Transmission Strategies for Parallel Relay Networks Based on Superposition Coding

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Abstract—**This paper is concerned with the transmission strategies and relay selection for decode-and-forward parallel relay networks (PRNs) based on superposition coding. Our focus is on the minimization of the aggregate transmission power that supports a given transmission rate. In this paper, two transmission strategies are investigated, i.e., an optimal-cooperation (OC) strategy requiring strict synchronization at the destination/relay nodes and a no-information-overlapping (NIO) strategy with no synchronization requirement for the ease of implementation using current technologies such as interleave-division multiple-access (IDMA). Furthermore, a reduced-relay-selection (RRS) strategy involving only 2-level superposition coding is given to trade off between complexity and performance. Numerical results show that the performance gap between OC and NIO is negligible, and selecting the best two relay nodes is adequate to achieve most performance gain even when the number of relay nodes is large.**

Keywords-parallel relay networks, superposition coding, transmission strategies, relay selection.

I. INTRODUCTION

Recently, there has been growing interest in relay assisted transmission schemes due to their potential of providing performance improvement in terms of reliability, diversity gain and achievable rate region [1]-[8]. The parallel relay network (PRN) was first introduced by Brett Schein [1] and has drawn a lot of attention. Different schemes, e.g., decode-forward (DF) ones and amplify-forward (AF) ones, can be adopted to implement the PRN. In an AF scheme, the relay nodes essentially act as analog repeaters, and therefore enhance the system noise. Although AF-based schemes can achieve the best performance under certain conditions [12], they are difficult to scale to large networks due to the strict synchronization requirement among different geographically located nodes. While in a DF-based scheme, full decoding and re-encoding are required at each relay node, which may therefore lead to higher decoding complexity and error propagation. However, these drawbacks in DF-based schemes can be compensated by strong channel coding with low-cost decoding technologies.

Our focus in this paper is only on the DF-based scheme in PRNs. Currently, most routing protocols [4]-[6] or power allocation schemes [3] have been developed based on the opportunistic relaying (OR) strategy [9], i.e., only the relay node with the best channel condition is selected to relay the information. In [7], the optimal transmission strategy, denoted as optimal cooperation (OC), is proposed for DF-based PRNs. However, it is difficult for implementation due to its strict synchronization requirement and high complexity when a large number of relay nodes are involved.

In this paper, we investigate simplified strategies to overcome the above two shortcomings. First, we propose a no-information-overlapping (NIO) strategy to exempt the strict synchronization requirement at relay/destination nodes. Then, an effective relay pre-exclusion method is given to reduce the optimization complexity of the NIO strategy. Finally, a reduced-relay-selection (RRS) strategy that only allows the best two relay nodes to take part in the transmission is introduced to further reduce the implementation complexity. Numerical results show that the performance gap between OC and NIO is negligible, and selecting the best two relay nodes is adequate to achieve most performance gain even when the number of relay nodes is large.

II. SYSTEM MODEL AND OPTIMAL TRANSMISSION STRATEGY

Consider the PRN shown in Fig. 1. The source node S wants to transmit information at rate R to the destination node D via L relay nodes. Assume fixed channel condition and full knowledge of the channel state information (CSI) at all nodes in the network. The source information is first encoded by an L-layer superposition encoder and broadcast to the L relay nodes [13]. Each relay node l then tries to recover as much information as possible and re-encodes and forwards the recovered information (or a part of it) to the destination node D. Finally, the signals from all relay nodes are overlapped at D and decoded using an iterative decoder.

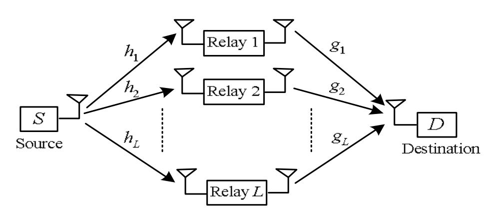

Fig. 1. The system model of a DF-based parallel relay network with L relay nodes.

