

How Does Correlation Affect the Capacity of MIMO Systems with Rate Constraints?

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Abstract—In this paper, we study the impact of antenna correlation at mobile units (MUs) on the capacity of multi-user multiple-input multiple-output (MIMO) systems with rate constraints. Both numerical results and asymptotic analyses show that antenna correlation can actually be regarded as a beneficial factor for such systems. It is also shown that the maximum eigenmode beamforming (MEB) based interleaved-division multiple-access (IDMA) system is an effective platform to exploit the potential gain provided by antenna correlation at MUs.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have attracted much research interest recently due to their significant capacity gains over single-antenna systems [1][2][3][5]. The work in [1][2] studies the capacity of a single-user/multi-user MIMO system under the assumption of independent and identically distributed (i.i.d.) fading at antenna elements. However, this assumption does not always hold in practical propagation environments when the antenna elements, especially those at a mobile unit (MU), cannot be spaced sufficiently far apart due to the limited physical size of the transmitter/receiver. Antenna correlation is then incurred in this case. Intuitively, antenna correlation has two consequences [3]. First, it reduces the effective dimensionality of the transmitter and/or receiver sides, with the related disadvantage most noticeable in the high rate/power regime. Second, it increases the potential of power focusing effect, with the related advantage most noticeable in the low rate/power regime. These two opposite effects result in a critical point: antenna correlation is advantageous below this point and *vice versa*. In a conventional single-user MIMO system, such a critical point occurs at a relatively low transmission rate, so antenna correlation is generally regarded as a negative factor [3][11].

In this paper, we investigate the impact of antenna correlation at MUs in multi-user MIMO systems with rate constraints, where minimizing the sum power under the given rate constraint of each MU is an appropriate objective [4]. The applications of these systems include delay-sensitive services such as speech and real-time video. Our focus is on MIMO multiple-access channels (MACs), but the results in this paper can be extended to MIMO broadcast channels (BCs) straightforwardly using the duality principle [5]. We show numerically that the critical point mentioned above occurs at a rate increasing with the number of MUs (denoted by K below).

This implies that the range where antenna correlation is advantageous becomes larger when K increases. We also quantify this advantage analytically in the limiting cases of $K \rightarrow \infty$ and/or $M \rightarrow \infty$, where M denotes the number of antennas at the base station (BS). Our results show that antenna correlation at MUs is potentially beneficial for multi-user MIMO systems with rate constraints. We also discuss implementation techniques. Related simulation results are provided to demonstrate the effectiveness of such techniques in exploiting the potential advantage provided by antenna correlation at MUs.

Our theoretical discussions on antenna correlation are based on capacity analysis (i.e., the theoretical limit of system performance), which distinguishes this paper from some existing work on related issues. For example, the focus of [6] is on a specific transmission strategy where the BS only transmits to one MU at a time; [7] is related to the zero-forcing beamforming strategy over BCs.

The notations used in this paper are as follows. \mathbf{I} denotes an identity matrix. For any matrix \mathbf{A} , \mathbf{A}^* and $\|\mathbf{A}\|$ are the conjugate transpose and 2-norm of \mathbf{A} respectively. For a Hermitian matrix \mathbf{A} , $\text{tr}(\mathbf{A})$, $\mathbf{A}^{1/2}$ and $\lambda_i(\mathbf{A})$ denote, respectively, its trace, principal square root and i th eigenvalue after sorting in descending order. In particular, $\lambda_{\max}(\mathbf{A})$ denotes the maximum eigenvalue of \mathbf{A} . $E\{\cdot\}$ is the expectation operator.

II. SYSTEM MODEL

Consider a K -user MIMO system over a quasi-static flat fading MAC with M antennas at the BS and N antennas at each MU. Denote by \mathbf{H}_k and \mathbf{x}_k the channel matrix and transmitted signal of MU k , respectively. The received signal \mathbf{y} can be represented as

$$\mathbf{y} = \sum_{k=1}^K \mathbf{H}_k \mathbf{x}_k + \mathbf{n} \quad (1)$$

where \mathbf{n} is a vector of complex additive white Gaussian noise samples with zero mean and unit variance. $\{\mathbf{H}_k\}$ are assumed to be perfectly known at all transmitters and the receiver.

