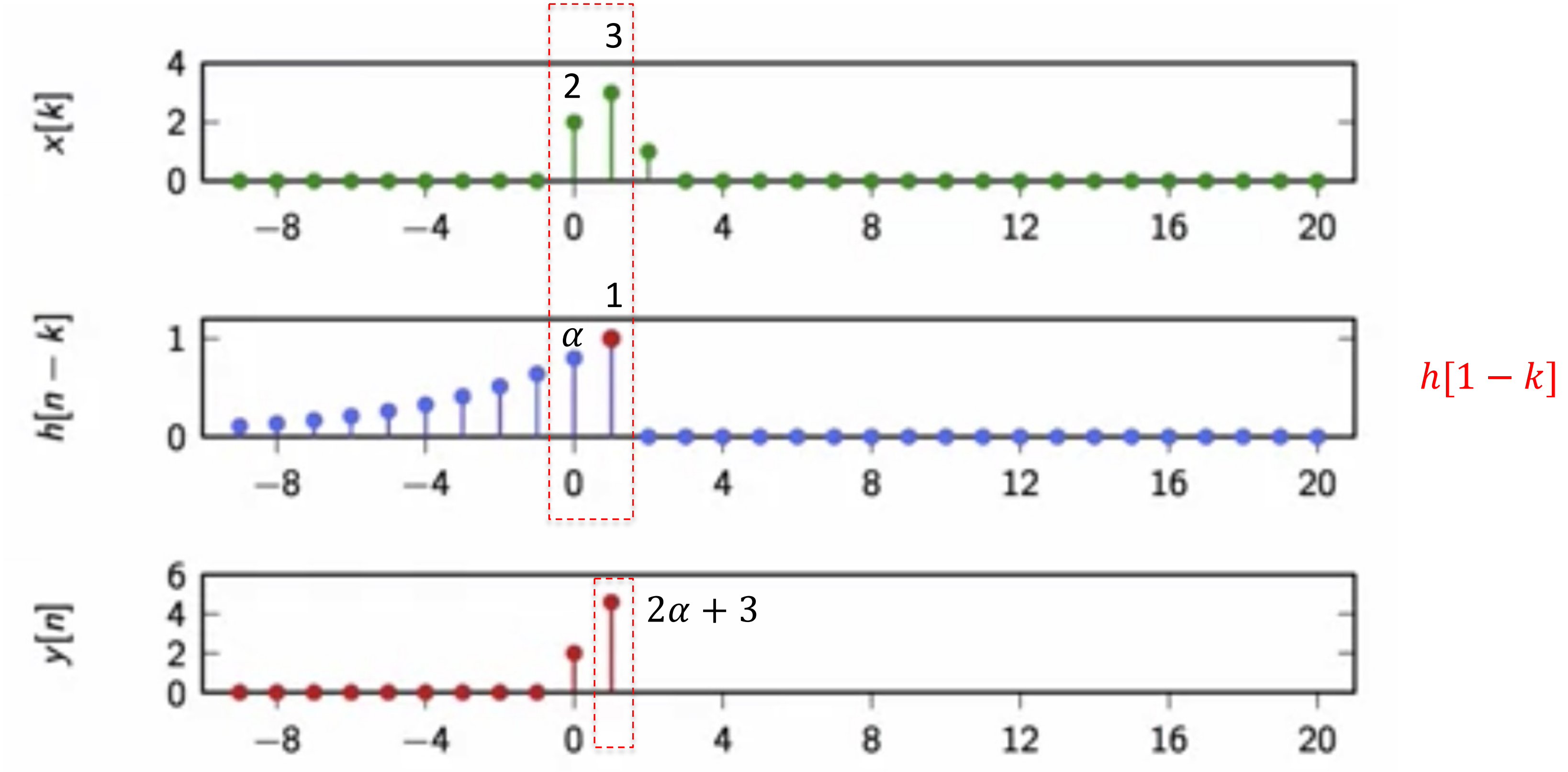


$h[-3-k]$

$x[n] * h[n]$ when n = 0

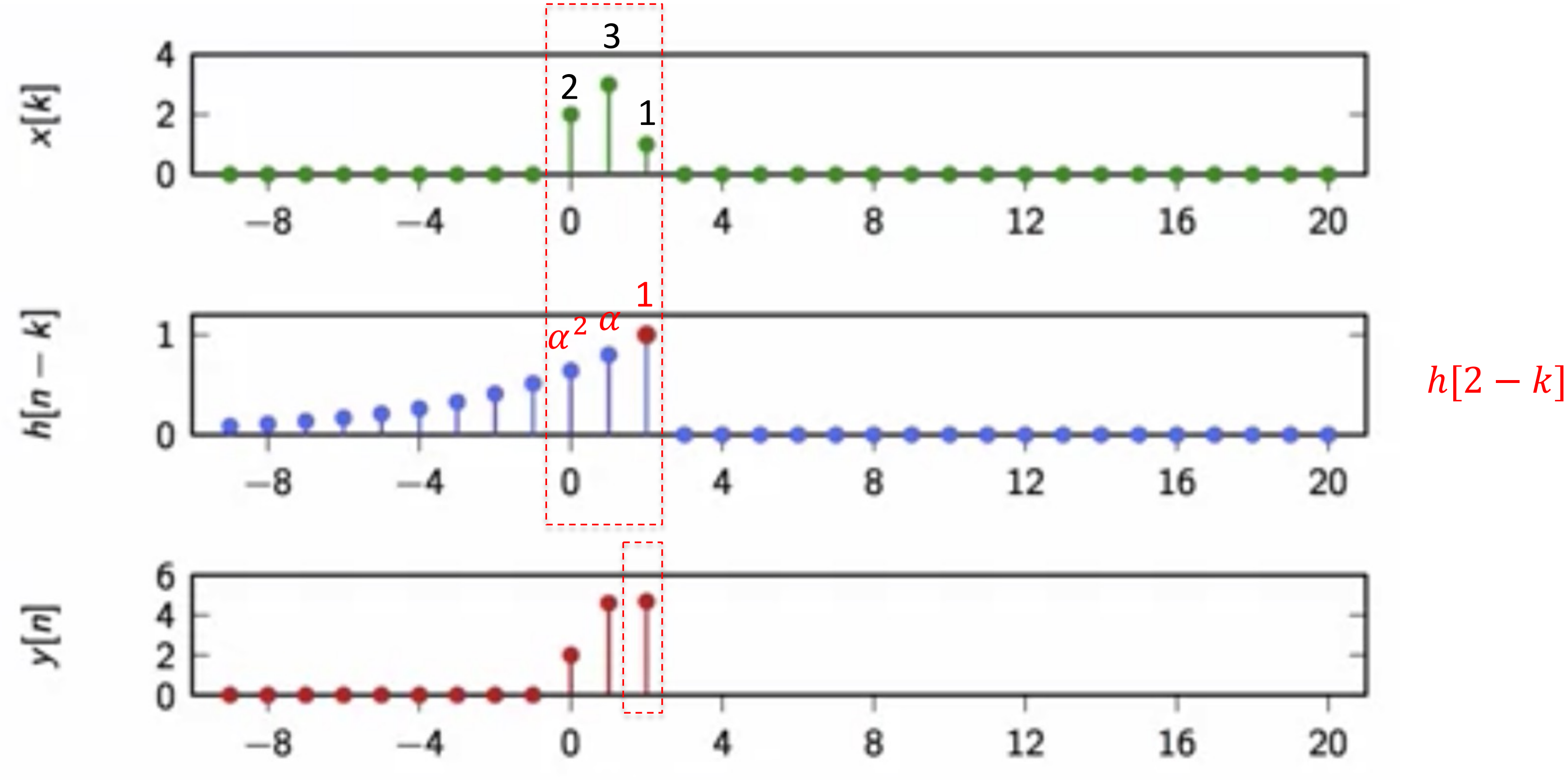


$x[n] * h[n]$ when n = 1

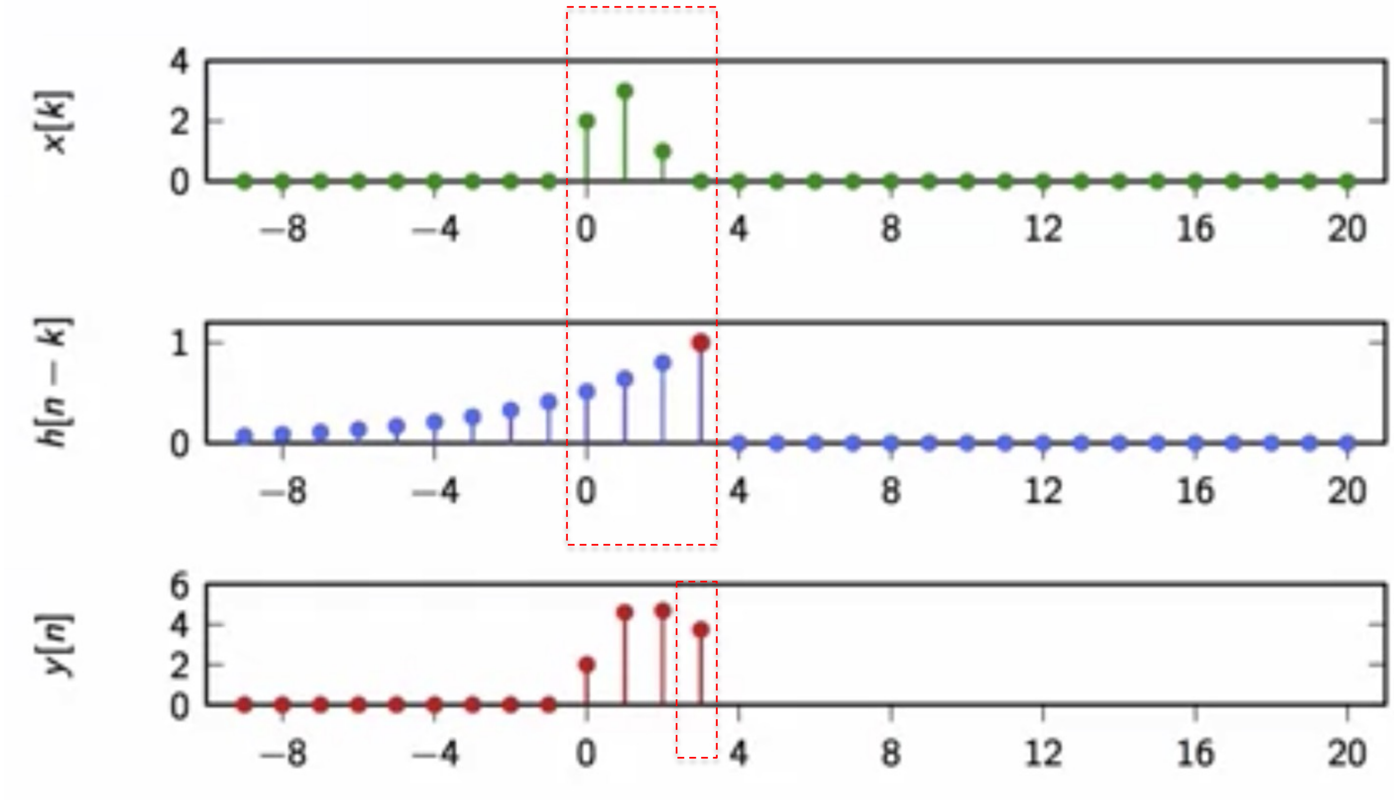


3
2
x[k]
4
2
0
−8
−4
0
4
8
12
16
20
1
α
h[n − k]
1
0
−8
−4
0
4
8
12
16
20
$h[1-k]$
y[n]
6
4
2
0
$2\alpha + 3$
−8
−4
0
4
8
12
16
