

# A NOVEL ORDERED-SPIHT FOR EMBEDDED COLOR IMAGE CODING

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## ABSTRACT

The magnitude distribution law of DWT coefficients of color image data in Karhunen-Loeve space is investigated. Considering the KLT matrix formed, the magnitude of DWT coefficients in components  $K_1$ ,  $K_2$  and  $K_3$ , in general, have the relation  $K_1 > K_2 > K_3$  where  $K_1$ ,  $K_2$ ,  $K_3$  are the three components of a color image in KL space. Based on this characteristic and other DWT coefficient features, a novel ordered-SPIHT for embedded color image coding (OCSPIHT) is proposed in this paper. The set of DWT coefficients of an image in KL space are split into several sets according to their magnitudes. For each specific threshold, only the sets with coefficients equal to or larger than the threshold will be encoded. This can save many bits and improve coding performance especially at low bit rate.

## 1. INTRODUCTION

Set Partitioning In Hierarchical Trees (SPIHT) [1] is an embedded image compression algorithm with encoding efficiency comparable to EBCOT [2] which is employed in JPEG2000 [3]. The lower complexity and computation requirement of SPIHT is particularly attractive.

In the implementation of SPIHT, the significant information is stored in three ordered lists -- list of insignificant sets (LIS), list of insignificant pixels (LIP) and list of significant pixels (LSP). SPIHT consists of two main passes -- sorting pass and refinement pass -- to encode an image. The LIS and LIP entries are encoded in the sorting pass while the LSP entries are encoded in the refinement pass.

Apart from being a still image coding algorithm, SPIHT has been extended to color images [4-6] and video compression [6,7]. In [6], SPIHT is applied to color images and gives embedded color image bitstream. In the scheme, LIP and LIS are initialized with the appropriate coordinates of LL subband in all three color planes, SPIHT is then used to code all the pixels and sets in LIP and LIS. Fig. 1 shows the initial internal structure of the LIP and LIS, where  $Y$ ,  $C_r$  and  $C_b$  stand for the coordinates of each root pixel in a color plane. The  $Y$ ,  $C_r$  and  $C_b$  components have the same size. This method gives fully embedded bitstream and has a precise rate control.

YYYYYYYYY  $C_r C_r C_r C_r C_r C_r C_r C_r C_r C_r$   $C_b C_b C_b C_b C_b C_b C_b C_b C_b C_b$

Fig.1 Initial structure of LIP and LIS,  $Y$ ,  $C_r$  and  $C_b$  are the same size.

Another embedded color image compression algorithm—KLT+SPIHT – is given in [4]. First Karhunen-Loeve transform is applied to the color components (RGB or  $Y C_r C_b$ ). Then the wavelet transform is applied to the three resulting color components,  $K_1$ ,  $K_2$ ,  $K_3$ . SPIHT algorithm is applied to the three KLT components as in [6]. KLT+SPIHT can give better coding performance because KL transform further decorrelates the three color components  $Y$ ,  $C_r$  and  $C_b$  [8].

Kassim and Lee [5] proposed an improved embedded color image compression algorithm, called CSPIHT, in  $Y C_r C_b$  space based on the distribution characteristic of DWT coefficients that the magnitude of DWT coefficients in the luminance plane ( $Y$ ) is generally larger than those in chrominance planes ( $C_r$  and  $C_b$ ). In CSPIHT, a comprehensive SOT (Spatial Orientation Tree) structure is created across different spectral planes. Luminance nodes in the LL subband that do not have any offspring (the node with \*) will have descendants in the LL subbands of the two chrominance planes, as shown in Fig.2. This allows 6 chrominance zerotrees and 8 pixels to be encoded

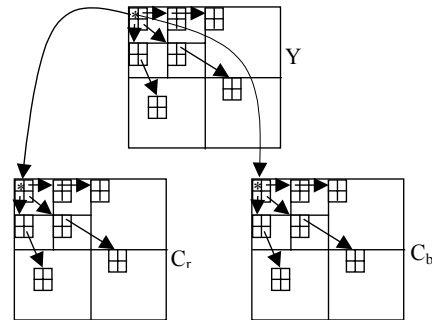


Fig.2 SOT structure in different spectral planes in CSPIHT scheme.

as a set. Coding performance can be improved especially at low bit-rate.