In this paper, we assume independent fading among antennas at the BS and only focus on the fading correlation among antennas at each MU due to its limited physical size. Based on the model in [3][7], \mathbf{H}_k can be written as

$$\mathbf{H}_k = \sqrt{s_k} \mathbf{W}_k \mathbf{T}_k^{1/2} \quad (2)$$

where s_k is a scalar denoting the gain of path loss and lognormal

fading¹, \mathbf{W}_k is a Rayleigh fading matrix whose entries are i.i.d. complex circular symmetric Gaussian variables with zero mean and unit variance. \mathbf{T}_k in (2) is a semi-positive definite matrix with $\text{tr}(\mathbf{T}_k) = N$ such that it characterizes correlation effect of antennas at MU k and does not alter the average gain of \mathbf{H}_k . For all MUs, $\{s_k\}$ are independently generated with the same distributions (i.e., $\{s_k\}$ are i.i.d. among different MUs), so are $\{\mathbf{W}_k\}$ and $\{\mathbf{T}_k\}$. These in turn lead to the i.i.d. property of $\{\mathbf{H}_k\}$ among different MUs. Note that \mathbf{T}_k is modeled as a random matrix with probability density function (PDF) denoted by \mathbb{T} (which is independent of k since $\{\mathbf{T}_k\}$ are i.i.d.). This is motivated by the fact that the channel statistics change over time due to the mobility of MUs and the variation of scattering environments [8].

III. POTENTIAL BENEFIT OF ANTENNA CORRELATION AT MUS

A. Preliminary

Let R be the system sum rate. For simplicity, we assume that each MU has the same rate constraint R/K (the discussions below can be extended to the case of different rate constraints straightforwardly). As mentioned in Section I, the design objective of a multi-user MIMO system with rate constraints is to minimize the transmitted sum power under the given rate constraints for each channel realization $\{\mathbf{H}_k\}$, which is given by

$$P = \min_{\{\mathbf{Q}_k\}} \sum_k \text{tr}(\mathbf{Q}_k) \quad (3)$$

subject to $\mathbf{R} \in C_{\text{MAC}}(\{\mathbf{H}_k\}, \{\mathbf{Q}_k\})$

where $\mathbf{R} = [R/K, \dots, R/K]$, $\mathbf{Q}_k \equiv \mathbb{E}\{\mathbf{x}_k \mathbf{x}_k^*\}$ is the input covariance matrix of MU k and $C_{\text{MAC}}(\{\mathbf{H}_k\}, \{\mathbf{Q}_k\})$ denotes the capacity region of a MIMO MAC given $\{\mathbf{H}_k\}$ and $\{\mathbf{Q}_k\}$ of all MUs [8].

Denote by $\bar{P}(K, R, \mathbb{T})$ the average minimum transmitted sum power (AMTSP) of the system in (1), i.e.,

$$\bar{P}(K, R, \mathbb{T}) = \mathbb{E}\{P\} \quad (4)$$

where the expectation is taken over the joint distribution of $\{\mathbf{H}_k\}$. Note that when a MU is under deep fading during a channel realization, extremely large transmission power may be required. To avoid this, a MU doesn't transmit in this case. Denote by ε the probability of this event for each MU. In general, there is no closed-form solution to (4), but it can be evaluated numerically using the iterative algorithms in [9][10].

B. Correlation Gain

For a given PDF \mathbb{T} , we define the ratio

$$G(M, K, R, \mathbb{T}) \equiv \frac{\bar{P}(K, R, \mathbb{I})}{\bar{P}(K, R, \mathbb{T})} \quad (5)$$

as the correlation gain that characterizes the impact of antenna correlation at MUs. The numerator in (5) is calculated with $\mathbf{T}_k = \mathbf{I}$, $\forall k$ for each channel realization (i.e., no correlation, NC). In this case, the PDF of \mathbf{T}_k is re-denoted by \mathbb{I} for clarity.

Figure 1 plots the correlation gain of MIMO systems with $M = N = 4$ and different K . All MUs are assumed to be independent and uniformly distributed in an edge-length-1 hexagon cell with fourth power path-loss law and the standard

deviation of their normalized lognormal fading is 8. We set ε at 0.01. $\{\mathbf{T}_k\}$ are i.i.d. rank-1 matrices, which is referred to as the full correlation (FC) scenario.