20

$x[n] * h[n]$ when n = 2

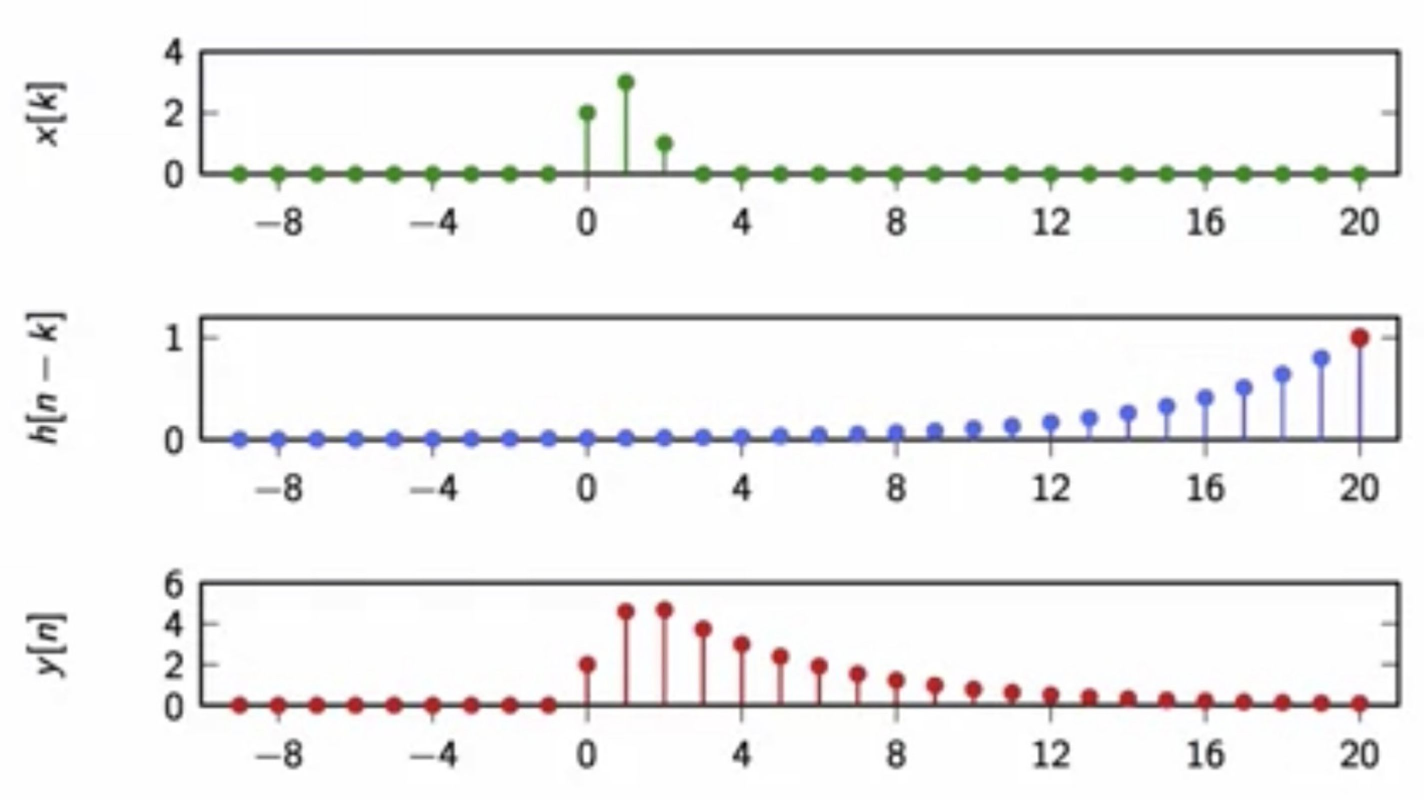


$x[n] * h[n]$ when n = 3



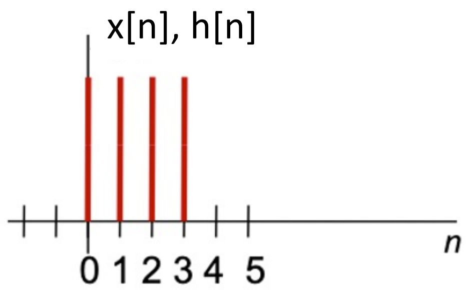
$h[3-k]$

$x[n] * h[n]$ for all n



Convolution Example 2

$$x[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad h[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

