Proceedings of 2006 IEEE Information Theory Workshop (ITW'06)

Power Allocation for Practically Coded IDMA Systems over Broadcast Channels

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***Abstract*— This paper presents a power allocation technique based on an interior-point method (IPM) for practically coded interleave-division multiple-access (IDMA) systems over broadcast channels. Numerical results show that the proposed technique is efficient and significant performance improvements (compared with orthogonal systems such as TDMA and FDMA) are achievable by applying iterative multi-user detection (MUD) to IDMA in fading channels, even with a fairly small number of simultaneous users. This finding has useful practical implications for broadcast systems.**

I. INTRODUCTION

It is known that the performance of multi-user systems can be significantly enhanced in fading channels by using random-waveform schemes such as code-division multiple-access (CDMA) and interleave-division multiple-access (IDMA) together with multi-user detection (MUD) [1]-[3]. The related advantage of CDMA and IDMA over orthogonal schemes (such as time-division multiple-access (TDMA) and frequency-division multiple-access (FDMA)) is referred to as multi-user gain (MUG) [4]. Un-equal power allocation and successive interference cancellation (SIC) are key strategies in achieving MUG based on ideal coding. Using SIC, analytic formulae have been developed for the optimal power levels in both MACs and BCs [4]-[7].

A power allocation strategy based on SIC is, however, effective only when ideal or nearly ideal channel codes are available. When a practical code (e.g., a convolutional code or a turbo/LDPC code with relatively short code length) is used, the SIC-based method becomes inefficient since practical codes may, compared with the capacity limit, require a few more dB of signal-to-noise-ratio (SNR) to achieve satisfactory performance and using SIC the accumulated extra SNR may incur an excessive power cost. Iterative MUD is a more efficient solution than SIC for practically coded systems [2][8]. Recently, there has been significant progress in the development of low-cost, high-performance iterative MUD techniques and excellent performance has been demonstrated at low-to-medium throughputs. When the system sum-rate is high, it has been shown that un-equal power allocation can also be a useful technique for practically coded systems with iterative MUD. In this case, finding the optimal power profiles is however quite a difficult issue. Linear programming has been employed in received power optimization for IDMA systems over multiple access channels (MACs) [8][9], and a two-step approach using a combined linear-programming and ordered-matching technique has been proposed to provide a sub-optimal solution to the transmitted power optimization problem for MACs. However, power optimization for practically coded broadcast systems still remains a challenging issue.

In this paper, we first formulate the power optimization problem for practically coded IDMA systems over broadcast channels (BCs) [3][6]. We develop an approach based on an interior-point method (IPM) [10]. Numerical results show that the IPM approach can provide good solutions and significant performance gains can be achieved using the proposed technique in fading environments. Our results in this paper demonstrate that the performance of practical wireless broadcast systems can potentially be greatly enhanced by exploiting the recent progress in multi-user information theory.

II. BROADCAST SYSTEMS

Fig. 1 below shows a K-user IDMA broadcast system. At the base station, for each user k, the information bit stream $\boldsymbol{d}_k$ is first encoded by a forward error correction (FEC) encoder (ENC$_k$) and interleaved by a unique interleaver π_k, and then the interleaved chip streams for all users are added together with proper power control factors $\{p_k\}$ and transmitted over a Gaussian BC.

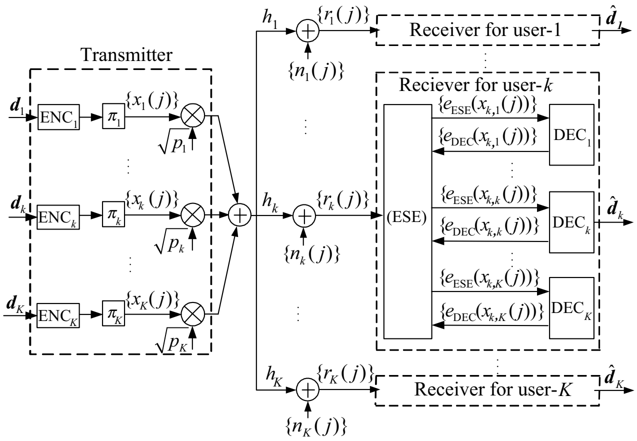

Fig. 1. An IDMA broadcast scheme with multi-user detection.