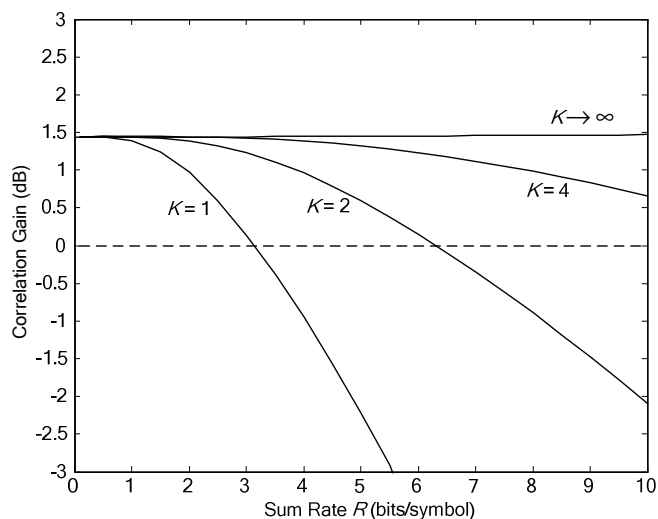


Fig. 1. The correlation gain of MIMO systems with different K in the FC scenario. $M = N = 4$ and $\varepsilon = 0.01$. The numbers of MUs K are marked on the curves.

From Fig. 1, we have the following observations.

- When K is finite, there is a critical point for each curve, i.e., the cross point of the 0-dB horizontal line and the correlation gain curve. Below this critical point, FC is advantageous and *vice versa*.
- The critical-point value on abscissa increases with K . This indicates that the advantage of FC becomes more noticeable when K increases. For example, when $K = 2$, the critical point occurs at about $R = 6.2$ bits/symbol; when $K = 4$, it occurs at about $R = 13.5$ bits/symbol (not shown in Fig. 1). Note that these rates are already high enough for MIMO systems with $M = N = 4$. Hence, with multi-user concurrent transmission in such systems, FC is always advantageous in the rate range of general interest in practice. The situation for $K \rightarrow \infty$ (the related curve is obtained by (13)) will be discussed in Section IV.

The above observations show that antenna correlation at MUs is potentially advantageous in multi-user MIMO systems. Although Fig. 1 is related to full correlation, we have observed in our simulations that partial correlation can also provide a potential advantage.

C. Asymptotic Analysis for $M \rightarrow \infty$

We now consider an asymptotic analysis for $M \rightarrow \infty$, which is more convincing and insightful.

Lemma 1: For the multi-user MIMO system with the channel model in (2), almost surely (i.e., with probability one) as $M \rightarrow \infty$, the sub-channels seen by all MUs are mutually orthogonal.

The proof of Lemma 1 is given in the Appendix. This indicates that all MUs can perform singular value decomposition together with water filling separately to achieve the optimal system performance and there is no interference among them. As a consequence, the original multi-user MIMO

¹ We assume equal path loss and lognormal fading for all the antenna links seen by a particular MU.

system with a sum rate R degrades to K independent single-user ones, each with a rate constraint R/K . Hence we have

$$\begin{aligned} G(\infty, K, R, \mathbb{T}) &= \lim_{M \rightarrow \infty} \frac{K\bar{P}(1, R/K, \mathbb{I})}{K\bar{P}(1, R/K, \mathbb{T})} \\ &= \lim_{M \rightarrow \infty} \frac{\bar{P}(1, R/K, \mathbb{I})}{\bar{P}(1, R/K, \mathbb{T})}. \end{aligned} \quad (6)$$

Since $\{\mathbf{H}_k\}$ are i.i.d. among different MUs, we will drop the subscript k when we are only interested in their distributions. Then (2) is rewritten as

$$\mathbf{H} = \sqrt{s}\mathbf{W}\mathbf{T}^{1/2} \quad (7)$$

where \mathbf{H} , s , \mathbf{W} and \mathbf{T} have, respectively, the same distributions as \mathbf{H}_k , s_k , \mathbf{W}_k and \mathbf{T}_k , $\forall k$.

For each MU, the gain of its i th sub-channel is given by $\lambda_i(\mathbf{H}\mathbf{H}^*)$. Then following the water-filling principle in MIMO systems [8], we have

$$\bar{P}(1, R/K, \mathbb{T}) = \mathbb{E} \left\{ \sum_i (p^* - 1/\lambda_i(\mathbf{H}\mathbf{H}^*))^+ \right\} \quad (8a)$$

where $(x)^+ \equiv \max(x, 0)$ and p^* satisfies

$$\sum_i \log_2 \left[1 + \lambda_i(\mathbf{H}\mathbf{H}^*) (p^* - 1/\lambda_i(\mathbf{H}\mathbf{H}^*))^+ \right] = R/K \quad (8b)$$

for each realization of \mathbf{H} .