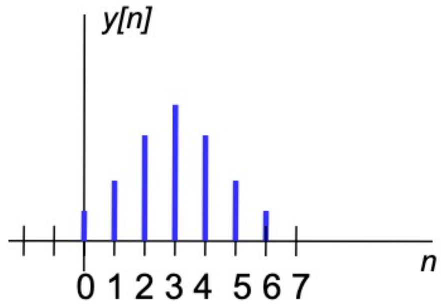


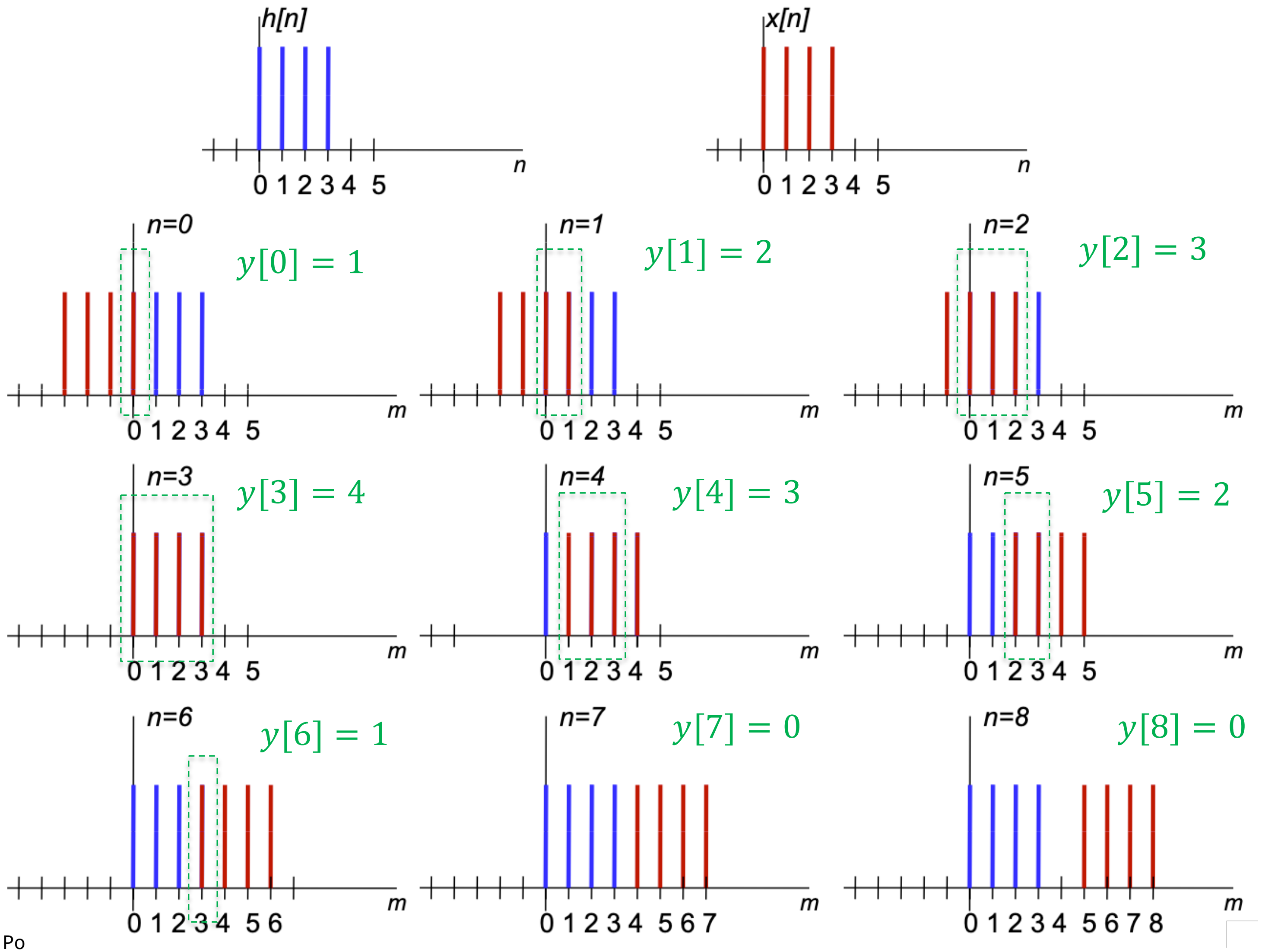
What is the output y[n] of the LTI system?

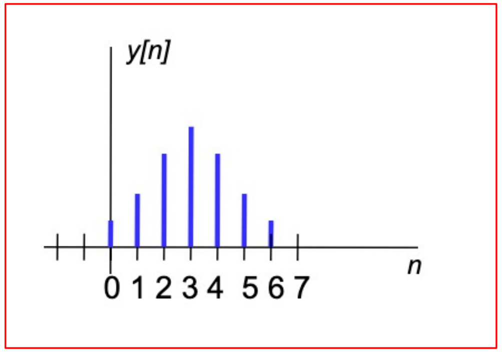
Solution:

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m]\; x[n-m]$$

$$= \begin{cases} \sum_{m=0}^{n} 1\cdot 1 = (n+1) & 0 \le n \le 3 \\ \sum_{m=n-3}^{3} 1\cdot 1 = (7-n) & 4 \le n \le 6 \\ 0 & n \le 0,\ n \ge 7 \end{cases}$$




h[n]
x[n]
n=0
y[0] = 1
n=1
y[1] = 2
n=2
y[2] = 3
n=3
y[3] = 4
n=4
y[4] = 3
n=5
y[5] = 2
n=6
y[6] = 1
n=7
y[7] = 0
n=8
y[8] = 0
n
m


y[n]
0 1 2 3 4 5 6 7
n

$$y[n] = h[n] * x[n]$$

Convolution Example 3

- The impulse response of an LTI system is of the form:
  - $h[n] = a^n u[n] \quad |a| < 1$
- And the input to the system is of form:
  - $x[n] = b^n\, u[n] \quad |b| < 1, b \neq a$
- Determine the output of the system using discrete convolution operation.

Solution:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$$

$$= \sum_{k=0}^{n} a^k b^{n-k} u[n] = b^n \sum_{k=0}^{n} a^k b^{-k} u[n] = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k u[n]$$

$$= b^n \left[\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}\right] u[n] = b^n \left[\frac{\frac{b^{n+1}-a^{n+1}}{b^{n+1}}}{\frac{b-a}{b}}\right] u[n] = \left[\frac{b^{n+1}-a^{n+1}}{b-a}\right] u[n]$$

Convolution Example 4

- Consider a discrete-time system with finite-duration input $x[n] = \{1,1,1,1\}$ and impulse response $h[n] = a^k u[n], \quad |a| < 1$.
- Determine the response $y[n]$ of this LTI system.