Denote by $\boldsymbol{x}_k = \{x_k(j)\}$ the length-J signal sequence for user k after encoding and interleaving. At the receiver for user-k, we can write the received signal as

$$r_k(j) = h_k \sum_{i=1}^{K} \sqrt{p_i}\, x_i(j) + n_k(j),\ \forall k,\ j = 1, 2, \ldots, J \qquad (1)$$

where h_k is the channel coefficient from the base station to the receiver for user-k, and $\{n_k(j)\}$ are samples of a zero-mean additive white Gaussian noise (AWGN) process. For simplicity, we assume that all the AWGN noise samples $\{n_k(j)\}$ at different receivers have the same variance $\sigma^2 =$

This work was supported by a grant from the Research Grant Council of the Hong Kong SAR, China [Project No. CityU 1314/04E].

$N_0/2$. But our results can be easily extended to the situation for non-uniform noise energy levels.

Each of the receivers in Fig. 1 performs an iterative interference cancellation process. It consists of an elementary signal estimator (ESE) and a bank of K single-user *a posteriori* probability (APP) decoders (DECs). We follow the iterative detection principle developed in [11], as outlined below. Without loss of generality, we only focus on the receiver for user-k, and the operations at other receivers are just the same. For simplicity, only binary $x_k(j) \in \{+1, -1\}$ and real $\{h_k\}$ are considered here.

- At the receiver for user-k, the ESE output for user-i is

$$e_{\text{ESE}}\left(x_{k,i}(j)\right) = \frac{2h_k\sqrt{p_i}\left(r_k(j) - \text{E}\left(\xi_{k,i}(j)\right)\right)}{\text{Var}\left(\xi_{k,i}(j)\right)} \quad (2)$$

  where $\{\xi_{k,i}(j)\}$ is the noise-plus-interference component in $r_k(j)$ in (1) with respect to $x_i(j)$, i.e.,

$$\xi_{k,i}(j) \equiv r_k(j) - h_k\sqrt{p_i}x_i(j) = h_k\sum_{m\neq i}\sqrt{p_m}x_m(j) + n_k(j) \quad (3)$$

  In (2), $\text{E}(\xi_{k,i}(j))$ and $\text{Var}(\xi_{k,i}(j))$ (the mean and variance of $\xi_{k,i}(j)$) can be estimated using (3) provided that all $\{\text{E}(x_m(j))\}$ and $\{\text{Var}(x_m(j))\}$ are known. The latter are initialized as $\text{E}(x_{k,m}(j))=0$ and $\text{Var}(x_{k,m}(j))=1$, $\forall m$, where the subscript k is added to distinguish the estimations of $x_m(j)$ at different receivers.
- Using $\{e_{\text{ESE}}(x_{k,i}(j))\}$ as the *a priori* information, DEC_i at the receiver for user-k generates extrinsic log-likelihood ratios (LLRs) $\{e_{\text{DEC}}(x_{k,i}(j))\}$, based on which $\{\text{E}(x_{k,i}(j))\}$, $\{\text{Var}(x_{k,i}(j))\}$, $\{\text{E}(\xi_{k,i}(j))\}$ and $\{\text{Var}(\xi_{k,i}(j))\}$ can be updated.
- The updated values of $\{\text{E}(\xi_{k,i}(j))\}$ and $\{\text{Var}(\xi_{k,i}(j))\}$ are used to repeat the ESE operation in (2). This process continues for a preset number of times L before a hard decision is made to produce the final output.
- The above detection procedure at each receiver can be realized by either a parallel or a serial schedule [12]. With the parallel schedule, ESE operations are conducted simultaneously for all users and then DEC operations for all users. With the serial schedule, the operations are conducted in a consecutive manner such as: ESE operations for user-1, DEC operations for user-1, ESE operations for user-2, DEC operations for user-2, …, and so on. For small L, the serial schedule outperforms the parallel one. And when L is large, both schedules lead to roughly the same performance.

