Automating Custom-Precision Function Evaluation for Embedded Processors

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Talk outline

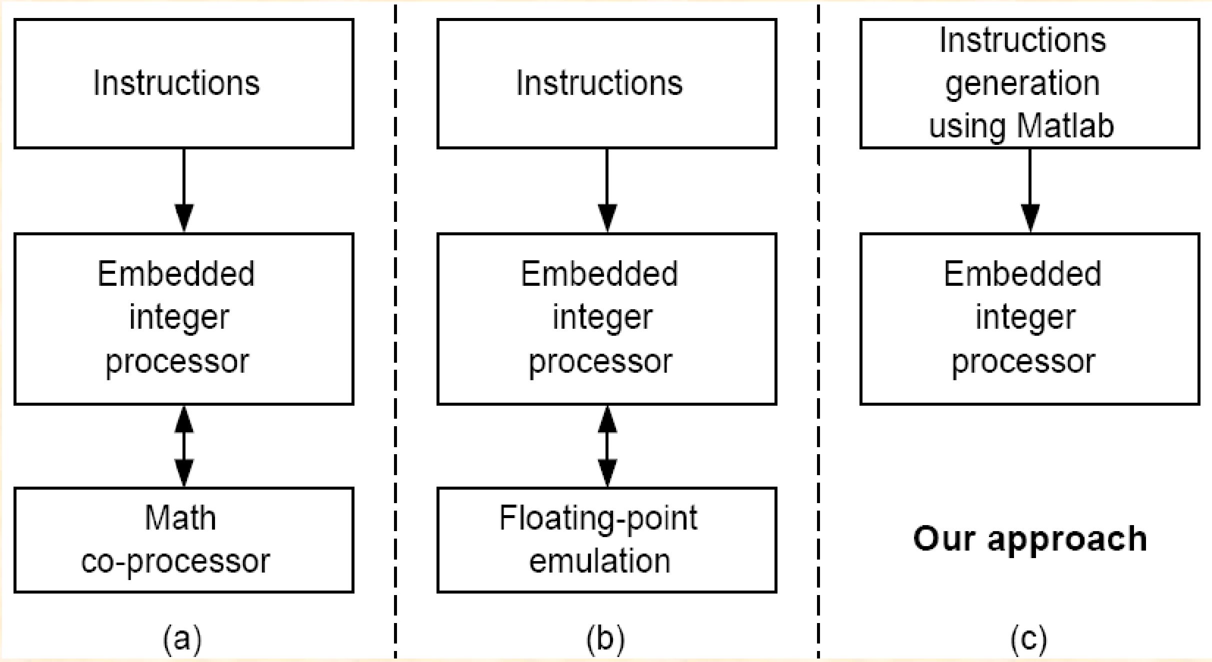
1. achievements
2. motivation
3. function evaluations
4. design tool flow
5. error analysis
6. performance evaluation
7. future work
8. summary

1. Achievements

- customizable library for floating-point function evaluation based on input integer instruction set
- automatic code generation using high-level Matlab model, and optimization for customizing precisions
- evaluation of this method with two elementary functions and Xilinx embedded design kit
- automating the selection of approximation method, polynomial degree for a given function, accuracy requirement and execution time