In this paper, we analyze the magnitude distribution characteristic of DWT coefficients in KL space, and propose a novel ordered-SPIHT for embedded color image coding with better coding performance than that in [4] and [5].

## 2. NOVEL ORDERED-SPIHT

### 2.1. The magnitude distribution law of KL-DWT coefficients

KL transform allows largest decorrelation among the three color components[8]. Encoding color image in KL space may

improve coding efficiency. We observed that the magnitude of DWT coefficients of the three KLT components are different.

Let  $K_1, K_2, K_3$  denote the three KLT components, the transform is expressed as

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = A \begin{bmatrix} a - m_a \\ b - m_b \\ c - m_c \end{bmatrix} \quad (1)$$

where  $a, b, c$  are color image row vectors representing RGB,  $YCbCr$  or YUV components. The row vectors  $m_a, m_b$  and  $m_c$  denotes the means of  $a, b$  and  $c$  respectively.  $A$  is the KLT matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

where  $[a_{i1} \ a_{i2} \ a_{i3}]$ ,  $i=1,2,3$  are eigenvectors of the covariance matrix  $C$  for the three color vector  $a, b$ , and  $c$  given as

$$C = \begin{bmatrix} \frac{1}{N}aa^T - m_a^2 & \frac{1}{N}ab^T - m_a m_b & \frac{1}{N}ac^T - m_a m_c \\ \frac{1}{N}ba^T - m_b m_a & \frac{1}{N}bb^T - m_b^2 & \frac{1}{N}bc^T - m_b m_c \\ \frac{1}{N}ca^T - m_c m_a & \frac{1}{N}cb^T - m_c m_b & \frac{1}{N}cc^T - m_c^2 \end{bmatrix} \quad (3)$$

where  $N$  is the total of image pixels.

Let  $[a_{i1}, a_{i2}, a_{i3}]$  and  $v_i, i=1,2,3$ , where  $v_1 \geq v_2 \geq v_3$ , be the eigenvector and eigenvalue of covariance matrix  $C$ . In general, we have  $MSB_1 \geq MSB_2 \geq MSB_3$ , where  $MSB_i, i=1,2,3$ , are the Maximum Significant Bit of DWT coefficients of component  $K_i, i=1,2,3$ .

Most wavelet based coding algorithms such as SPIHT and EZW utilize the property of DWT coefficients that the DWT coefficients in LL subband is generally larger than that in high frequency subbands. This property can also be observed from many color images as shown in Table 1 and 2.

Table 1: the MSB of Peppers (512\*512)

DWT L	4-level DWT				5-level DWT			
	LL	LH	HL	HH	LL	LH	HL	HH
K1	10	9	9	8	11	10	10	10
K2	9	8	8	7	10	10	9	9
K3	9	7	7	7	9	9	8	7

Table 2: the MSB of Lena (512\*512)

DWT L	4-level DWT				5-level DWT			
	LL	LH	HL	HH	LL	LH	HL	HH
K1	10	8	9	9	11	9	10	9
K2	8	7	7	6	9	7	8	7
K3	7	5	5	5	8	6	6	6

Table 1 and Table 2 give the MSB of different parts of DWT coefficients of images Peppers and Lena. The DWT coefficients of each KLT component ( $K_1, K_2, K_3$ ) are disparted into 4 parts, LL, LH, HL and HH. LL refers to all the coefficients in the lowest subband, i.e. "LL" in Fig 3. LH refers to all the coefficients in the vertical high frequency subbands, i.e. "LH1" and "LH2" in Fig 3. HL refers to all the coefficients in the horizontal high frequency, i.e. "HL1" and "HL2" in Fig 3. HH refers to coefficients in the diagonal high frequency subbands, i.e. "HH1" and "HH2" in Fig. 3.