III. PERFORMANCE EVALUATION

The performance of the above iterative process can be quickly evaluated using the following signal to noise-plus-interference ratio (SNIR) evolution techniques [11]. Denote by $\{\gamma_{k,i}^{(l)}\}$ the average SNIR with respect to user-i for the outputs of the ESE at the receiver for user-k after the l-th iteration. Let $f_k(\gamma_{k,i}^{(l)})$ be the average variance of the outputs of DEC_k driven by an input sequence with SNIR $\gamma_{k,i}^{(l)}$. In [11], it is shown that $\gamma_{k,i}^{(l)}$ can be approximately tracked by the following recursion:

Parallel Schedule: For the first iteration,

$$\gamma_{k,i}^{(1)} = \frac{p_i|h_k|^2}{|h_k|^2\sum_{m\neq i}p_m + \sigma^2}, \forall k,i, \quad (4a)$$

and thereafter,

$$\gamma_{k,i}^{(l)} = \frac{p_i|h_k|^2}{|h_k|^2\sum_{m\neq i}p_m f_m(\gamma_{k,m}^{(l-1)}) + \sigma^2}, \forall k,i. \quad (4b)$$

Some comments on (4) are as follows.

- In (4a), $\gamma_{k,i}^{(1)}$ is the SNIR for the received signal $r_k(j)$ at the receiver for user-k with respect to $x_i(j)$.
- In (4b), $f_m(\gamma_{k,m}^{(l-1)})$ is the average variance at the output of DEC_m for given input SNIR $\gamma_{k,m}^{(l-1)}$. It represents the remaining average uncertainty about $\{x_m(j)\}$ in the output of DEC_m at the receiver for user-k after the (l-1)-th iteration, which determines the residual interference to other users.
- The $f_m(\cdot)$ function is only determined by the code used. For a rate-R ideal code, the $f(\cdot)$ function (the subscript m is omitted) is obtained based on the Shannon capacity formula, i.e., if the input SNIR $\gamma < 2^{2R}$-1, the output is random binary error (in BPSK format) and so the variance is 1. Otherwise the output is error free and so the variance is 0. Mathematically, we have

$$f(\gamma) = \begin{cases} 1 & \text{if } \gamma < 2^{2R} - 1 \\ 0 & \text{if } \gamma \geq 2^{2R} - 1 \end{cases}. \quad (5)$$

  And for a practical code, it can be obtained by the Monte-Carlo method [8]. Fig. 2 shows some examples of $f(\cdot)$ functions.

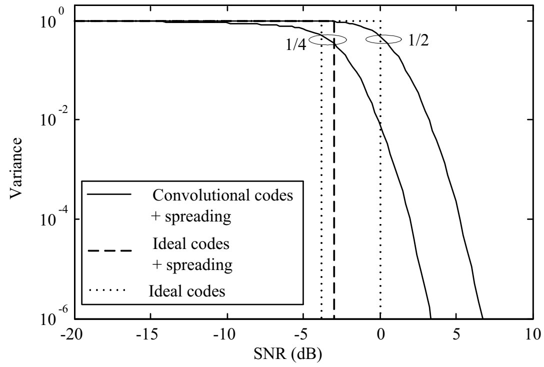

Fig. 2. Some examples of $f(\cdot)$ functions. The corresponding codes are a rate-1/2 convolutional code (The code generators are $(23, 35)_8$.) with length-1 and 2 spreading, the rate-1/2 ideal code with length-1 and 2 spreading and a rate-1/4 ideal code. The numbers marked beside the curves are the corresponding coding rates.