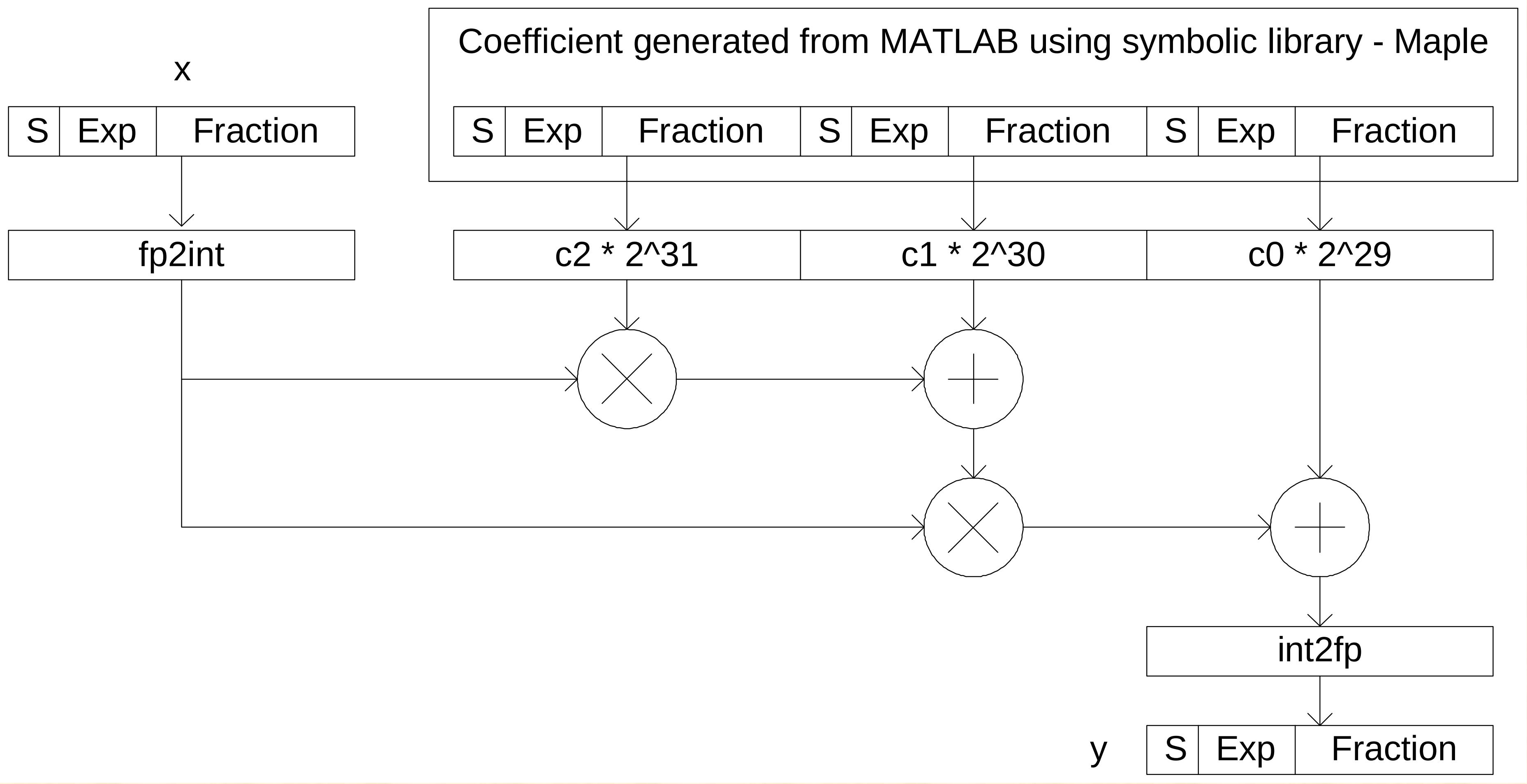
2. Motivation

- embedded systems are usually space and time critical, a dedicated co-processor and a larger memory for instruction are infeasible
- previous work on math co-processor and floating point emulation
- automated code generation for mathematical function library targeting customizable precision (depending on the error requirements)