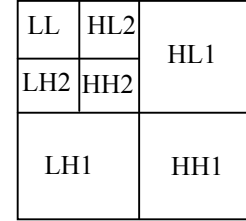


Fig3. 2-level DWT

From Table 1 and Table 2, we can see that: 1). The magnitude of DWT coefficients in components  $K_1, K_2$  and  $K_3$ , in general, have the relation  $K_1 > K_2 > K_3$ . 2). The magnitude of DWT coefficients in LL subband is larger than that in high frequency subbands, i.e. LH, HL and HH bands.

## 2.2. The proposed OCSPIHT

For KLT+SPIHT, the LIP and LIS are initialized as in Fig. 1. For each threshold, all the nodes in LIP and all the sets in LIS are coded using SPIHT. Based on the distribution law of KL-DWT coefficients introduced in section 2.1, for larger threshold, only some parts of DWT coefficients in three KLT components have significant coefficients.

For example, with threshold =  $2^0$  for peppers at 4-level DWT in Table 1, only  $K_1$ -LL part has significant coefficients. All the DWT coefficients in the other 11 parts of the three KLT components are insignificant; With threshold =  $2^9$ , only five parts ( $K_1$ -LL,  $K_1$ -LH,  $K_1$ -HL,  $K_2$ -LL,  $K_3$ -LL) have significant coefficients. All the DWT coefficients in the other 7 parts are insignificant. As it is not required to encode these insignificant parts, many bits can be saved. Based on this idea, we proposed a novel OCSPIHT to improve the coding efficiency. Similar to the two SPIHT, two versions of OCSPIHT will be described - the binary-uncoded version and the arithmetic coding version.

### 2.2.1. Binary-uncoded version OCSPIHT

We call the OCSPIHT scheme using binary-uncoded version SPIHT as binary uncoded version OCSPIHT. In this scheme, the DWT coefficients of each component,  $K_1, K_2, K_3$ , are disparted into 4 parts, LL, LH, HL and HH, as explained in section 2.1. SPIHT's SOT structure is used for all spectral planes. At the beginning, only the nodes in  $K_1$ -LL is encoded. Only the parts that have coefficients equal to or larger than the threshold will be encoded. It is very simple to accomplish the

OCSPIHT coding scheme. The implementation process is as follows.

**1). Initialization:**

output  $n = \lfloor \log_2(\max_{(i,j)} \{c_{i,j,1}\}) \rfloor$ ,  $c_{i,j,1}$  represents the DWT coefficient value of  $K_1$ ; add the nodes in  $K_1$ -LL to LIP, and LIS and LSP are set as empty list.

**2). Significance estimation of each part.**

2.1) Output  $n_k = \lfloor \log_2(\max_{(i,j)} \{c_{i,j,k}\}) \rfloor$ ,

$k = \{K_2 - LL, K_3 - LL\}$ ,  $c_{i,j,k}$  represents the DWT coefficient value in part k,  $K_2 - LL$  and  $K_3 - LL$  represent the LL subband of  $K_2$  and  $K_3$  respectively. If  $n_k = n$ , add nodes in part k to LIP.

2.2) Output  $n_k = \lfloor \log_2(\max_{(i,j)} \{c_{i,j,k}\}) \rfloor$ ,

$k = \{K_1 - LH, K_1 - HL, K_1 - HH, K_2 - LH, K_2 - HL, K_2 - HH, K_3 - LH, K_3 - HL, K_3 - HH\}$ , If  $n_k = n$ , add sets in part k to LIS.

**3). Bit-plane coding:**

Code LIP, LIS and LSP using SPIHT operation.

**4). Quantization-Step Update:**

Decrement n by 1 and go to step 2.

**2.2.2. Arithmetic coding version OCSPIHT**

We call the OCSPIHT scheme using arithmetic coding version SPIHT as arithmetic coding version OCSPIHT. In this scheme the DWT coefficients of each component  $K_1, K_2, K_3$ , are disparted into 2 parts -- the lowest subband (LL) and the high frequency subbands (all the coefficients except those in LL subband) because the significance of four sets grouped as 2\*2 needed to be arithmetic coded as a single symbol in this scheme. The implementation of arithmetic coding version OCSPIHT is similar to that of binary-uncoded version OCSPIHT.