Define $\tilde{g}_i \equiv \lambda_i(\mathbf{H}\mathbf{H}^*)/M$, $\forall i$. From (7), we have

$$\tilde{g}_i = s\lambda_i(\mathbf{W}\mathbf{T}\mathbf{W}^*)/M.$$

It is proved in [11] that for a given \mathbf{T} , the distribution of $\lambda_i(\mathbf{W}\mathbf{T}\mathbf{W}^*)$, $\forall i$ converges to a Gaussian one with mean $M\lambda_i(\mathbf{T})$ and variance $M(\lambda_i(\mathbf{T}))^2$ when $M \rightarrow \infty$. Then $\lambda_i(\mathbf{W}\mathbf{T}\mathbf{W}^*)/M$, $\forall i$ converges to $\lambda_i(\mathbf{T})$ almost surely as $M \rightarrow \infty$ for a given \mathbf{T} . Therefore, we have the following lemma.

Lemma 2: Almost surely as $M \rightarrow \infty$, \tilde{g}_i converges to $\lambda_i(\mathbf{T})s$, $\forall i$ for given \mathbf{T} and s .

Returning to (8) and on the basis of this lemma, we have

$$\begin{aligned} \lim_{M \rightarrow \infty} M\bar{P}(1, R/K, \mathbb{T}) &= \mathbb{E} \left\{ \lim_{M \rightarrow \infty} \sum_i (Mp^* - 1/\tilde{g}_i)^+ \right\} \\ &= \mathbb{E} \left\{ \frac{1}{s} \sum_i (\tilde{p}^* - 1/\lambda_i(\mathbf{T}))^+ \right\} \end{aligned} \quad (9a)$$

where \tilde{p}^* is independent of s and is the solution to

$$\sum_i \log_2 \left[1 + \lambda_i(\mathbf{T}) (\tilde{p}^* - 1/\lambda_i(\mathbf{T}))^+ \right] = R/K \quad (9b)$$

for each realization of \mathbf{T} . Based on (9), equation (6) is further written as

$$\begin{aligned} G(\infty, K, R, \mathbb{T}) &= \lim_{M \rightarrow \infty} \frac{M\bar{P}(1, R/K, \mathbb{I})}{M\bar{P}(1, R/K, \mathbb{T})} \\ &= \frac{\mathbb{E} \left\{ \frac{1}{s} N(2^{R/KN} - 1) \right\}}{\mathbb{E} \left\{ \frac{1}{s} \sum_i (\tilde{p}^* - 1/\lambda_i(\mathbf{T}))^+ \right\}}. \end{aligned} \quad (10)$$

In particular, we consider the FC scenario where \mathbf{T} is a random rank-1 matrix (i.e., $\{\mathbf{T}_k\}$ are i.i.d rank-1 matrices) with $\lambda_i(\mathbf{T}) = 0$, $\forall i \neq 1$. Due to the fact of $\sum_i \lambda_i(\mathbf{T}) = \text{tr}(\mathbf{T})$, $\lambda_1(\mathbf{T})$ reduces to a constant N . Then (10) can be rewritten as

$$G(\infty, K, R, \mathbb{T}) = \frac{N^2(2^{R/NK} - 1)}{(2^{R/K} - 1)}. \quad (11)$$

When both K and N are finite and $N \neq 1$, it is straightforward to prove that (11) is a strictly monotonously decreasing function of R with

$$\lim_{R \rightarrow 0} G(\infty, K, R, \mathbb{T}) = \lim_{R \rightarrow 0} \frac{N^2(2^{R/NK} - 1)}{(2^{R/K} - 1)} = N > 1 \quad (12a)$$

and

$$\lim_{R \rightarrow \infty} G(\infty, K, R, \mathbb{T}) = \lim_{R \rightarrow \infty} \frac{N^2(2^{R/NK} - 1)}{(2^{R/K} - 1)} = 0 < 1. \quad (12b)$$

Therefore, equation $G(\infty, K, R, \mathbb{T}) = 1$ have a unique solution within $[0, \infty)$. This indicates that there exists a critical point: FC is beneficial below this point and *vice versa*, which is consistent with the observations in Fig. 1.