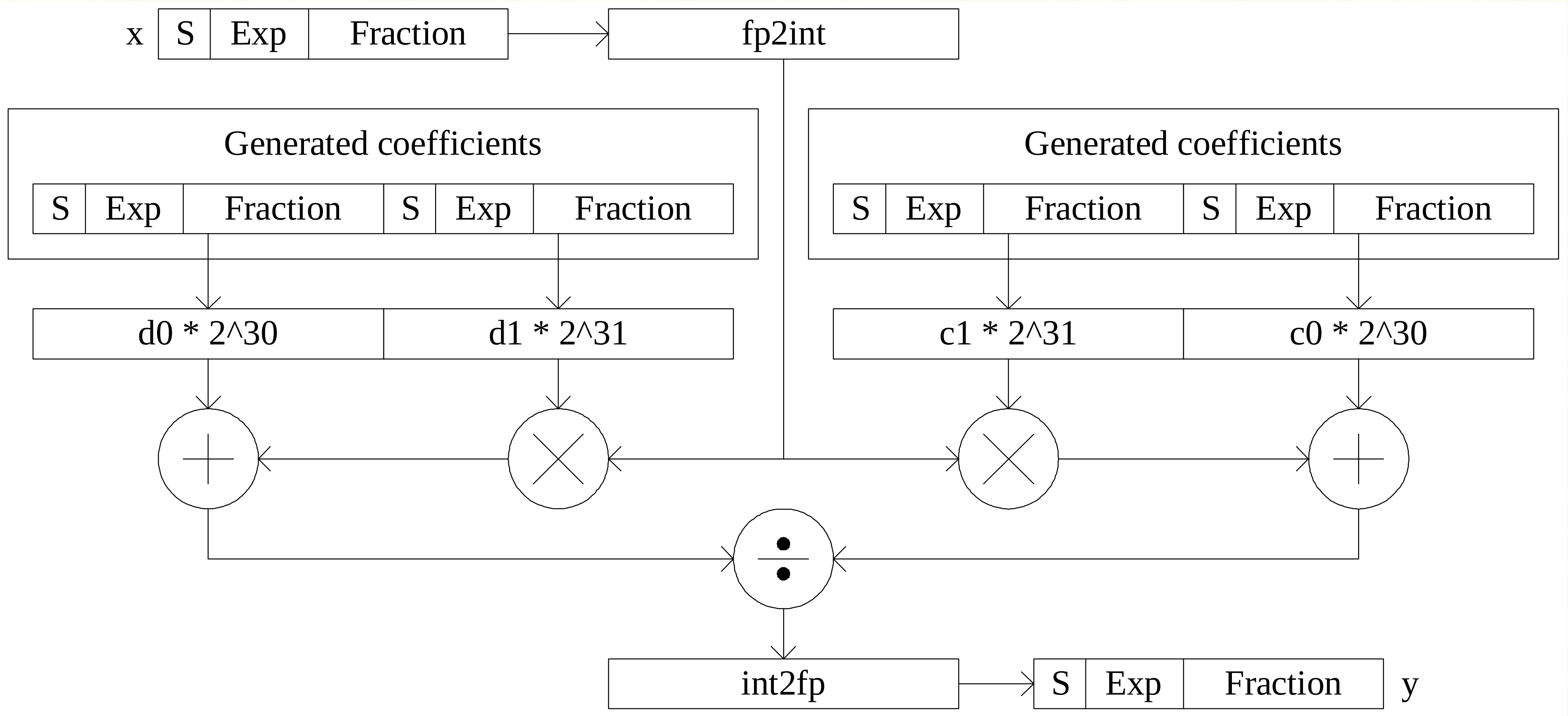
3. Function approximation

- Polynomial method: approximation with a single polynomial



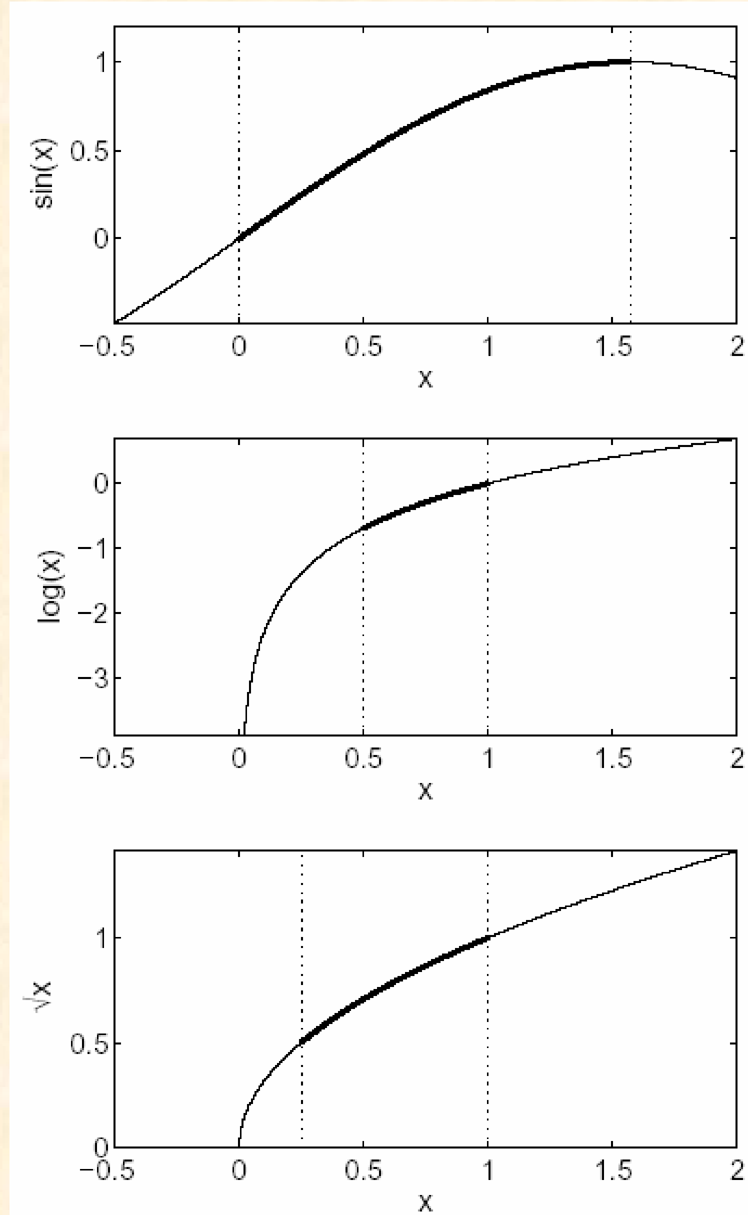
Rational approximation

- Rational method: with two polynomials (same degree)



Range Reduction

- f(x) where x=[a,b]
  - (1) range reducing x to a more convenient interval y=[a',b']
  - (2) function approximation on the reduced interval
  - (3) range reconstruction: expanding the result back to the original result range



Example: Evaluating log(x)

Evaluating $f(x) = \log(x)$

```
// Range Reduction
input.sng_as_flt = x;
exp = input.sng_as_fld.exp - 126;
ix = fp2int(input); // y = ix;

// Evaluation Method
// f(y) where y = [0.5, 1)
// e.g. degree-3 polynomial
```

