EE4015 Digital Signal Processing

Mid-Term Exam

Date: 8th November 2022 (Tuesday)

Answer **ALL** questions:

Question 1 **[30 marks]**

A discrete-time LTI system with input $x[n]$ and output $y[n]$ is described by the following relationship:

$$y[n] = 3x[n] + x[n-1] - 3x[n-2]$$

(a) Is the system memoryless? Justify your answer. **[3 marks]**

(b) Is the system causal? Justify your answer. **[3 marks]**

(c) Is the system BIBO stable? Justify your answer. **[3 marks]**

(d) Compute the impulse response $h[n]$ of the system. **[3 marks]**

(e) Determine whether the system is an FIR system or an IIR system based on the impulse response $h[n]$. Justify your answer.

[4 marks]

(f) Is it a linear-phase system? Justify your answer based on the impulse response.

[4 marks]

(g) Compute $y[n]$ when $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$. **[6 marks]**

(h) Determine and sketch the magnitude response $|H(e^{j\omega})|$ of the system. Based on the amplitude response, determine which type of frequency selective filter (lowpass, highpass, bandpass or bandstop) the system should belong to? **[4 marks]**

Question 2 **[20 marks]**

Consider a continuous-time (CT) signal $x(t)$ expressed as

$$x(t) = 2\cos(30\pi t) + 4\cos(80\pi t)$$

(a) Determine the Nyquist frequency and Nyquist rate of the signal $x(t)$. **[4 marks]**

(b) Determine the Continuous-Time Fourier Transform (CTFT) $X(j\Omega)$ of the signal $x(t)$.

[6 marks]

(c) The signal $x(t)$ is sampled at $50Hz$ to become a Discrete-Time signal $x[n]$. Determine the mathematical expression of $x[n]$. **[6 marks]**

(d) If we use an ideal anti-imaging lowpass reconstruction filter, what is the CT signal $y(t)$ that we can reconstruct from the sampled signal? **[4 marks]**

<u>Question 3</u> [30 marks]

A causal LTI system is characterized by the following transfer function $H(z)$:

$$H(z) = \frac{1 - 0.24z^{-1}}{1 - 0.36z^{-2}}$$

(a) Let the system input and output be $x[n]$ and $y[n]$, respectively. Write down the difference equation that relate $x[n]$ and $y[n]$. **[4 marks]**

(b) Find all pole and zero locations of $H(z)$ and determine the stability of the system based on the region of convergence (ROC). **[6 marks]**

(c) Based on the pole and zero locations of $H(z)$ to determine whether the inverse system with transfer function of $G(z) = 1/H(z)$ is exist or not. Explain your answer. **[2 marks]**

(d) Determine the system impulse response $h[n]$. **[8 marks]**

(e) Compute the system output $y[n]$ when the input is $x[n] = (0.24)^n u[n-1]$ using z-transform. **[5 marks]**

(f) Compute the system frequency response $H(e^{j\omega})$ and then determine its magnitude and phase responses. **[5 marks]**

<u>Question 4</u> [20 marks]

The impulse response of a LTI discrete-time filter is:

$$h[n] = \begin{cases} \sin^2(\pi(n+1)/4), & n = 0,1,2 \\ 0, & otherwise \end{cases}$$

(a) Is the filter linear phase? Explain your answer. **[6 marks]**

(b) Compute the filter output $y[n]$ when the input is $x[n] = u[n-1]$. **[6 marks]**

(c) Determine the Discrete Fourier transform (DFT) of $h[n]$. **[8 marks]**