Solution:

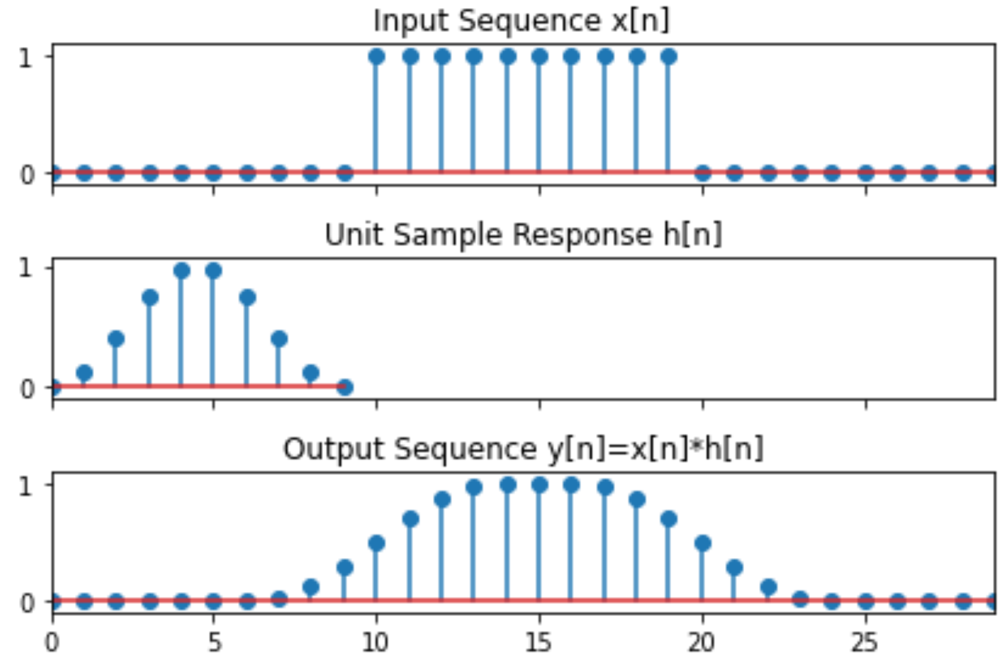
We recognize that x[n] can be written as the different between two unit-step sequences, i.e. $x[n] = u[n] - u[n-4]$. Hence, we can solve for $y[n]$ as the difference between the output of the system with a step input and the output of the system with a delayed step input. Thus, we solve for the response to a unit step as:

$$y_1[n] = h[n] * u[n] = \sum_{k=-\infty}^{\infty} a^k u[k] u[n-k] = \sum_{k=0}^{n} a^k u[n] = \left[\frac{1-a^{n+1}}{1-a}\right] u[n]$$

$$y[n] = h[n] * u[n] - h[n] * u[n-4] = y_1[n] - y_1[n-4] = \left[\frac{1-a^{n+1}}{1-a}\right] u[n] - \left[\frac{1-a^{n-3}}{1-a}\right] u[n-4]$$

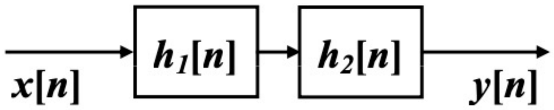
Convolution : Python Code

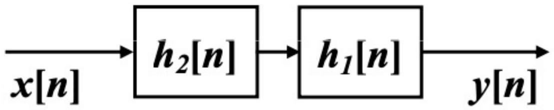
```
import matplotlib.pyplot as plt
from scipy import signal
import numpy as np

x = np.repeat([0., 1., 0.], 10)
h = signal.hann(10)
y = signal.convolve(x, h, mode='same') / sum(h)

fig, (ax_x, ax_h, ax_y) = plt.subplots(3, 1, sharex=True)
ax_x.stem(x)
ax_x.set_title('Input Sequence x[n]')
ax_x.margins(0, 0.1)
ax_h.stem(h)
ax_h.set_title('Unit Sample Response h[n]')
ax_h.margins(0, 0.1)
ax_y.stem(y)
ax_y.set_title('Output Sequence y[n]=x[n]*h[n]')
ax_y.margins(0, 0.1)
fig.tight_layout()
fig.show()
```

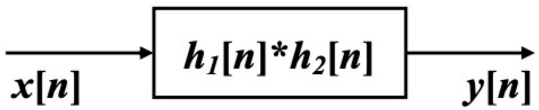


https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.convolve.html

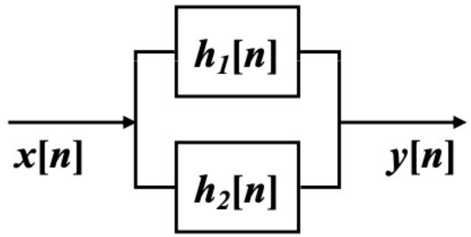
Convolution Properties

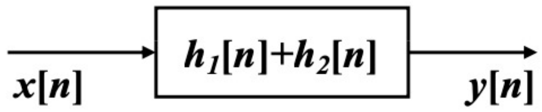






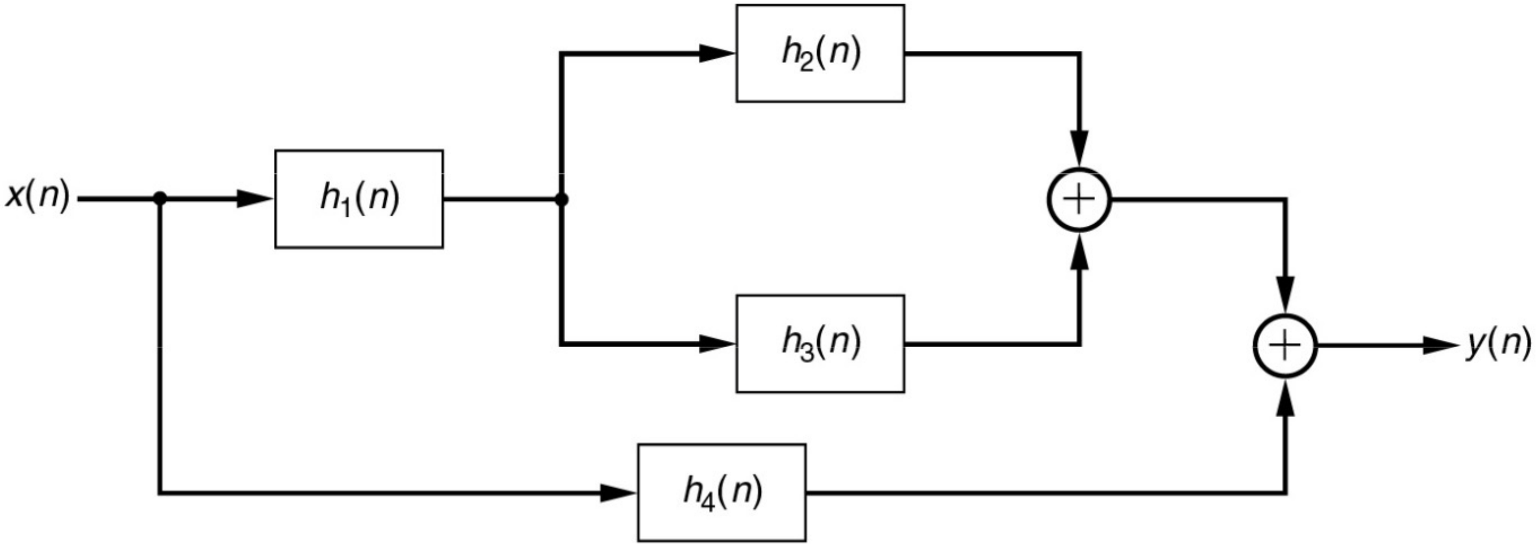
$$h_1[n]*h_2[n]= h_2[n]*h_1[n]$$





$$h_1[n]+h_2[n]= h_2[n]+h_1[n]$$

More Complex System Interconnections



$$y[n] = x[n] * h_c[n]$$

$$h_c[n] = h_1[n] * (h_2[n] + h_3[n]) + h_4[n]$$

Stability Analysis

Stability Analysis of LTI Systems

- Similar to the continuous-time LTI systems, we can analyze the stability of the system based on its impulse response.
- A Discrete-Time LTI system is Bounded-Input Bounded-Output (BIBO) stable if and only if its unit sample response $h[k]$ is absolutely summable.