$f1 = ((c_3 \times y + c_2) \times y + c_1) \times y + c_0;$

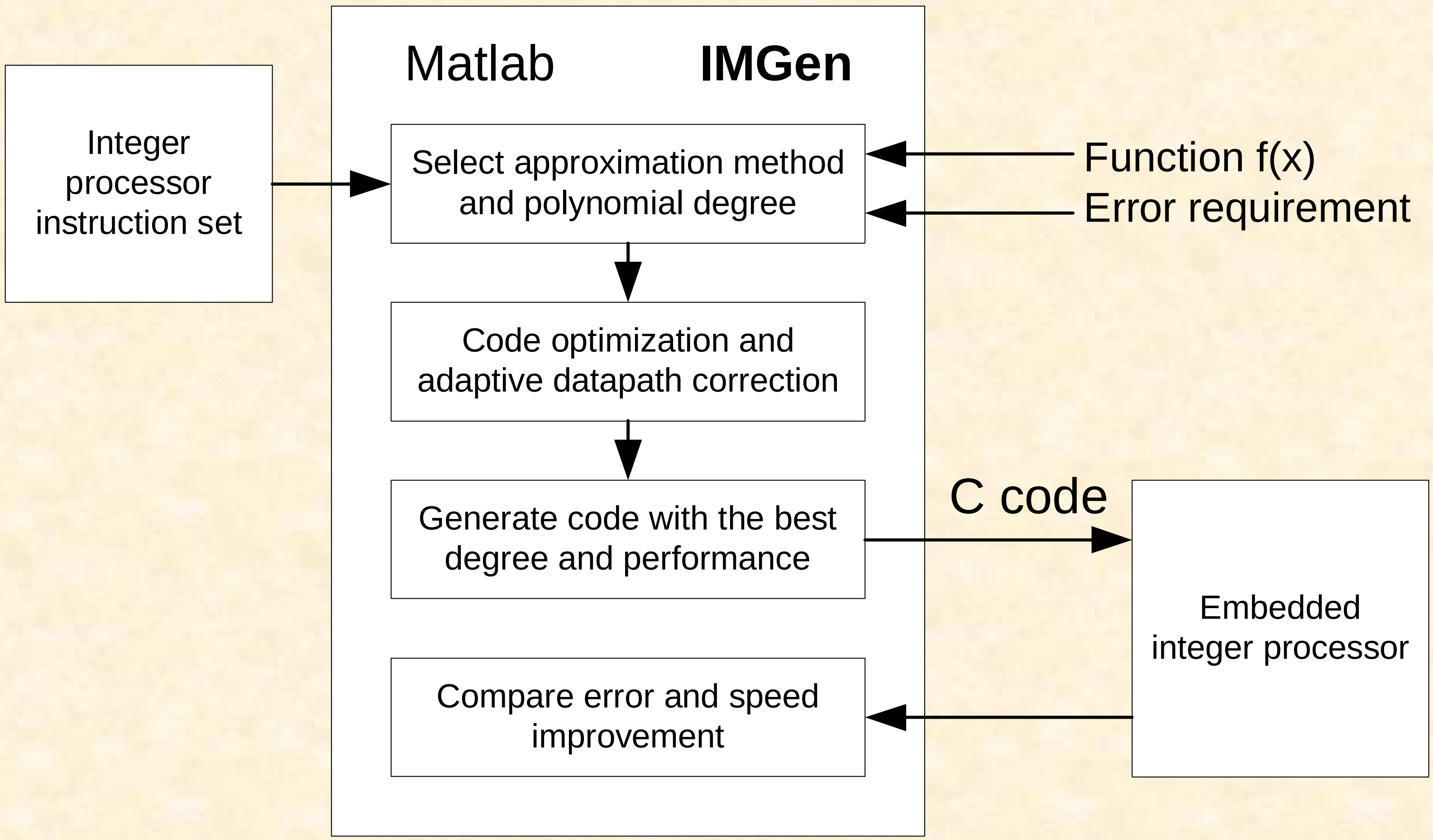
```
// Range Reconstruction
s1 = range(exp); // find exp × log(2)
f1 = (f1 >> overflow) + s1;
output = int2fp(f1);
output.sng_as_fld.exp += overflow;
```

input x

Adjust the output exponent

4. Design tool flow using Matlab

- technology-independent flow
  - use the embedded PowerPC as an example



IMGen – 3 steps

- automation:
  - user error requirement → function evaluation implementation
  - select rational / polynomial approximation
  - select rational / polynomial degree
  - select 32-bit / 64-bit datapath
- generation: using custom precision code
  - Add / multiply / divide / shift operations
- optimization: e.g. loop unrolling techniques

Code optimization

- code generation optimization example

```
long ic[2] = {1488522235, -1456492463};
#define CORRECTION if (j==0) s3 = s3 << 1;
```

generated coefficient

adaptive datapath correction

unoptimized code

```
ix = fp2int(input);
iy = ic[0];
for (j=0; j<degree; j++){
   mhw(ix, iy, s3);
   CORRECTION
   iy = s3 + ic[j+1];
}
```

using loop-unrolling technique

```
ix = fp2int(input);
mhw(ix, ic[0], s3);
s3 = s3 << 1;
iy = s3 + ic[1];
```

Floating-point – fixed-point conversion

- input and output are both floating-point format
- internal computation is transparent to users

C-code: Data structure for input/output

```
typedef struct sng_flds {
unsigned sgn : 1; // 0x8000 0000
unsigned exp : 8; // 0x7F80 0000(bias 127)
unsigned val : 23; // 0x007F FFFF
} SNG_FLD;
```

C-code: *fp2int* - floating-point to integer

```
output = input.val << 8;
output += 0x80000000;
output = output >> 1;
```