The evolution characterization of the serial schedule is as follows.

Serial Schedule: For the first iteration,

$$\gamma_{k,i}^{(1)} = \frac{p_i|h_k|^2}{|h_k|^2\left(\sum_{m<i}p_m f_m(\gamma_{k,m}^{(1)}) + \sum_{m>i}p_m\right) + \sigma^2}, \forall k,i, \quad (6a)$$

and thereafter,

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$$\gamma_{k,i}^{(l)} = \frac{p_i |h_k|^2}{|h_k|^2 \left(\sum_{m<i} p_m f_m(\gamma_{k,m}^{(l)}) + \sum_{m>i} p_m f_m(\gamma_{k,m}^{(l-1)}) \right) + \sigma^2}, \forall k, i \quad (6b)$$

The parallel schedule has more concise expressions and so it is often used for illustration. However, all of the numerical results in this paper are based on the serial schedule due to its superior performance. Since the optimization method discussed below applies to both parallel and serial schedules, we will not distinguish them unless necessary. Let $\boldsymbol{p}=\{p_k\}$ be a power profile, and $\boldsymbol{\gamma}_k=\{\gamma_{k,i}^{(L)}\}$ be the corresponding SNIR values obtained by either (4) or (6) at the receiver for user k. We introduce the following functions to characterize $\{\boldsymbol{\gamma}_k\}$.

$$\boldsymbol{\gamma}_k = \Omega_k(\boldsymbol{p}), \forall k. \quad (7)$$

Since the K receivers in Fig. 1 operate with different input SNIR (the corresponding h_k values are different), their output SNIR values, i.e. $\{\boldsymbol{\gamma}_k\}$, are also different.

IV. TRANSMITTED POWER OPTIMIZATION

We now consider minimizing the sum transmitted power $\sum_k p_k$ of an IDMA system over a BC while achieving required performance $\gamma_{k,k}^{(L)} \geq \Gamma_k$, $\forall k$, for pre-specified $\{\Gamma_k\}$. We assume that $\{h_k\}$ are fixed and known. The optimization problems are formulized as follows.

Transmitted power optimization (TPO) for BC: Find the distribution $\{p_k\}$ that minimizes

$$\Phi = \sum_k p_k \quad (8)$$

subject to $\gamma_{k,k}^{(L)} \geq \Gamma_k$, $\forall k$, where $\gamma_{k,k}^{(L)}$ is the k-th element in $\boldsymbol{\gamma}_k = \Omega_k(\boldsymbol{p})$.

The problem of (8) is generally nonlinear and non-convex. We call a power profile feasible if it satisfies the system requirement, and define the ***feasible region*** as the set of all feasible power profiles. Clearly, if a power profile $\{p_k\}$ is within the feasible region,

$$\gamma_{k,k}^{(L)} \geq \Gamma_k, \forall k. \quad (9)$$

Then the power allocation problem under consideration requires a search for minimizing sum-power solutions within feasible regions. When the number of users K is small, the optimal solution can be found by an exhaustive search. However, for large K such an exhaustive search method becomes impractical. We therefore consider alternative approaches.

A. Interior-Point Method (IPM)

The interior-point method (IPM) [10] is a very promising technique for this type of constrained optimization problem. The key principle of IPM is to modify (8) to the following form.

TPO with IPM (TPO-IPM) for BC: Find the distribution $\boldsymbol{p}=\{p_k\}$ that minimizes

$$\Phi = \sum_k p_k |h_k|^2 + \alpha \cdot \varphi(\boldsymbol{p}). \quad (10)$$

In TPO-IPM, the constraints of (8) (i.e., $\gamma_{k,k}^{(L)} \geq \Gamma_k$, $\forall k$, where $\gamma_{k,k}^{(L)}$ is the k-th element in $\boldsymbol{\gamma}_k = \Omega_k(\boldsymbol{p})$) are incorporated in $\varphi(\boldsymbol{p})$ that is a barrier function to be discussed in more detail below. The parameter α ($\alpha \geq 0$) is a constant used to control the impact of the barrier function. When $\alpha = 0$, the target functions in TPO-IPM and TPO are the same. By gradually reducing α close to zero, we can therefore guide the solution of the TPO-IPM problem towards the desired TPO solution.