D. Numerical Results for Different M

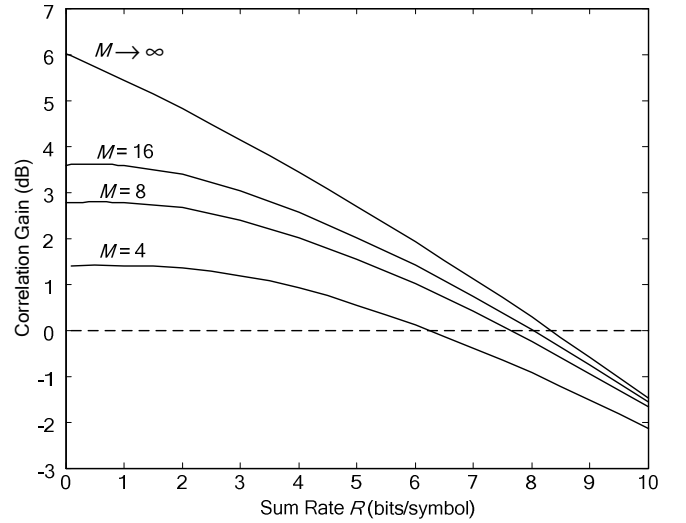


Fig. 2. The correlation gain of MIMO systems with different M in the FC scenario. $\varepsilon = 0.01$, $K = 2$ and $N = 4$. The curves for $M = 4, 8$ and 16 are obtained numerically; while that for $M \rightarrow \infty$ is obtained using (11). The numbers of antennas at the BS M are marked on the curves.

In Fig. 2, we show the correlation gain of MIMO systems with different M . The setting of \mathbb{T} and other parameters of channel fading are the same as those in Fig. 1. K and N are set at 2 and 4, respectively. It is observed in Fig. 2 that there is a critical point for each curve and the critical-point value on abscissa increases with M . However, compared with Fig. 1, the increment of the critical-point value on abscissa is very small. For example, when M is doubled from 4 to 8, the related increment is only about 1.3 bits/symbol (i.e., from $R = 6.2$ bits/symbol to $R = 7.5$ bits/symbol). This indicates that, compared with increasing K , increasing M is not an efficient way to enlarge the rate range where antenna correlation is advantageous.

IV. CORRELATION GAIN FOR $K \rightarrow \infty$ AND $M \rightarrow \infty$

This section focuses on the correlation gain of multi-user MIMO systems with infinite K and M .

It is shown in [4] that when $K \rightarrow \infty$, the limit of (4) is given by

$$\bar{P}(\infty, R, \mathbb{T}) = \int_{\varepsilon}^1 \frac{R \ln 2 \cdot 2^{R(t-\varepsilon)/M}}{F^{-1}(t, \mathbb{T})} dt \quad (13)$$

where $F(\cdot)$ is the cumulative distribution function of $\lambda_{\max}(\mathbf{H}\mathbf{H}^*)$

that depends on \mathbb{T} and $F^{-1}(\cdot)$ is its inverse.

Substituting (13) into the definition of the correlation gain in (5), we can obtain

$$G(M, \infty, R, \mathbb{T}) = \frac{\int_{\varepsilon}^1 2^{R(t-\varepsilon)/M} / F^{-1}(t, \mathbb{I}) dt}{\int_{\varepsilon}^1 2^{R(t-\varepsilon)/M} / F^{-1}(t, \mathbb{T}) dt}. \quad (14)$$

Based on Lemma 2, the following equality holds for a given \mathbf{T} when $M \rightarrow \infty$.

$$F^{-1}(t, \mathbb{T}) = \lambda_{\max}(\mathbf{T}) \cdot F^{-1}(t, \mathbb{I}), \quad \forall t.$$

Since \mathbf{T} is a semi-positive definite random matrix with $\text{tr}(\mathbf{T}) = N$, it is true that $\lambda_{\max}(\mathbf{T}) \geq 1$ for each realization of \mathbf{T} . Then for infinite M , we have

$$F^{-1}(t, \mathbb{T}) \geq F^{-1}(t, \mathbb{I}), \quad \forall t. \quad (15)$$

Based on (14) and (15), the following can be shown.