5. Error analysis

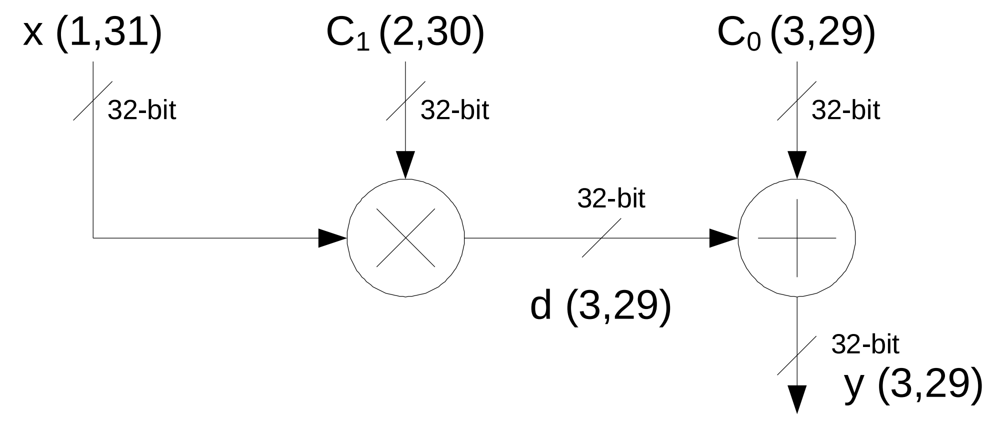
- approximation error (error of approximating a function, e.g. using minimax)
- quantization error induced by: (1) the multiply-add datapath, (2) range reconstruction (function dependent)
- rational approximation has a much lower approximation error

$$E_{total} = E_{approximation} + E_{quantization}$$
$$E_{quantization} = E_{poly\_approximation} + E_{range\_reconstruction}$$

Degree	Polynomial approx.	Rational approx.
1	0.02983005	0.0008607941
2	0.00342398	0.0000017146
3	0.00044161	0.0000000032

Error analysis example

- quantization error analysis of degree-one log(x)
- x(1,31) = 31 fraction bits
- E_d = error accumulated at signal d
- higher degree → higher error



$$E_y = E_{poly\_approximation}$$
$$E_y = E_{accum} + E_{quantization_y}$$
$$E_y = (E_d + E_{C_0}) + 2^{-29}$$

$$E_d = E_{C_1} + 2^{-29}$$
$$E_{C_1} = 2^{-31}$$
$$E_{C_0} = 2^{-30}$$

$$Therefore,$$
$$E_y = 2^{-31} + 2^{-29} + 2^{-30} + 2^{-29}$$
$$E_y = 5.12227 \times 10^{-9}$$

System automation

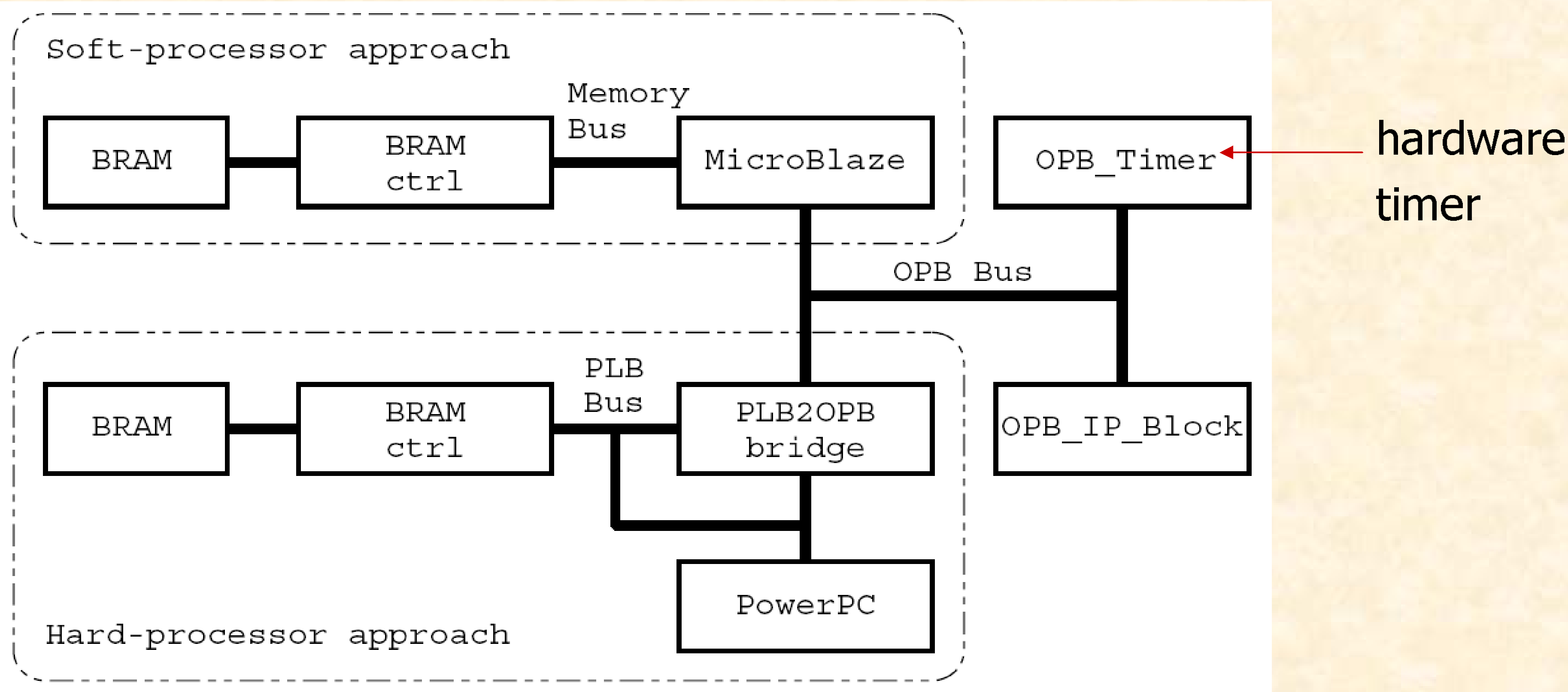
- input / output via Matlab, remote execution on the embedded system board

```
User input >> genlib('log', 0.01)

Phase 1: Maple command generates polynomial coefficients
Phase 2: Static error analysis calculates quantization error
Phase 3: Select polynomial/rational approximation
Phase 4: Select 32-bit/64-bit implementation
Phase 5: Generate embedded C code
         and execute in embedded integer processor
Phase 6: Output performance data and statistical error
      text   data   bss    dec    hex   filename
     44232   4296    48  48576   bdc0   TestApp/executable.elf
     cycle count for the Xilinx math library: 63335
     cycle count for the bus overhead:        60
     cycle count for the IMGen library:       618
     average speedup: 1.13e+002
     maximum error  : 0.0034241
     IMGen is generated and tested in 4.904e+001 seconds
```