For simplicity, we use the standard steepest descend method [10] in TPO-IPM and the required derivative values $\{\mathrm{d}\Phi/\mathrm{d}p_k\}$ can be obtained using numerical methods. In the following, we discuss some of the details involved, in particular the selections for the barrier function and the initial power profile.

B. The Barrier Function

To confine searching within the feasible region, we can use a barrier function $\varphi(\boldsymbol{p})$ that establishes a "lofty wall" on the border of the feasible region. Such a function can be constructed as follows.

$$\varphi(\boldsymbol{p}) = \sum_k \varphi_k(\gamma_{k,k}^{(L)}) \quad (11a)$$

where $\{\gamma_{k,k}^{(L)}\}$ is the k-th element in $\boldsymbol{\gamma}_k = \Omega(\boldsymbol{p})$ and

$$\varphi_k(\gamma_{k,k}^{(L)}) = \begin{cases} \log\left(\dfrac{\gamma_{k,k}^{(L)}}{\gamma_{k,k}^{(L)} - \Gamma_k} \right), & \gamma_{k,k}^{(L)} > \Gamma_k \\ +\infty, & \gamma_{k,k}^{(L)} \leq \Gamma_k \end{cases}. \quad (11b)$$

In the interior of a feasible region, $\gamma_{k,k}^{(L)} > \Gamma_k$, $\forall k$. If $\boldsymbol{p}$ is in the area far away from the border of the feasible region, $\gamma_k^{(L)} >> \Gamma_k$, $\forall k$, so $\varphi(\boldsymbol{p}) \approx 0$. However, if $\boldsymbol{p}$ is close to the border, i.e., $\gamma_k^{(L)} \to \Gamma_k$ for some k, then $\varphi(\boldsymbol{p}) \to \infty$. Thus introducing $\varphi(\boldsymbol{p})$ in (10) can effectively ensure that searching is confined to the feasible region provided that a proper starting point is selected.

C. Power Profile Initialization

In the optimization process, an initial power profile $\boldsymbol{p}$ within the feasible region is required, which can be obtained as follows. Recall the serial SNIR evolution process in (6). Setting L=1 $\gamma_{k,k}^{(L)} = \Gamma_k$, $\forall k$, we have

$$\Gamma_k \leq \frac{p_k |h_k|^2}{|h_k|^2 \left(\sum_{i<k} p_i f_i(\gamma_{k,i}^{(1)}) + \sum_{i>k} p_i \right) + \sigma^2}, \forall k. \quad (12)$$

Without loss of generality, we assume that $|h_1|^2 \leq |h_2|^2 \leq \cdots \leq |h_K|^2$. Then we have

$$\gamma_{k,i}^{(1)} \leq \gamma_{i,i}^{(1)}, \quad \forall k < i. \quad (13)$$

It is because that if the signal for a user can be validly detected at one receiver, it can surely be detected more reliably at other receivers with larger channel gains [7].

Based on (12) and (13), we can let

$$\Gamma_k = \frac{p_k |h_k|^2}{|h_k|^2 \left(\sum_{i<k} p_i f_i(\Gamma_i) + \sum_{i>k} p_i \right) + \sigma^2}, \forall k. \quad (14)$$

We rewrite (14) in matrix form as

$$\boldsymbol{A} \cdot \boldsymbol{p}^{\mathrm{T}} = \sigma^2 \cdot \boldsymbol{\Gamma}^{\mathrm{T}} \quad (15)$$

where $\boldsymbol{A}=\{A_{k,i}\}_{K \times K}$ satisfies

$$A_{k,i}=\begin{cases}-\Gamma_k\,|h_k|^2\,f_i(\Gamma_i), & k<i\\ |h_k|^2, & k=i\\ -\Gamma_k\,|h_k|^2, & k>i\end{cases} \tag{16}$$

and $\boldsymbol{\Gamma}=[\Gamma_1, \Gamma_2, \ldots, \Gamma_K]$.