$$\lim_{M \rightarrow \infty} G(M, \infty, R, \mathbb{T}) \geq 1. \quad (16)$$

Note that in the above derivation, we let $K \rightarrow \infty$ first and then $M \rightarrow \infty$. If $M \rightarrow \infty$ first and then $K \rightarrow \infty$, from (10) in Section III.C, we have

$$\begin{aligned} \lim_{K \rightarrow \infty} G(\infty, K, R, \mathbb{T}) &= \lim_{K \rightarrow \infty} \frac{\mathbb{E}\{N(2^{R/KN} - 1)/s\}}{\mathbb{E}\{(2^{R/K} - 1)/(s \cdot \lambda_{\max}(\mathbf{T}))\}} \\ &= \frac{\mathbb{E}\{1/s\}}{\mathbb{E}\{1/(s \cdot \lambda_{\max}(\mathbf{T}))\}} \geq 1 \end{aligned} \quad (17)$$

where the last inequality can be proved by following the similar procedures in (14)-(16).

In addition, when K and M go to infinite at the same speed, i.e., $K \rightarrow \infty$ with $K/M = \alpha$ (α is a finite constant), we can adopt the same bounding technique in [4] and obtain the limit of (4) in this case as

$$\lim_{\substack{K \rightarrow \infty \\ K/M = \alpha}} \bar{P}(K, R, \mathbb{T}) = \lim_{M \rightarrow \infty} \int_{\varepsilon}^1 \frac{R \ln 2}{F^{-1}(t, \mathbb{T})} dt. \quad (18)$$

Then the correlation gain in (5) reduces to

$$\lim_{\substack{K \rightarrow \infty \\ K/M = \alpha}} G(M, K, R, \mathbb{T}) = \lim_{M \rightarrow \infty} \frac{\int_{\varepsilon}^1 1/F^{-1}(t, \mathbb{I}) dt}{\int_{\varepsilon}^1 1/F^{-1}(t, \mathbb{T}) dt}. \quad (19)$$

Similar to (16) and (17), we can obtain

$$\lim_{\substack{K \rightarrow \infty \\ K/M = \alpha}} G(M, K, R, \mathbb{T}) \geq 1. \quad (20)$$

Combining (16), (17) and (20), we can show the following theorem.

Theorem 1: For a multi-user MIMO system with $K \rightarrow \infty$ and $M \rightarrow \infty$, the asymptotic correlation gain $G(\infty, \infty, R, \mathbb{T}) \geq 1$.

This theorem shows that, when the dimensionality of the BS is high enough and there exists a sufficiently large number of concurrent transmission MUs, antenna correlation at MUs is beneficial in all the rate range.

Since $\{\lambda_i(\mathbf{T})\}$ are all non-negative and $\lambda_{\max}(\mathbf{T})$ is bounded by $\sum_i \lambda_i(\mathbf{T}) = \text{tr}(\mathbf{T}) = N$ for a given \mathbf{T} , from (14), (17) and (19), we can obtain

$$\max_{\mathbb{T}} G(\infty, \infty, R, \mathbb{T}) = N \quad (21)$$

where the maximum is achieved when \mathbf{T} is a random rank-1 matrix (i.e., $\{\mathbf{T}_k\}$ are i.i.d. rank-1 matrices) representing the FC

scenario. Hence the following can be shown.

Theorem 2: For a multi-user MIMO system with $K \rightarrow \infty$ and $M \rightarrow \infty$, the maximum correlation gain is N , which is achieved by the FC scenario.

V. PRACTICAL IMPLEMENTATION OF THE CORRELATION GAIN

The previous sections are focused on the capacity analysis. We now consider the practical implementation aspect of the correlation gain.

We adopt interleave-division multiple-access (IDMA) [13] as a platform and use the maximum eigenmode beamforming (MEB) strategy [4] to exploit the potential correlation gain in practically coded multiple-access MIMO systems. Consider a 4-user MIMO system with $M = 8$ and $N = 4$. A rate-1/2 (23, 35)₈ convolutional code with information length 4096 followed by length-2 spreading and QPSK modulation is employed for all MUs. We set the system sum rate R at 6 bits/symbol. Based on superposition coding principle, multiple coded streams may be assigned to each MU to guarantee the system sum rate. Following the MEB strategy, each MU only transmits in its maximum eigenmode direction. Other transmitting, receiving and power allocation principles are the same as those in IDMA systems (refer to [13] for details).