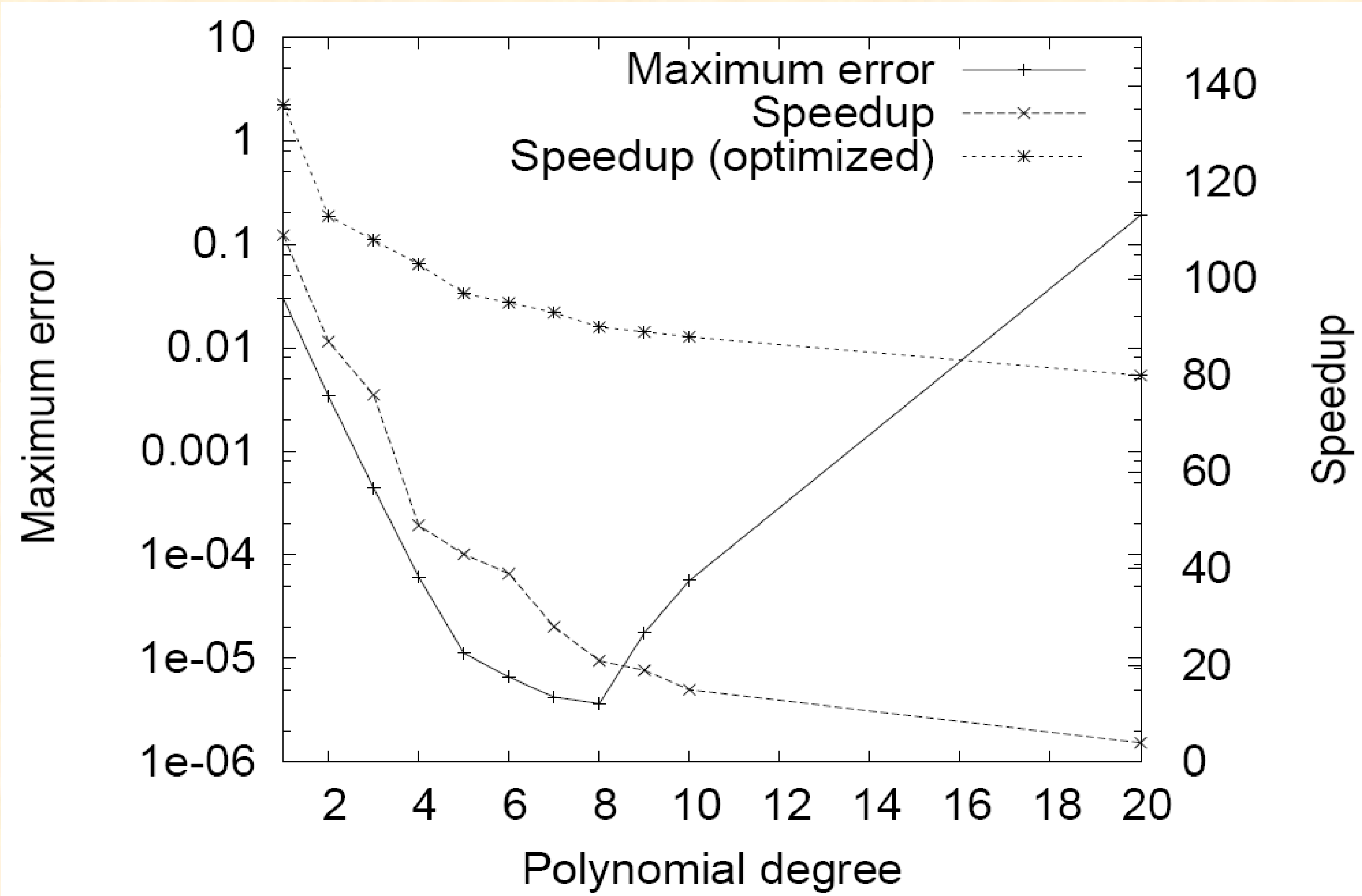
Embedded system under test

- use Xilinx ML310 system, XC2VP30 device, with two embedded PowerPC chips
- can target Xilinx MicroBlaze soft integer processor