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Denote by h_l and g_l the fading coefficients of the *S*-to-relay and relay-to-*D* links for relay node *l*, respectively. Without loss of generality, we assume $|h_1|^2 \le |h_2|^2 \le \ldots \le |h_L|^2$ and normalize the variances of the Gaussian noises at both each relay node *l* and the destination node *D* to one. The optimal transmission strategy, referred to as optimal-cooperation (OC) [7], is implemented in two phases by viewing the PRN in Fig. 1 as a concatenation of a broadcast channel (BC) and a multiple access channel (MAC). The details are summarized below.

At the BC part, an *L*-layer superposition-coded sequence is transmitted from the source node *S*. Let r_l be the coding rate of the *l*-th layer. The minimum transmitted power required at the source node *S*, which is denoted by P_s, is achieved when descent decoding order on *l* is applied at all relay nodes (i.e., layer-*L*'s signal is decoded and peeled out first and layer-1's is decoded last) and each relay node *l* happens to successfully decode the signals of layers $\{i, i \le l\}$, i.e.,

$$P_s = \sum_{l=1}^{L}\left(\frac{2^{2r_l}-1}{|h_l|^2} + \left(2^{2r_l}-1\right)\sum_{i=l+1}^{L}\left(\frac{2^{2r_i}-1}{|h_i|^2}\prod_{j=l+1}^{i-1}2^{2r_j}\right)\right) \quad (1)$$

At the MAC part, each relay node *l* successfully recovers the information for layers $\{i, i \le l\}$ and then re-encodes and forwards *all* of them to *D*. Note that in the OC strategy, all the relay nodes $\{i, i \ge l\}$ are involved in relaying layer-*l*'s information, and hence the corresponding replicas related to layer-*l*'s information should be in perfect synchronization at the destination node *D* to minimize the transmit power. Such an overlapped-information transmission scheme in MAC is equivalent to a non-overlapped one in a MAC with channel gains $\{|g_{equ(l)}|^2\}$ where $|g_{equ(l)}|^2 = \sum_{i\ge l} |g_i|^2$ for all *l*. According to the definition of $\{|g_{equ(l)}|^2\}$, we always have $|g_{equ(1)}|^2 \ge |g_{equ(2)}|^2 \ge \ldots \ge |g_{equ(L)}|^2$ and therefore ascent decoding order on *l* should be applied at *D* to achieve the minimum transmission power, i.e.,

$$P_{MAC}^{OC} = \sum P_l^{OC} = \sum_{l=1}^{L}\left(2^{2r_l}-1\right)2^{2\sum_{i<l} r_i} / \left|g_{equ(l)}\right|^2, \quad (2)$$

where P_l^{OC} is the total required power to transmit layer-*l*'s information, which is shared by relay nodes $\{i, i \ge l\}$.

In summary, the OC strategy involves the following optimization problem

$$P^* = \min_{\{r_l\}} \left(P_s + P_{MAC}^{OC}\right) \quad (3)$$

subject to $\sum r_l = R$, $r_l \ge 0$, $l = 1, 2, \ldots, L$.

The problem (3) is convex and can be solved using standard convex optimization tools. Some comments regarding to the OC strategy are listed below.

- When r_l equals to zero for some *l*, the relay node *l* is still required to re-encode and forward the information it recovers, i.e., the layers $\{i, i < l\}$, to *D*. For example, if the optimal rate vector $\boldsymbol{r}^* = (r_1, r_2, \ldots, r_L) = (R, 0, \ldots, 0)$, all relay nodes should forward the first layer to *D* to achieve the minimum transmit power.
- When the same information are forwarded by multiple relay nodes, the related signals are required to be perfectly in phase when they arrive at *D*, which is somewhat like the beamforming technique used in multiple antenna systems. As a consequence, the power to transmit layer *l*'s signal from relay node *i* ($i \ge l$) is $P_l^{OC} |g_i|^2/|g_{equ(l)}|^2$.