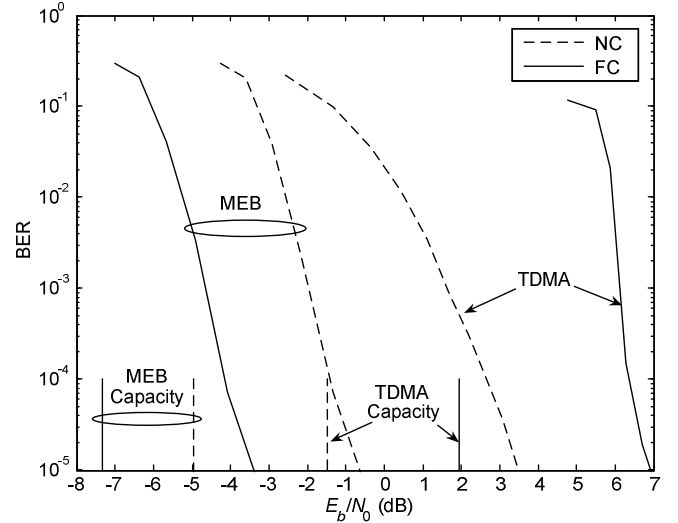


Fig. 3. The BER performance of MEB-based IDMA systems and TDMA-based MIMO systems in the NC and FC scenarios. $\varepsilon = 0.01$, $K = 4$, $M = 8$, $N = 4$ and $R = 6$ bits/symbol.

Figure 3 shows the bit-error-rate (BER) performance of 4-user MEB-based IDMA systems in the NC and FC scenarios. The performance of 4-user time-division multiple-access (TDMA) based MIMO systems is also plotted for comparison. Clearly, its performance is the same as that of single-user MIMO systems. In the NC scenario, following the water-filling principle, such TDMA-based systems require a variety of codes with different coding rates. For convenience, we adopt artificial codes in this case. Specifically, compared to a rate- r ideal code (i.e., capacity-achieving code), a rate- r artificial code requires the same additional power to achieve the same BER as that required by the rate-1/2 convolutional code above. The parameters of channel fading in Fig. 3 are the same as those in

Fig. 1. The capacities (i.e., the performance limits) of MEB-based IDMA and TDMA, which are obtained by (4), are also plotted as references.

From Fig. 3, we can clearly observe the performance loss due to antenna correlation in TDMA (about 3.4 dB at BER = 10⁻⁵) and the significant correlation gain achieved in MEB-based IDMA (about 2.6 dB at BER = 10⁻⁵). The values of such loss and gain are both in line with those predicted by their corresponding capacity lines. The observation above indicates that the MEB-based IDMA system is an effective platform to exploit the correlation gain defined in (5). It is also seen in Fig. 3 that, the NC scenario of MEB-based IDMA significantly outperforms that of TDMA. The related gain is referred to as multi-user gain (see [4] for detailed discussions on this issue).

VI. CONCLUSIONS

In this paper, we show that antenna correlation at MUs is potentially beneficial for multi-user MIMO systems with rate constraints. We also show that the MEB-based IDMA system is an effective platform to exploit such a benefit. The findings of this paper are useful in practical systems where minimizing the physical size of a MU is highly desirable and antenna correlation may result consequently. Intuitively, the potential benefit provided by antenna correlation mainly comes from two aspects. First, the loss of space dimensionality due to antenna correlation can be compensated by the diversity related to the independent fading of all MUs. Second, antenna correlation enables focusing power.

Although this paper focuses on MIMO systems with rate constraints, we have observed similar results in our simulations for systems with power constraints. Quantifying the potential correlation gain in this case analytically is an interesting future research topic.