$$S = \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

- Let $x[n]$ be a bounded input sequence {i.e. $|x[n]| < B_x$ for all n, where B_x is a finite number}. We must show that the output is bounded when S is finite. To this end, we work again with the convolution formula.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

- If we take the absolute value of both sides of this equation, we obtain

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]x[n-k]\right|$$

- Now, the absolute value of the sum of terms is always less than or equal to the sum of the absolute values of the terms

$$|y[n]| \leq B_x \sum_{k=-\infty}^{\infty} |h[k]| \leq B_x S$$

- Hence, since both B_x and S are finite, the output is also bounded, i.e., an LTI system is stable if its unit sample response is absolutely summable.

LTI System Stability Example (1)

- Check the stability of the first-order recursive system shown below:
  - $y[n] = a\, y[n-1] + x[n]$
- The impulse response of this system is
  - $h[n] = a^n u[n]$ for all formula
- Its stability factor *S* is

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \sum_{n=0}^{\infty} |a|^n$$

LTI System Stability Example (2)

- It is obvious that S is unbounded for $|a| \geq 1$, since each term in the series are greater than or equal to 1.
- For $|a| < 1$, we can apply the infinite geometric sum formula, to find

$$S = \frac{1}{1 - |a|} \quad for\ |a| < 1$$

- Since S is finite for $|a| < 1$, the system is stable.

Block Diagram Representation

Representations of LTI Discrete-Time Systems

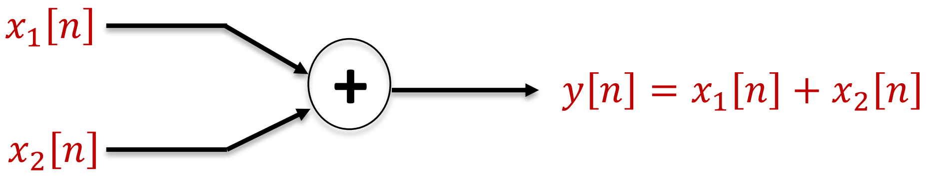
In general, LTI discrete-Time systems can be represented by

- **Unit Sample Response** : $h[n]$
- **Difference Equations** with or without feedback :
  - $y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x\{n-2]$
  - $y[n] = a_1 y[n-1] + a_2 y[n-1] + b_0 x[n] + b_1 x[n-1] + b_2 x\{n-2]$
- **Block Diagram** with basic operation elements
  - Adders, Constant Multipliers, and Unit Delay Elements
- **Transfer Function** in z-domain :
  - $H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$

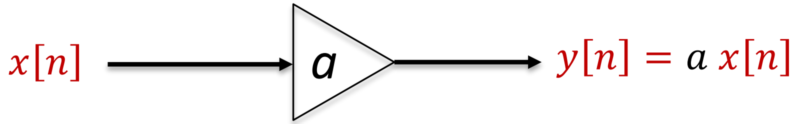
Block Diagram Representation of DT System

Some basic blocks that can be interconnected to form complex systems.

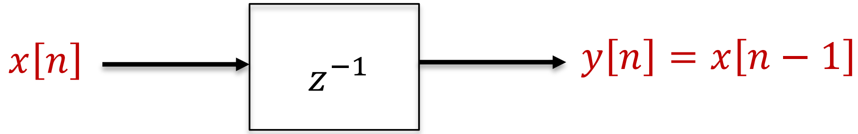
- **An Adder** : A system that performs the addition of two signal sequences to form another sequence



- **A Constant Multiplier** :This operation simply represents applying a scale factor on the input $x[n]$.

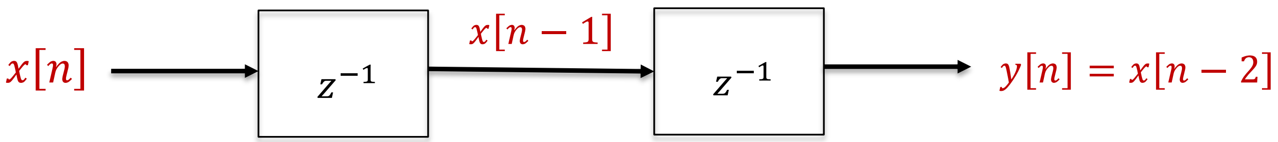


- **A Unit Delay Element** : The unit delay is a special system that simply delays the signal passing through it by one sample.