III. The No-Information-Overlapping Strategy

In this section, we propose a no-information-overlapping (NIO) strategy to avoid the synchronization requirement in implementation. Later we will show by numerical results that the NIO strategy is only slightly inferior to the optimal OC strategy, even when the number of relay nodes *L* is large.

A. No-Information-Overlapping Strategy

The implementation of the NIO strategy is similar to that of the OC one except the MAC part. Different from the OC strategy that relay node *l* should re-encode and forward all the information it recovers, i.e., layers $\{i, i \le l\}$, we only allow it to re-encode and forward layer-*l*'s signal in the NIO strategy. In this case, the destination node *D* will receive each layer's information from only one relay node and synchronization is therefore unnecessary. The separation of information (no information overlapped among relay nodes) can be easily implemented based on the interleave-division multiple-access (IDMA) principle [11].

For a given rate allocation scheme $\boldsymbol{r} = (r_1, r_2, \ldots, r_L)$, the minimum required power at the BC side is still determined by (1). To achieve the minimum required power at the MAC side, we need apply the optimal decoding order at *D* according to $\{|g_i|^2\}$. Let $\pi(\cdot)$ be a permutation such that $|g_{\pi(1)}|^2 \le |g_{\pi(2)}|^2 \le \ldots \le |g_{\pi(L)}|^2$. The minimum required power is achieved when layer-$\pi(L)$ is decoded first and layer-$\pi(1)$ is decoded last, i.e.,

$$P_{MAC}^{NIO} = \sum P_{\pi(l)}^{NIO} = \sum_{l=1}^{L}\left(2^{2r_{\pi(l)}}-1\right)2^{2\sum_{i<l} r_{\pi(i)}} / \left|g_{\pi(l)}\right|^2, \quad (4)$$

where $P_{\pi(l)}^{NIO}$ is the required power to transmit layer-$\pi(l)$'s information from relay node $\pi(l)$ to *D*.

In summary, the calculation of the minimum transmit power required in the NIO strategy involves the following optimization problem

$$P^* = \min_{\{r_l\}} \left(P_s + P_{MAC}^{NIO}\right) \quad (5)$$

subject to $\sum r_l = R$, $r_l \ge 0$, $l = 1, 2, \ldots, L$.

which is also convex and can be solved through convex optimization techniques.

B. An Example: NIO for a 2-Relay PRN

To better understand the NIO strategy, in this part we consider a PRN with only two relay nodes where the optimal rate allocation can be determined in a very simple way. Without loss of generality, we still assume $|h_1| \le |h_2|$. Depending on the channel conditions, we can deal with such a PRN case by case as follows.

Case 1: $|h_1| \le |h_2|$ and $|g_1| < |g_2|$ (or $|h_1| < |h_2|$ and $|g_1| \le |g_2|$).

In this case, the NIO strategy degrades to the OR strategy, i.e., $\boldsymbol{r}^* = (0, R)$. This is so since relay node 2 has larger channel gains than relay node 1 at both the BC and MAC sides (or at least one side) and relaying information from node 2 is always more economical than from node 1.

Case 2: $|h_1| = |h_2|$ and $|g_1| = |g_2|$.

In this case, the two relay nodes are equal important and we can conclude that $r_1 = r_2 = R/2$.

Case 3: $|h_1| < |h_2|$ and $|g_1| > |g_2|$.

In this case, it is a bit difficult to see the optimal rate allocation vector directly. However, when L = 2, the target function in (5) can be rewritten into an explicit form as

$$P_s + P_{MAC}^{NIO} = \frac{(2^{2r_2}-1)2^{2r_1}}{|h_2|^2} + \frac{2^{2r_1}-1}{|h_1|^2} + \frac{(2^{2r_1}-1)2^{2r_2}}{|g_1|^2} + \frac{2^{2r_2}-1}{|g_2|^2}, \quad (6)$$

then we can apply the standard convex optimization tool to solve (5), which leads to $\boldsymbol{r}^* = ([r_1]_0^R, [r_2]_0^R)$ where