The K equations in (15) are linear with respect to the K variables $\{p_k\}$. If $f_k(\Gamma_k) = 0$, $\forall k$, then the matrix $\boldsymbol{A}$ is upper-triangular and so the solution of $\boldsymbol{p}$ is unique. In practice, $\{\Gamma_k\}$ are usually selected to ensure very small $\{f_k(\Gamma_k)\}$ (corresponding to very low BERs), which implies good solvability of (15). (We have observed that (17) is solvable in all our simulations.) Note that further iterations can only improve performance, so for any $L > 1$ the $\boldsymbol{p}$ obtained using (15) must be inside the feasible region and is a valid initial power profile.

V. NUMERICAL RESULTS

So far, we are not able to provide a comprehensive analysis of the convergence properties of the IPM proposed above. We therefore demonstrate its efficiency using numerical results. We consider a K-user IDMA broadcast system with QPSK modulation over a quasi-static fading BC. The channel is modelled as $h_k = A(\rho_k^{-\nu}10^{\xi_k/10})^{1/2}\chi_k$, where A is a constant, ρ_k $(0 \le \rho_k \le 1)$ the normalized distance between user-k and the receiver, ν the path-loss exponent, ξ_k a zero-mean Gaussian random variable with variance σ_s^2 that characterizes shadow fading, and χ_k the complex Gaussian variable with unit power. For simplicity, we set $A = 1$, so the amplitude of h_k is just a relative value. We set $\nu = 4$, σ_s= 8 and the $\{\rho_k\}$ are generated under the assumption that all users are uniformly located in a normalized circular cell area with unit radius.

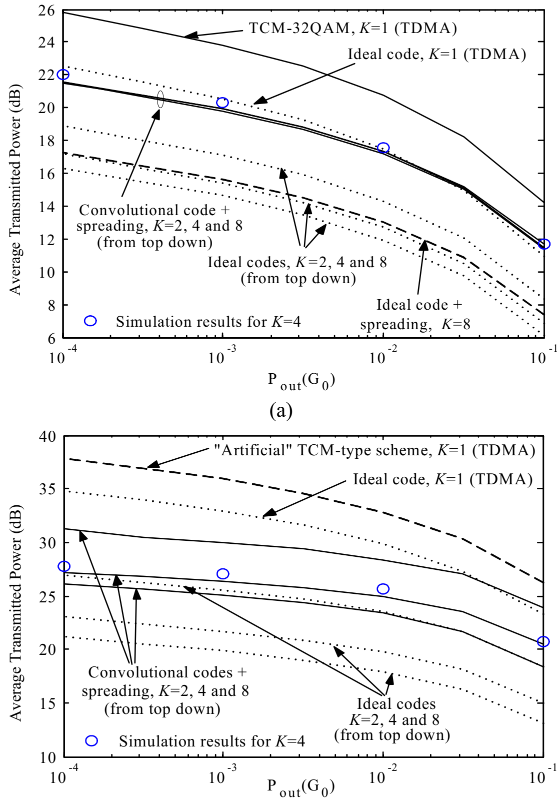
Considering deep fading, we allow transmission outage for users with channel gains below a given fading threshold G_0. That is to say, if $|h_k|^2 < G_0$, an outage is declared for user-k and p_k is set to zero (a similar strategy is adopted in [8]). Then power allocation is applied to the remaining active users. We define the outage probability $P_{\text{out}}(G_0) = \Pr(|h_k|^2 < G_0)$, $\forall k$. The target final SINRs for the active users are set to ensure $\text{BER}_k \le 10^{-4}$.