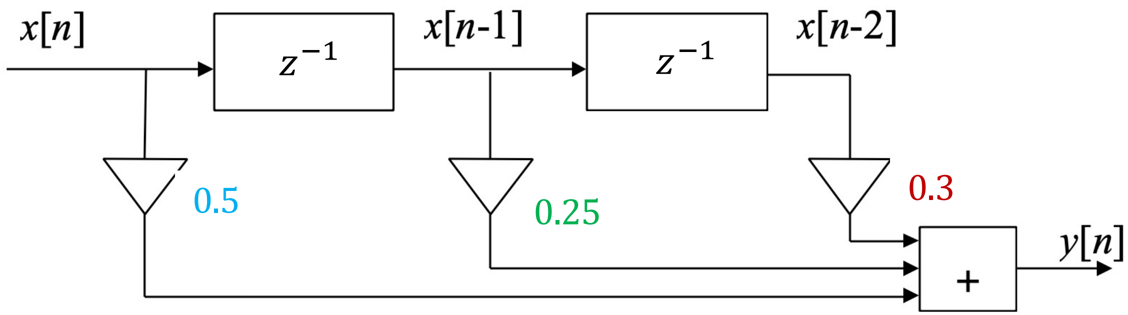


Difference Equation Representation of DT System

- Example 1: $y[n] = x[n-2]$

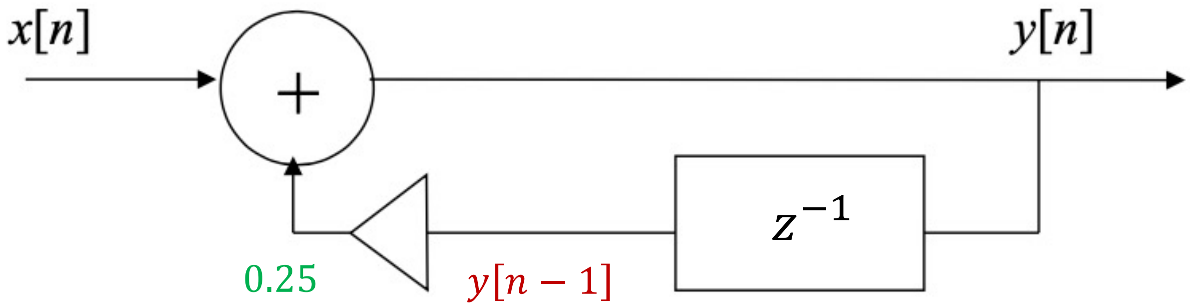


- Example 2: $y[n] = 0.5x[n] + 0.25x[n-1] + 0.3x[n-2]$



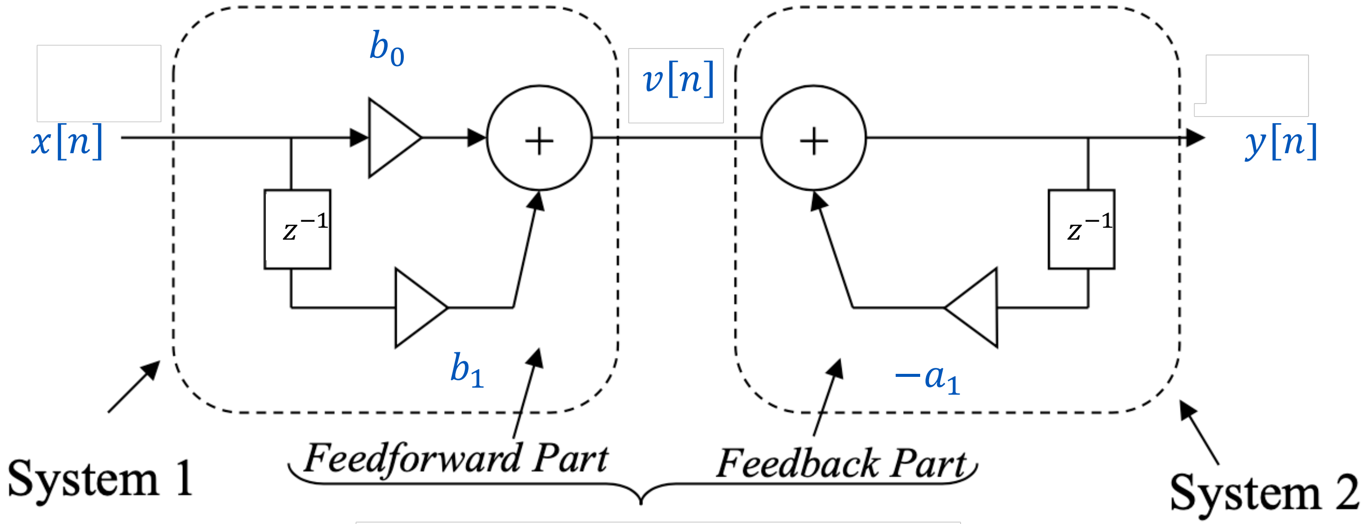
Difference Equation with Feedback

- Example 3: $y[n] = x[n] + 0.25y[n-1]$



Direct Form Structure (or Direct Form I)

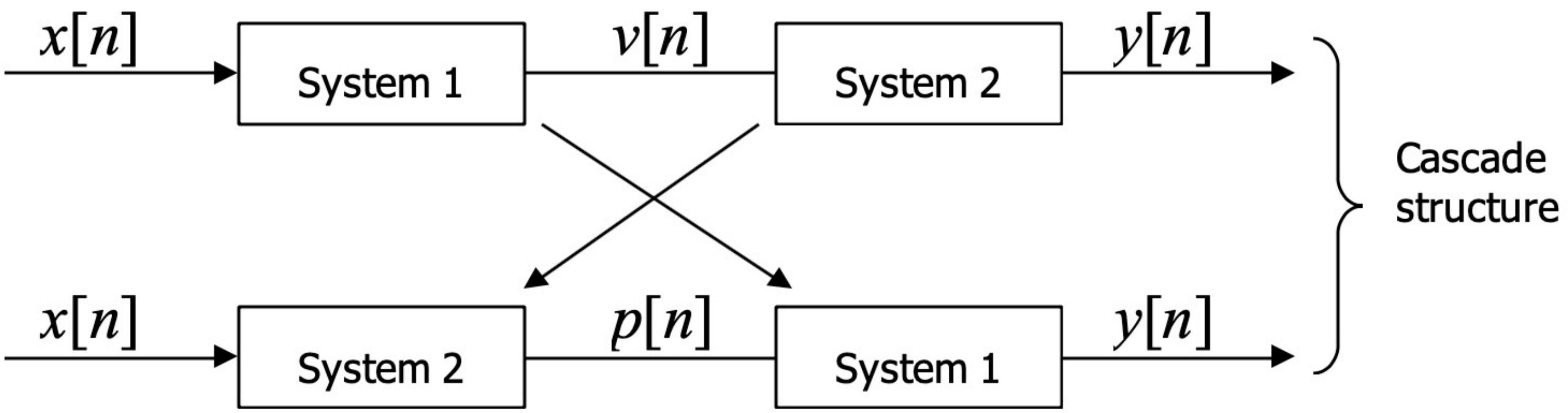
- Draw a system implementation for the below difference equation.
  - $y[n] = b_0 x[n] + b_1 x[n-1] - a_1 y[n-1]$
- We can write the above difference equation as a set of two equations
  - $v[n] = b_0 x[n] + b_1 x[n-1]$ and $y[n] = v[n] - a_1 y[n-1]$



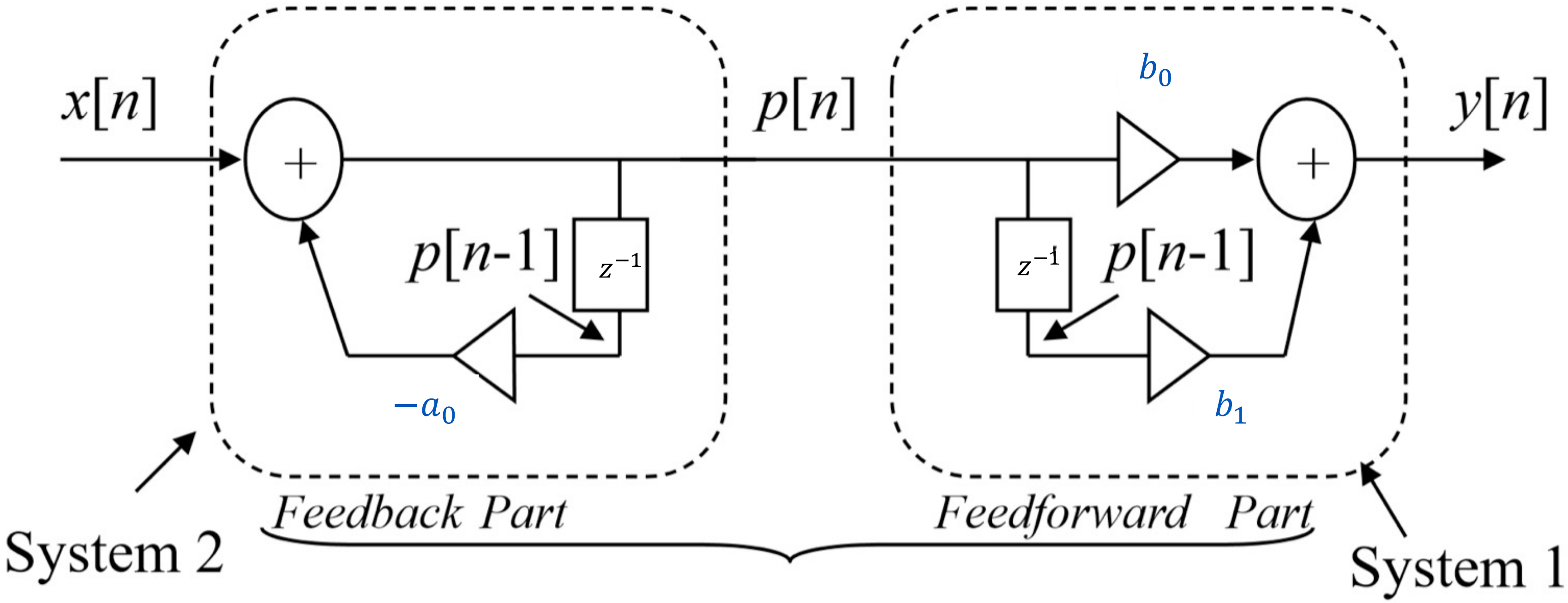
Direct Form I Structure

Cascade Structure

- Without changing the input-output relationship, we can reverse the ordering of the two systems in the cascade representation.