$$r_1 = \frac{1}{2}R + \frac{1}{4}\log_2\left(\left(1/|g_2|^2 - 1/|g_1|^2\right) \Big/ \left(1/|h_1|^2 - 1/|h_2|^2\right)\right) \quad (7a)$$

$$r_2 = \frac{1}{2}R - \frac{1}{4}\log_2\left(\left(1/|g_2|^2 - 1/|g_1|^2\right) \Big/ \left(1/|h_1|^2 - 1/|h_2|^2\right)\right) \quad (7b)$$

and

$$[x]_0^R = \begin{cases} R, & x > R \\ x, & 0 \le x \le R \\ 0, & x < 0 \end{cases}. \quad (8)$$

The details are omitted here for brevity.

IV. Pre-Exclusion Method and Reduced-Relay-Selection

In an NIO-based PRN, the optimization complexity increases with the number of relay nodes L in the network. It can be computationally costly when L is large. However, we notice that there are always many zero entries in $\boldsymbol{r}^*$ after optimization, indicating that the corresponding relay nodes are inactive and prevented from information transmission. For the ease of discussion, we define the remaining active relay node as the *optimal relay group*, which is denoted by $\mathfrak{R}$. In this section, we first propose a simple relay pre-exclusion method to eliminate the inactive relay nodes so as to reduce the optimization complexity. A reduced-relay-selection (RRS) scheme is then proposed to select the best two relay nodes from $\mathfrak{R}$ to further reduce the implementation complexity.

A. Pre-Exclusion Method

We first consider a PRN with only two relay nodes. From the example in last section, we have the theorem below.

Theorem 1: *For an NIO-based PRN with two relay nodes, if 1 $\notin \mathfrak{R}$ (or 2 $\notin \mathfrak{R}$) for a given target rate* R*, we will always have 1 $\notin \mathfrak{R}$ (or 2 $\notin \mathfrak{R}$) when* R *decreases.*

The following corollaries hold as a direct consequence of theorem 1.

Corollary 1: For an NIO-based PRN with two relay nodes. If 1 $\notin \mathfrak{R}$ (or 2 $\notin \mathfrak{R}$) for a given target rate R, then we always have 1 $\notin \mathfrak{R}$ (or 2 $\notin \mathfrak{R}$) when a third relay node is added to the network and the target rate R remains unchanged.

Corollary 2: For an NIO-based PRN with L relay nodes. If $l \notin \mathfrak{R}$ for a given target rate R, then we always have $l \notin \mathfrak{R}$ when the $(L+1)$-th relay node is added to the network and the target rate R remains unchanged.

Next, we propose the pre-exclusion method to eliminate some inactive relay nodes before solving the optimization problem (5). For convenience, we define a potential active relay group, which is denoted by $\mathfrak{R}$`. The basic principle of the pre-exclusion method is to check each relay node one by one based on the theorem and corollaries above, which is described as follows:

1) Randomly select a relay node l and initialize $\mathfrak{R}$` to be $\{l\}$.
2) For a newly selected relay node i from outside $\mathfrak{R}$`, we randomly select one relay node $j$ from $\mathfrak{R}$` and form a 2-relay PRN with relay node set $\{i, j\}$.
3) In 2), if i is an inactive node (the rate allocate result is $(0, R)$ in this current 2-relay PRN $\{i, j\}$, we simply drop it and check another relay node from outside $\mathfrak{R}$` until all the relay nodes are checked.
4) In 2), if i is an active node in this 2-relay PRN, we should select another relay node from $\mathfrak{R}$` and form a new 2-relay PRN. Then we re-do 2) until $i$ is dropped or all the relay nodes in $\mathfrak{R}$` have been selected. Note that if j happens to be an inactive node in some steps, we need drop it as well.
5) Update $\mathfrak{R}$`, then go back to 2) until all the relay nodes outside $\mathfrak{R}$` are checked.