Fig. 3 shows the required average transmitted power versus $P_{\text{out}}(G_0)$ with the IPM for an IDMA broadcast system where the system sum-rate is fixed at 4 and 8 bits/symbol. A rate-1/2 convolutional code with different spreading lengths is used by all users and the corresponding $f(\cdot)$ functions are shown in Fig. 2. User-specific chip-level interleavers are used which are all generated randomly and independently. In this case, it can be shown that a simple repetition code can be used as the spreading sequences for all users without affecting performance.

For comparison, we have included the results of ideally coded CDMA [4] (The ideally coded CDMA with $K = 1$ is equivalent to ideally coded TDMA.) and the results of practical TDMA with trellis-coded modulation (TCM) for corresponding rates of 4 and 8 bits/symbol respectively. The rate-4 bits/symbol TCM-32QAM code is cited from [13]. For the rate of 8 bits/symbol, we cannot identify a suitable practical coding scheme for TDMA from the existing literature. For simplicity, we therefore assume an "artificial" TCM-type scheme that operates at 3 dB away from capacity, as shown in Fig. 3(b).

Without spreading, the basic rate per user is 1 bit/symbol with rate 1/2 convolutional coding followed by QPSK modulation. If rate higher than 1 bit/symbol is required for a user, we can assign multiple codes to him. For example, in Fig. 3(a), two codes are assigned to each user in the case of K = 2 to achieve a sum-rate of 4 bits/symbol, which follows the super-position coding scheme discussed in [14].

The circles in Fig. 3 are obtained by numerical simulation (the data length is 4096) of convolutionally coded CDMA systems for K = 4 and can be seen to be close to those obtained by evolution.



(a)

(b)

Fig. 3. The required average transmitted power versus $P_{\text{out}}(G_0)$ for the system shown in Fig. 1. A = 1, ν = 4, σ_s = 8 and the system sum-rates in (a) and (b) are 4 and 8 bits/symbol respectively.

From Figs. 3(a) and (b), we can make several interesting observations:

- The MUG discussed in [4, p. 253] based on capacity region analysis can be observed in Fig. 3 by comparing the performance for the ideally coded CDMA for different K values, relative to K = 1. (Note: A Broadcast system with K = 1 is equivalent to an orthogonal system such as TDMA.).
- The MUG can also be observed for convolutionally coded IDMA systems compared with TCM coded TDMA. At a sum-rate of 4 bits/symbol, the MUG is about 4~5 dB for the convolutionally/spreading coded IDMA over the TCM based TDMA. Most MUG is achieved at K = 2 and increasing K only leads to marginal improvement.

- At a sum-rate of 8 bits/symbol, the gap between the convolutionally/spreading coded IDMA (*K* = 8) and the TCM based TDMA increases to about 10~13 dB, which is quite significant. This indicates the potential performance advantage of multi-user systems for very high rate applications. Also we can see that in Fig. 3(b), the curves for different *K* values become more distinguishable than those in Fig. 3 (a), with most gain achieved when *K* = 4.
- Note the "cluster" behavior of the curves for the convolutionally coded MUD scheme in Fig. 3(a), i.e., the curves for different *K* are very close together. This can be explained as follows. For the curves of *K* = 4 and 8, we can see that when a rate-1/2 ideal code is used and spreading operations is involved for *K* = 8, the two curves are also very close together, which is due to the fact that spreading operations for *K* = 8 do not produce coding gain, so it is not strange that the "cluster" behavior occurs in the corresponding conlutionally coded MUD scheme. Therefore, to maximize the MUG by increasing *K*, we should employ low-rate codes with good coding gain instead of conventional spreading. IDMA [11][12] provides a good framework for this purpose. As for the curves for *K* = 2 and 4, it seems that the "cluster" behavior is caused by the use of the convolutional code that has relatively weak BER performance, we may expect that these two curves can be more separated if a more powerful channel code is used.