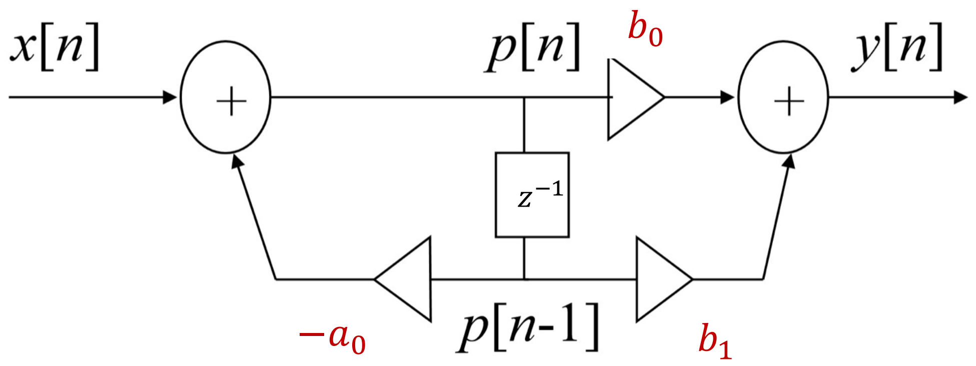
Canonic Form Structure (or Direct Form II)



Direct Form II Structure

Canonical Form

- There is no need for two delay operations in the Direct Form II structure, they can be combined into a single delay.

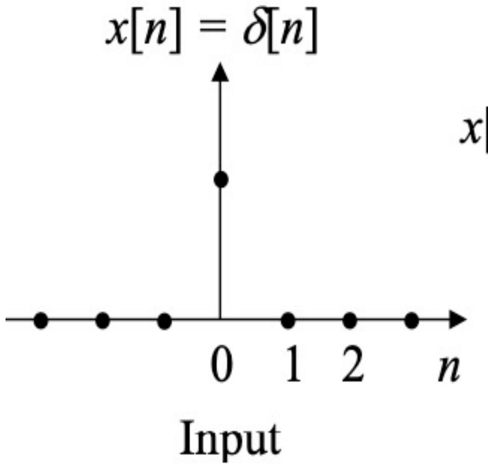


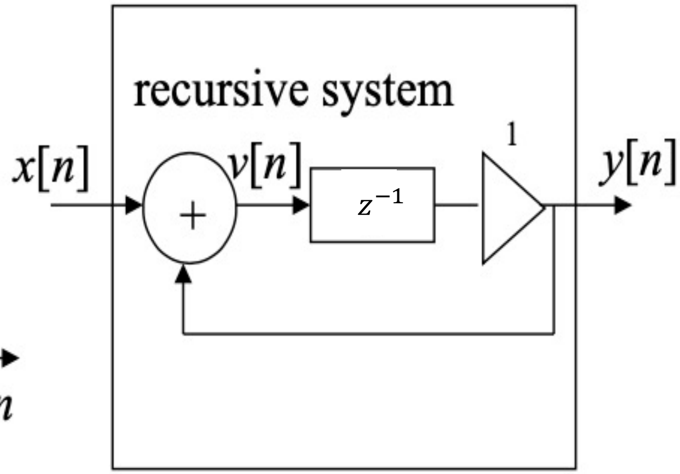
- Since delay operations are implemented with memory in a computer, this Canonical Form implementation would require less memory compared to the implementation of Direct Form I and II structure.

IIR and FIR Systems

Infinite Impulse Response (IIR) System

- If the impulse response of an LTI system is of infinite duration, the system is said to be an Infinite Impulse Response (IIR) system.
- Example
  - $v[n] = x[n] + y[n]$
  - $y[n] = v[n-1]$
  - $y[n] = x[n-1] + y[n-1]$





- If $x[n] = \delta[n]$, calculate $h[n]$ for n =0,1,2,...

Find the Impulse Response based Difference Equation

- Find the impulse response $h[n]$ of the following first-order recursive system.

$$y[n] = \begin{cases} ay[n-1] + x[n] & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- To find $h[n]$, we let $x[n] = \delta[n]$ and apply the zero-initial condition.

$$n = 0, y[0] = h[0] = ay[-1] + \delta[0] = 1$$

$$n = 1, y[1] = h[1] = ay[0] + \delta[1] = a$$

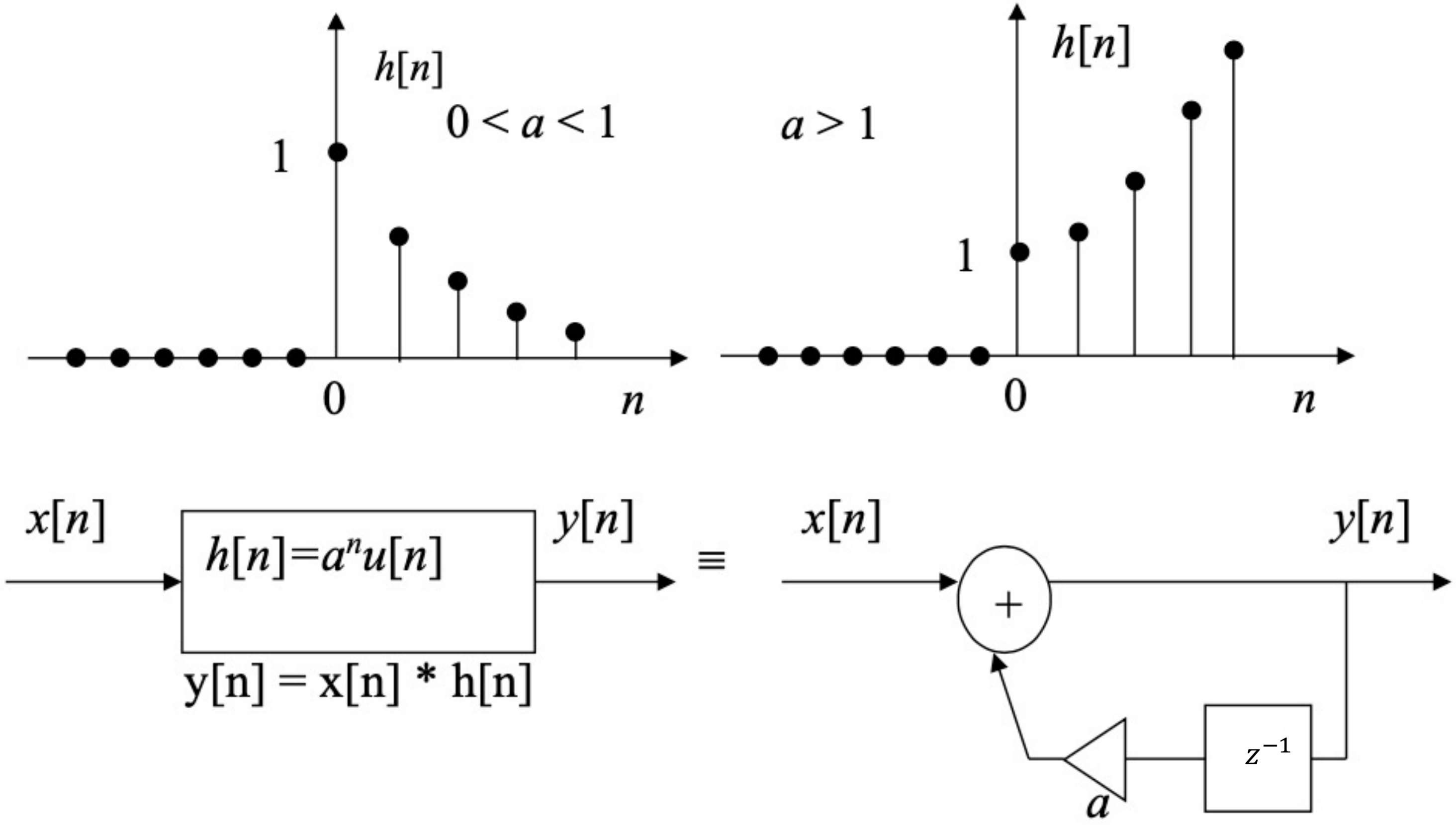
$$n = 2, y[2] = h[2] = ay[1] + \delta[2] = a^2$$