It is easy to see that, using the pre-exclusion method above, the resultant potential active relay group $\mathfrak{R}$` must contain the optimal relay group $\mathfrak{R}$ as a sub-set, i.e., $\mathfrak{R} \subseteq \mathfrak{R}$`. However, the resultant $\mathfrak{R}$` is dependent on the order of the node selection. As in the above pre-exclusion method, we can not make sure arbitrary two relay nodes in the network to form a 2-relay PRN. For example, let R = 1 for a PRN with three relay nodes, $\{|h_1|^2, |h_2|^2, |h_3|^2\} = \{1/3, 1/2, 1\}$ and $\{|g_1|^2, |g_2|^2, |g_3|^2\} = \{1, 1/9, 1/10\}$. First, assume the node selection order (1, 2, 3), we have $\mathfrak{R}$` = {1, 3} according to the above procedures. Then, consider the order {3, 1, 2}, we have $\mathfrak{R}$` = {1}. In practice, we use the following node selection order:

Denote by P_l the required power when all information is relayed by *relay node l*. Hence, we can get an ordered relay link $P_L^\pi \le P_{L-1}^\pi \le \ldots \le P_1^\pi$ according to P_l, and the corresponding permuted relay nodes be $\{L^\pi, L\text{-}1^\pi, \ldots, 1^\pi\}$. Ascent order on l^π is used, i.e., 1^π is checked first and L^π is checked last.

The effectiveness of the Pre-exclusion method for the NIO strategy can be measured by the ratio between the average (with respect to the channel distribution) sizes of $\mathfrak{R}$ and $\mathfrak{R}'$, i.e.,

$$\eta = \frac{E[|\mathfrak{R}|]}{E[|\mathfrak{R}'|]}, \quad (9)$$

where $E[|\mathfrak{R}|]$ is the average size of $\mathfrak{R}$ for the optimal rate vector and $E[|\mathfrak{R}'|]$ is the average size after pre-exclusion method.

B. Reduced-Relay-Selection Scheme

After optimization, the implementation complexity of the NIO-strategy is dependent on $|\Re|$, i.e., a $|\Re|$-layer superposition coding scheme is required for the information transmission. When $|\Re|$ is large, the system complexity also becomes unacceptable. Besides that, we find that there are always many small rate entries in $\boldsymbol{r}^*$, indicating that the contribution of the corresponding relay nodes on reducing the total transmit power is limited and the performance degradation will be marginal when these relay nodes do not take part in the transmission. To further reduce the system complexity, we propose the reduced-relay-selection (RRS) strategy. In this strategy, we only select 2 relay nodes from $\Re$ to help relaying information in the PRN. As a consequence, only a 2-layer superposition coding to involved in implementation, which significantly reduces the implementation complexity of the PRN. There are several rules to select the wanted 2 relay nodes. In this paper, we only consider a straightforward rule, i.e., we only choose the two relay nodes with the largest two rate entries in $\boldsymbol{r}^*$.

V. NUMERICAL RESULTS

In this section, we present some numerical results to illustrate the performance of the proposed transmission strategies. We consider a PRN with L relay nodes that are independent and uniformly distributed in a circle with radius 1 (as shown in Fig. 2, note that we only use this simulation model to verify the efficiency of our proposed transmission strategies). The channel coefficients for relay nodes l is modeled as $h_l = d_{Sl}^{-v}\chi_{Sl}$ and $g_l = d_{lD}^{-v}\chi_{lD}$, where d_{Sl} (or d_{lD}) is the distance between relay node l and source node S (or destination node D), v the path-loss exponent (v = 2 in our paper) and χ_{Sl} (or χ_{lD}) a complex Gaussian random variable with zero mean and unit variance that characterizes flat Rayleigh fading.

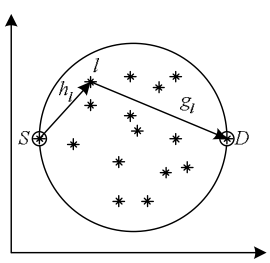

Fig. 2. Relay nodes distribution in a PRN.