In OCSPIHT, the SOT structure of all color components is same as that in SPIHT without comprehensive SOT structure across the different spectral planes as in CSPIHT[5]. OCSPIHT has less computation complexity than that of KLT+SPIHT because this scheme leads to fewer initialized nodes in LIP and LIS. Suppose that the LL subband has dimension X by Y, OCSPIHT requires XY nodes in initialized LIP, and 0 nodes in initialized LIS. CSPIHT[5] requires XY nodes in initialized LIP and XY nodes in initialized LIS. KLT+SPIHT requires 3XY nodes in initialized LIP and 9XY/4 nodes in initialized LIS. Thus, OCSPIHT has the fewest initial entries in LIP and LIS.

According to the OCSPIHT's coding scheme we can see that OCSPIHT would get better color compression efficiency and get a fully embedded bitstream with the feature of precise rate control.

**3. SIMULATION RESULTS**

OCSPIHT is applied to color images Peppers. The performance is compared against other two algorithms in  $YCbCr$  space.

Fig. 4 and Fig. 5 give the compression results. OCSPIHT is the ordered-SPIHT for embedded color image coding method proposed in this paper, KLT+SPIHT is introduced in [4], and CSPIHT is proposed in [5]. Two versions SPIHT (binary-

uncoded version and arithmetic coding version) are used in our simulations.

**3.1 Simulation results analysis for binary-uncoded version OCSPIHT**

From Fig 4, we can see that the coding performance of OCSPIHT is better than KLT+SPIHT and CSPIHT, especially at low bit-rate. The reasons are: 1). the DWT coefficients of each component  $K_1, K_2, K_3$ , are disparted into 4 parts (LL, LH, HL, HH), only the parts with coefficients larger than the threshold are encoded. This can save many bits as compared with KLT+SPIHT. 2). KL transform further decreases the correlation among the three color components ( $Y, C_r, C_b$ ) so OCSPIHT can get better performance than that of CSPIHT.

From Fig. 4, the coding performance of CSPIHT is better than KLT+SPIHT, which is the conclusion in [5].

**3.2 Simulation results analysis for arithmetic coding version OCSPIHT**

From Fig. 5 we can see that OCSPIHT is the best one in these three coding algorithms while KLT+SPIHT is better than CSPIHT. The reason that OCSPIHT is better than KLT+SPIHT is discussed in section 3.1. There are two reasons why CSPIHT is worse than KLT+SPIHT with arithmetic coding version SPIHT. They are 1). 4 pixels (sets) grouped as 2\*2 is encoded as a single symbol in arithmetic coding version SPIHT -- fewer saved symbols are saved in CSPIHT. 2). The coding efficiency arithmetic coding is affected by the probability distribution of the symbols. The larger the difference among the symbols' probability is, the higher the coding efficiency is. For CSPIHT, comprehensive SOT is a new set. The difference of probability between its two symbols is not large. Thus, the coding efficiency is not high.

**4. CONCLUSION**

KL transform is studied. It is found that if the KLT matrix is formed with the row vectors being the eigenvectors in the order of decreasing eigenvalue, the magnitude of DWT coefficients in components  $K_1, K_2$  and  $K_3$  will, in general, have the relation  $K_1 > K_2 > K_3$ . For each component, the magnitudes of DWT coefficients in LL subband are generally larger than that in high frequency subbands, and the magnitudes in different high frequency subbands are different.

Based on these distribution properties of DWT coefficients, a novel ordered-SPIHT for embedded color image coding, OCSPIHT, is proposed. The DWT coefficients of each color components  $K_1, K_2, K_3$ , are disparted into several parts (4 parts for binary-uncoded version OCSPIHT and 2 parts for arithmetic coding version OCSPIHT). Only the parts with coefficients larger than the threshold are encoded using SPIHT. This can save more bits. Simulation results show that OCSPIHT has higher coding efficiency than KLT+SPIHT and CSPIHT, especially at low bit-rate.