APPENDIX: PROOF OF LEMMA 1

Proof: For the channel model in (2), we perform eigenvalue decomposition on \mathbf{T}_k as $\mathbf{T}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^*$ and define $\mathbf{W}_k = [\mathbf{w}_{k,1}, \dots, \mathbf{w}_{k,N}]$, where $\{\mathbf{w}_{k,i}\}$ are column vectors. Then \mathbf{H}_k is written as

$$\begin{aligned} \mathbf{H}_k &= \sqrt{s_k} [\mathbf{w}_{k,1}, \dots, \mathbf{w}_{k,N}] \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{U}_k^* \\ &= \sqrt{s_k} [\bar{\mathbf{w}}_{k,1}, \dots, \bar{\mathbf{w}}_{k,N}] \mathbf{A}_k \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{U}_k^* \end{aligned} \quad (22)$$

where $\bar{\mathbf{w}}_{k,i} = \mathbf{w}_{k,i} / \|\mathbf{w}_{k,i}\|$, $\forall i$ and \mathbf{A}_k is a diagonal matrix, i.e., $\mathbf{A}_k = \text{diag}\{\|\mathbf{w}_{k,1}\|, \dots, \|\mathbf{w}_{k,N}\|\}$.

When $M \rightarrow \infty$, we have

$$\mathbf{A}_k = \text{diag}\{\sqrt{M} + o_1(\sqrt{M}), \dots, \sqrt{M} + o_N(\sqrt{M})\}$$

where $o_i(\sqrt{M}) / \sqrt{M} \rightarrow 0$ as $M \rightarrow \infty$, $\forall i$ [11] and (22) can be rewritten as

$$\begin{aligned} \mathbf{H}_k &= \sqrt{s_k} [\bar{\mathbf{w}}_{k,1}, \dots, \bar{\mathbf{w}}_{k,N}] (\sqrt{M} \mathbf{I}) \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{U}_k^* + \tilde{\mathbf{H}}_k \\ &= [\mathbf{q}_{k,1}, \dots, \mathbf{q}_{k,N}] (\sqrt{M s_k} \mathbf{D}_k^{1/2}) \mathbf{U}_k^* + \tilde{\mathbf{H}}_k. \end{aligned} \quad (23)$$

In (23), $\mathbf{q}_{k,j}$ is defined as

$$\mathbf{q}_{k,j} = \sum_{i=1}^N \bar{\mathbf{w}}_{k,i} u_{k,i,j} \quad \forall j = 1, \dots, N \quad (24)$$

where $u_{k,i,j}$ is the entry in the i th row and the j th column of \mathbf{U}_k , and

$$\begin{aligned} \tilde{\mathbf{H}}_k &= \sqrt{s_k} [\bar{\mathbf{w}}_{k,1}, \dots, \bar{\mathbf{w}}_{k,N}] \\ &\quad (\text{diag}\{o_1(\sqrt{M}), \dots, o_N(\sqrt{M})\}) \mathbf{U}_k \mathbf{D}_k^{1/2} \mathbf{U}_k^*, \end{aligned}$$

which is ignorable compared with the first term in (23) when M is sufficiently large.

It is proved in [12, Lemma 1] that, for $\forall i \neq i'$ or $\forall k \neq k'$, $E\{|\bar{\mathbf{w}}_{k,i}^* \bar{\mathbf{w}}_{k',i'}|^2\} = 1/M$. Then we have

$$\lim_{M \rightarrow \infty} |\bar{\mathbf{w}}_{k,i}^* \bar{\mathbf{w}}_{k',i'}| = 0 \quad (25)$$

almost surely. Hence asymptotically, $[\mathbf{q}_{k,1}, \dots, \mathbf{q}_{k,N}]$ is a reduced unitary matrix, i.e., the directions of MU k 's sub-channels are determined by $\{\mathbf{q}_{k,j}, j = 1, 2, \dots, N\}$, which are clearly orthogonal to each other. On the other hand, for $\forall k \neq k'$, we can obtain

$$\begin{aligned} \lim_{M \rightarrow \infty} |\mathbf{q}_{k,j}^* \mathbf{q}_{k',j'}| &= \lim_{M \rightarrow \infty} \left| \left(\sum_{i=1}^N \bar{\mathbf{w}}_{k,i} u_{k,i,j} \right)^* \left(\sum_{i'=1}^N \bar{\mathbf{w}}_{k',i'} u_{k',i',j'} \right) \right| \\ &\leq \sum_{i=1}^N \sum_{i'=1}^N |u_{k,i,j}^* u_{k',i',j'}| \lim_{M \rightarrow \infty} |\bar{\mathbf{w}}_{k,i}^* \bar{\mathbf{w}}_{k',i'}| = 0 \end{aligned} \quad (26)$$

where the convergence is almost sure. This ends the proof.

ACKNOWLEDGMENT

This work was supported by a grant from the Research Grant Council of the Hong Kong SAR, P. R. China [Project No. 117508].

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