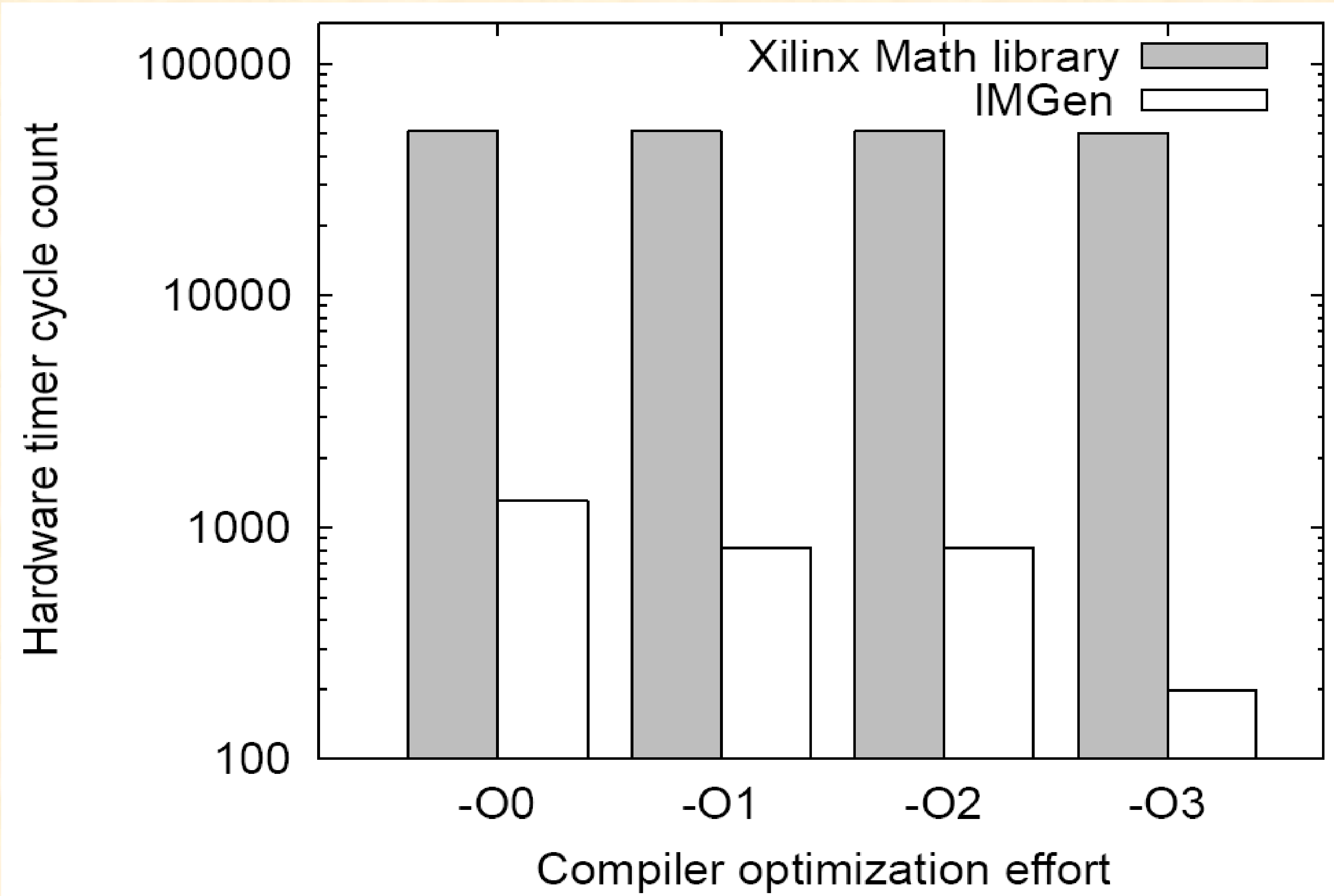
6. Performance evaluation

- compare with Xilinx emulated math library



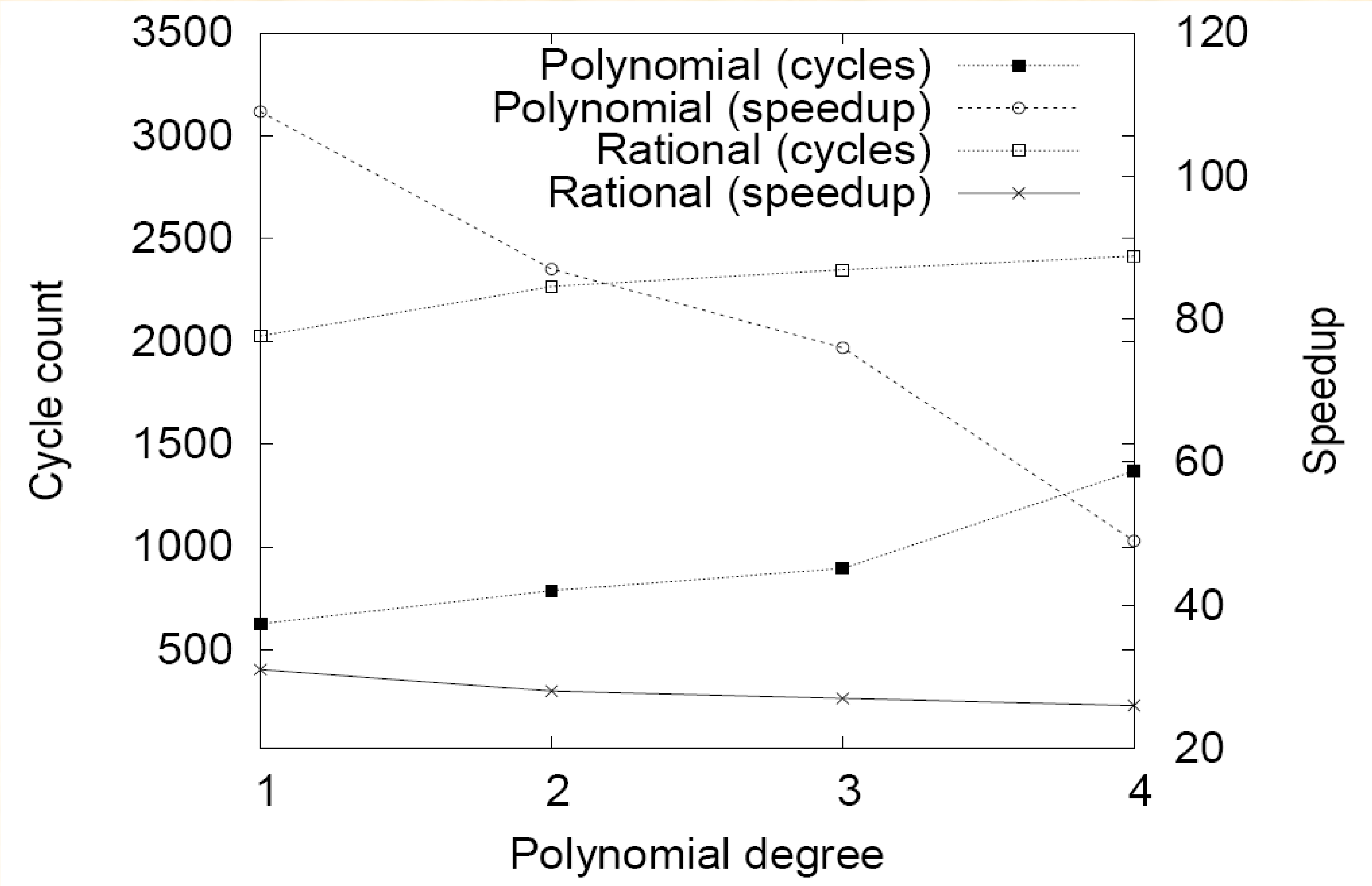
Compile time optimization

- study the effect of compiler optimization



Polynomial vs. rational

- we measure the bus latency, measure an accurate speedup factor $Speedup = (T_{math\_library} - T_{overhead})/(T_{IMGen} - T_{overhead})$



Performance comparisons

	32-bit	32-bit (opt.)	64-bit
Xilinx Math (cycles)	63759	63699	64370
Bus latency (cycles)	60	60	60
IMGen (cycles)	1369	672	1921
Speedup factor	48x	103x	34x
Measured error	0.00005886	0.00005920	0.00000159

tradeoff between speed and accuracy

	$\log(x)$	$\sqrt{x}$
Xilinx Math (cycles)	62725	9159
Bus latency (cycles)	60	60
IMGen (cycles)	1696	467
Speedup factor	38x	22x
Measured error	0.000004313	0.0008375

7. Future work

- code generation for more integer processors
- comparison with floating-point coprocessor
- use better range reduction technique for software implementation
- use run-time reconfiguration to configure soft-processors such as MicroBlaze

8. Summary

- customizable library for floating-point function evaluation based on input integer instruction set
- automatic code generation using high-level Matlab model, and optimization for customizing precisions
- evaluation of this method with two elementary functions and Xilinx embedded design kit
- automating the selection of approximation method, polynomial degree for a given function, accuracy requirement and execution time
- embedded code generator
  - cope with speed/code-size/error trade-off