**5. ACKNOWLEDGMENTS**

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## 6. REFERENCES

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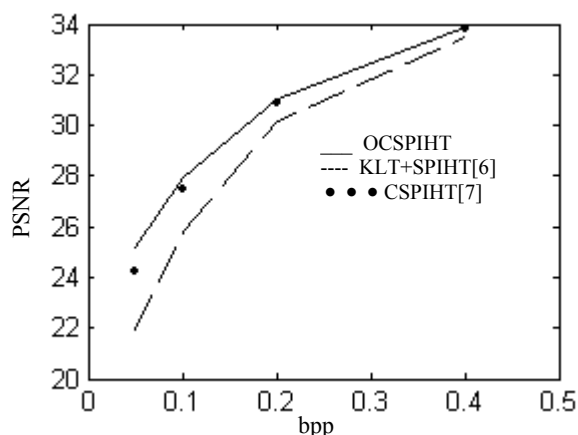
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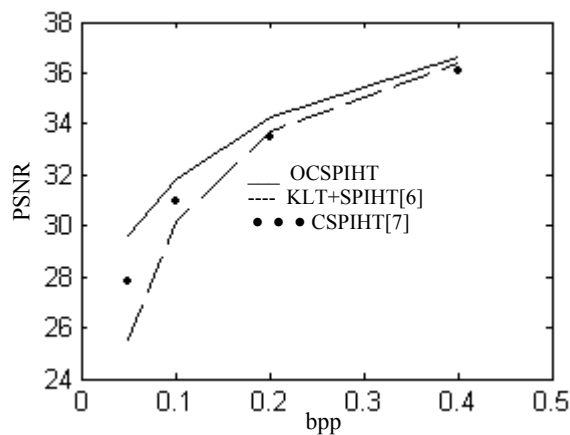
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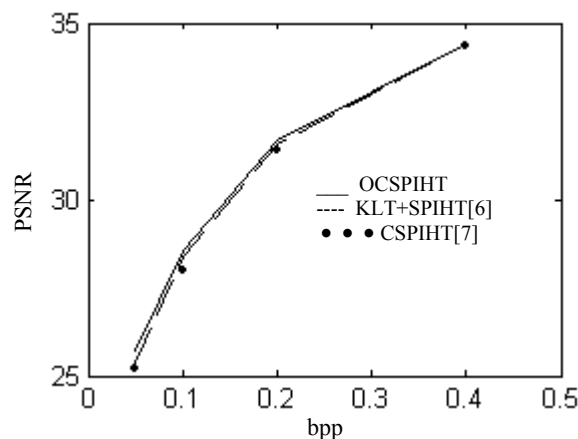


a). Luminance  $Y$

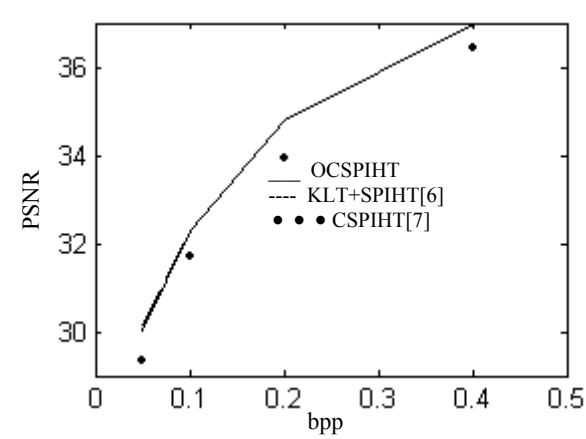


b). Chrominance  $C_r C_b$

Fig. 4. Performance compare for peppers (512\*512) at 4-level DWT without arithmetic coding



a). Luminance  $Y$



b). Chrominance  $C_r C_b$

Fig. 5. Performance compare for peppers (512\*512) at 4-level DWT with arithmetic coding