Table. 1 depicts the efficiency of the pre-exclusion method proposed in this paper. As expected, the average number of relay nodes in $\Re$ is quite small compared with L, which indicates there are many redundant nodes in a PRN. As shown in Table. 1, the redundant nodes after pre-exclusion method is within 2 nodes even when L = 30 and R = 4. The complexity of finding $\Re$ based on pre-exclusion method can be reduced significantly.

Table. 1. The efficiency of the pre-exclusion (PE) method in different target throughput and relay numbers. $E[|\Re|]$ and $E[|\Re'|]$ are compared.

L \ R	1			2			4		
	$E[\lvert\Re\rvert]$	$E[\lvert\Re'\rvert]$	η	$E[\lvert\Re\rvert]$	$E[\lvert\Re'\rvert]$	η	$E[\lvert\Re\rvert]$	$E[\lvert\Re'\rvert]$	η
5	1.670	1.717	97.3%	2.160	2.273	95.0%	2.787	2.946	94.6%
7	1.764	1.849	95.4%	2.387	2.578	92.6%	3.187	3.442	92.6%
10	1.826	1.952	93.5%	2.639	2.904	90.9%	3.604	4.026	89.5%
20	2.098	2.333	89.9%	3.138	3.679	85.3%	4.410	5.299	83.2%
30	2.132	2.384	89.4%	3.244	3.975	81.6%	4.798	6.018	79.7%

Fig. 3 shows the average required aggregate transmit power that supports a given target rate R versus the number of relays nodes L under different transmission strategies, i.e., the OR strategy, the OC strategy, the NIO strategy, and the NIO strategy with RRS. From Fig. 3 we can make the following observations:

- The performance gap between the NIO and OC strategies is neglectable for a wide range of R and L, indicating that the former is only slightly inferior to the later.
- When the number of relay nodes L in the network increases, the power gains of OC and NIO strategies over OR one also increases, especially when R is large.
- When R is low, the OR strategy is favorable as the gain achieved by OC or NIO is limited. However, for a high throughput R, a substantial amount of power is saved by employing OC or NIO with respect to OR.
- When the RRS strategy is applied, only a small of amount of power savings is lost when only 2 relay nodes take part in the transmission. Hence the RRS strategy is adequate to achieve most of the performance gain while with much lower complexity in implementation.

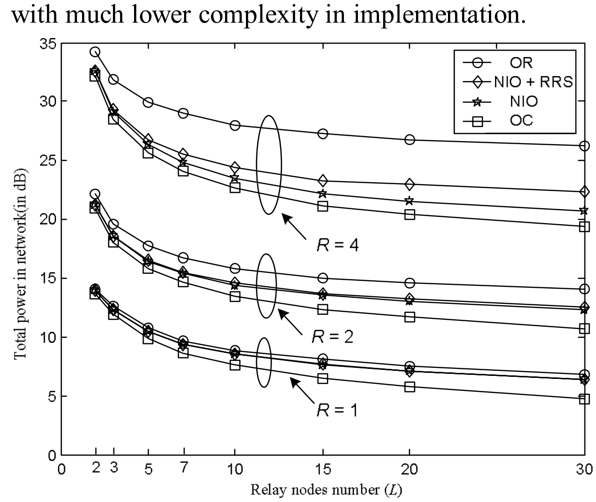

Fig. 3. The average aggregate transmit power versus the number of relay nodes in different transmission strategies.

VI. CONCLUSION

In this paper, we investigate different transmission strategies for DF PRNs based on superposition coding. Compared with the OC strategy, the NIO one proposed in this paper is only slightly inferior to the former without synchronization requirement. We also proposed a pre-exclusion method and a RRS strategy to reduce the optimization complexity and the implementation complexity, respectively. Numerical results show that considerable power savings can be obtained by the OC and NIO strategies with respect to the OR strategy. The pre-exclusion method is effective and the RRS strategy can achieve most of the performance gain in a quasi-static Rayleigh fading channel.

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