$$\vdots$$

$$n = n, y[\mathrm{n}] = h[\mathrm{n}] = a^n \quad \text{for } n \geq 0$$

Infinite-duration unit sample response

- $y[n] = h[n] = 0$ for $n < 0$, because $\delta[n]$ is zero for $n < 0$ and $y[-1] = 0$.
- Hence, $h[n] = a^n u[n]$ for all n



Finite Impulse Response (FIR) System

- Find the impulse response $h[n]$ of the following fourth order non-recursive system.

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + a_3 x[n-3] + a_4 x[n-4]$$

- To find $h[n]$,we let $x[n] = \delta[n]$

$n=0 \Rightarrow h[0] = a_0\delta[0] + a_1\delta[-1] + a_2\delta[-2] + a_3\delta[-3] + a_4\delta[-4] = a_0$

$n=1 \Rightarrow h[1] = a_0\delta[1] + a_1\delta[0] + a_2\delta[-1] + a_3\delta[-2] + a_4\delta[-3] = a_1$

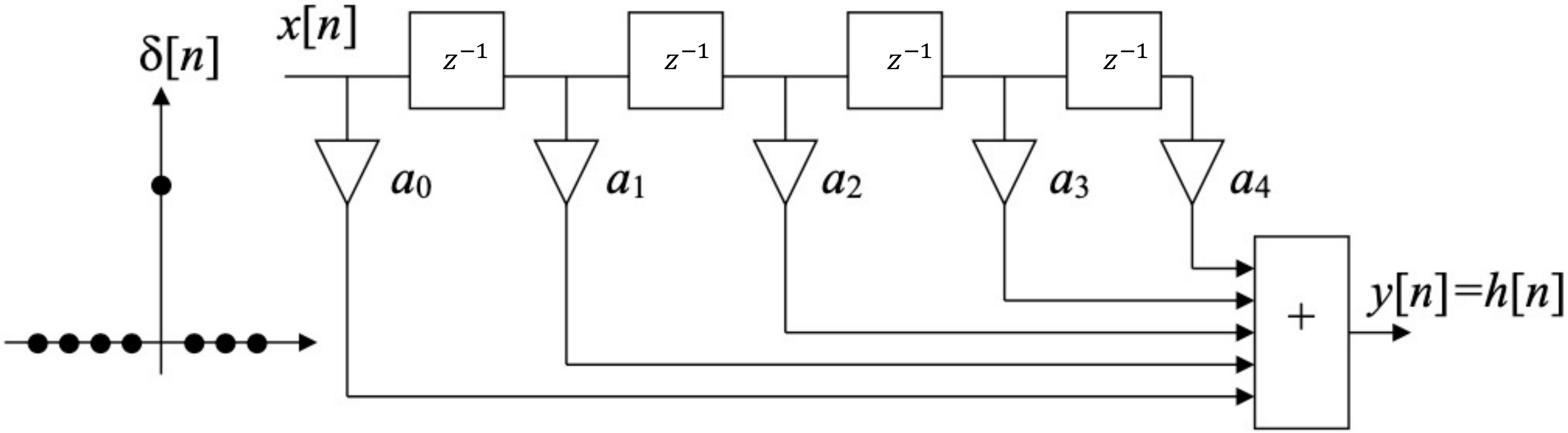
$n=2 \Rightarrow h[2] = a_0\delta[2] + a_1\delta[1] + a_2\delta[0] + a_3\delta[-1] + a_4\delta[-2] = a_2$

$n=3 \Rightarrow h[3] = a_0\delta[3] + a_1\delta[2] + a_2\delta[1] + a_3\delta[0] + a_4\delta[-1] = a_3$

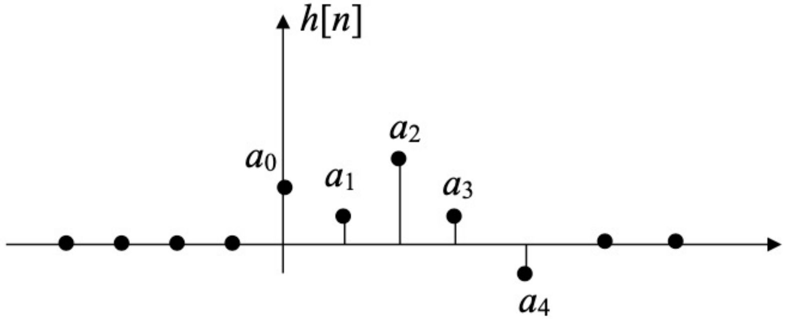
$n=4 \Rightarrow h[4] = 0 + 0 + 0 + 0 + a_4\delta[0] = a_4$

$n=5 \Rightarrow h[5] = 0 + 0 + 0 + 0 + a_4\delta[1] = 0$

- For $n \geq 5$, $h[n] = 0$, since the nonzero value of $\delta[n]$ has moved out of the memory of this system.



If the impulse response of an LTI system is of finite duration, the system is said to be an finite Impulse Response (FIR) system. In addition, non-recursive systems have finite impulse responses.



a_0, a_1, a_2, a_3 and a_4 are called coefficients (+ or -) or constants.

Summary (1)

- The definition of a discrete-time system
- The definition of the memory, Time Invariant, Linear, Causal and Stable Systems
- Interpretation of a discrete-time signal as a weighted sum of delayed impulses
- Definition and understanding of convolution (including hand and graphical computation of convolution)
- The impulse response of a linear and time-invariant system, and how to calculate it from a difference equation

Summary (2)

- Basic blocks of a discrete-time system: the adder, multiplier and unit delay
- How to draw the block diagram of a discrete-time system given its difference equation
- How to write the difference equation of a discrete-time system given its block diagram
- The difference between Direct Forms I and II and Canonical Form
- The difference between an FIR and an IIR system. In particular, that the impulse responses of FIR systems have identical values to the coefficients of the difference equation.
- Given the impulse responses of two cascaded systems, be able to compute the overall impulse response.