The above observations point to some interesting directions to improve the performance of wireless systems.

- For medium to high system sum-rates, an IDMA system with MUD can lead to significant performance enhancement in fading environments.
- For practical codes, an IDMA system with a small number (such as two or four) of users can achieve most gain compared with TDMA. This implies that the spreading factor of the system in Fig. 1 can be fairly small and thus the complexity involved can be reduced. (However, in cellular environments, a large spreading factor may be necessary to mitigate cross-cell interference and for other purposes.)
- When the system sum-rate increases, the achievable MUG also increases. Potentially, very significant gains can be achieved by IDMA with MUD at a high sum-rate.
- We may envisage that a mixed TDMA/IDMA or FDMA/IDMA scheme may achieve a good trade-off between complexity and MUG. In such schemes, the users are divided into groups. The users in the same group share a common carrier in the IDMA manner to achieve the MUG discussed above, and different groups of users can be supported in either TDMA or FDMA. This avoids the high complexity of conducting MUD for too many users.

The "cluster" behaviour seen in Fig. 3 has important practical implications. In a broadcast system, only the signal designated to a particular user is wanted at each receiver. Thus MUD in a BC always leads to a cost overhead. Thus it is highly desirable to minimize the number of users involved in MUD for a BC.

VI. Discussions and Conclusions

In this paper, we have proposed a power optimization technique for practically coded IDMA systems over BCs based on an interior-point method. The key findings are listed as follows.

- Considerable performance improvements (e.g., MUG) are achievable by the optimized IDMA schemes in fading environments over orthogonal schemes such as TDMA and FDMA.
- The MUG becomes significant when the sum-rate is high.
- A considerable proportion of the potential MUG can be achieved by a fairly small number (such as two or four) of simultaneous users.

Consequently, IDMA appears a very attractive option even for BCs. Although a cost overhead is inevitable for MUD in BCs, it can be justified by the performance gain.

Acknowledgment

The authors wish to acknowledge Wei Yu for stimulating discussions.

References

[1] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Trans. Inform. Theory*, vol. 43, pp. 858–871, May 1997.

[2] M. Moher, "An iterative multiuser decoder for near-capacity communications," *IEEE Trans. Commun.*, vol. 46, pp. 870–880, July 1998.

[3] C. Schlegel and Z. Shi, "Turbo performance of low-complexity CDMA iterative multiuser detection," in *Proc 2003 Int. Symp. Turbo Codes*, Brest, France, Sept. 2003, pp. 203–206.

[4] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge: Cambridge University Press, 2005.

[5] W. Yu and J. M. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 1875–1892, Sept. 2004.

[6] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 768–783, May 2004.

[7] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[8] L. Liu, J. Tong, and Li Ping, "Analysis and optimization of CDMA systems with chip-level interleavers," *IEEE J. Select. Areas Commun*, vol. 24, pp. 141-150, Jan., 2006.

[9] G. Caire, R. Mueller, and T. Tanaka, "Iterative multiuser joint decoding: optimal power allocation and low-complexity implementation," *IEEE Trans. Inform. Theory*, vol. 50, pp. 1950–1973, Sept. 2004.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[11] Li Ping, L. Liu, K. Y. Wu, and W. K. Leung, "Interleave-division multiple-access," *IEEE Trans. Wireless Commun.*, vol. 5, no. 4, pp. 938-947, April 2006.

[12] Li Ping, L. Liu, and W. K. Leung, "A simple approach to near-optimal multiuser detection: interleave-division multiple access," in *Proc. WCNC'03*, New Orleans, LA, Mar. 2003, pp. 391–396.

[13] C. Schlegel and L. Perez, *Trellis and Turbo Coding*, New York: Wiley, 2004, p. 13.

[14] X. Ma and Li Ping, "Coded modulation using superimposed binary codes," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3331–3343, Dec. 2004.