Towards a Better Diversity-Multiplexing Tradeoff in MIMO Systems

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MIMO Systems

H: Only one dimension!

Diversity Gain
multiple replicas

Transmit diversity

Receive diversity

Channel dimensions increase!

MIMO Channel

• Diversity gain ✓
• Multiplexing gain ✓
Diversity Gain

Error probability \( p \sim SNR^{-d} \)

\[ \leq N_r \quad \text{Receive diversity} \]

\[ \leq N_t \quad \text{Transmit diversity} \]

\[ \leq N_t \cdot N_r \quad \text{MIMO systems} \]

**How to achieve the maximum diversity gain?**

- Space-Time Trellis Codes (STTC) [Tarokh’98]
- Space-Time Block Codes (STBC) [Alamouti’98]
- ……
Multiplexing Gain

Capacity \[ C \sim \min \{N_t, N_r\} \log \text{SNR} \]

- Bell Lab Layered Space-Time (BLAST) [Foschini’98]
- ……

Data rate increases!

Spatial Multiplexing

Multiplexing-based Schemes
MIMO systems can provide both diversity gain and spatial multiplexing gain.

For a system with $m$ transmit and $n$ receive antennas, the optimal diversity-multiplexing tradeoff curve is given by [Zheng&Tse'03]:

$$d^*(r) = (m-r)(n-r)$$

- Alamouti’s scheme (m=2, n=1)
- D-BLAST (MMSE, ignoring the overhead)
How to Achieve the Optimal Tradeoff?

- Explicit code construction
  - [Yao’03]: a structured coding scheme for two-transmit two-receive antenna systems with code duration two
  - [Ma’02]: full-rate-full-diversity codes based on LCF coding and ML decoding
  - [Gamal’04]: LAttice Space-Time (LAST) codes under generalized minimum Euclidean distance lattice decoding
How to Achieve the Optimal Tradeoff?

• Signal Processing
  – [Tarokh’99]: combined array processing and space-time coding
    • [Prasad’01]: LSTBC
    • [Kim’03]: variable-rate STBC
    • [Tao’04]: optimized receiver design

Group Transmission and Detection

a good tradeoff between two gains

low complexity
Group Transmission and Detection

For each group $\mathcal{G}_i, i = 1, ..., G$

- MLD inside the group
- Interference cancellation among the groups

Group Zero Forcing (GZF)
Group Successive Interference Cancellation (GSIC)

A good tradeoff

B = $[b_1, b_2, ...]$
Our Contributions

• Derive the optimal diversity-multiplexing tradeoff of a group detector.

• Propose two novel group-transmission-group-detection based schemes where much better diversity-multiplexing tradeoff can be achieved compared to the existent schemes.
Optimal Tradeoff Function of a Group Detector

**Theorem 1:** When the block length $L \ge n - |\mathcal{G}_x| + |\mathcal{G}_x| - 1$, where $\mathcal{G}_x = \arg \max_{i=1,...,G} |\mathcal{G}_i|$, the diversity-multiplexing tradeoff of an $m$-transmit-$n$-receive system with GZF (GSIC) is given by

$$d(r) = \left\{ \min_{i=1,...,G} \left\{ d_{\mathcal{G}_i,\text{out}}(r_{\mathcal{G}_i}) \right\}; \sum_{i=1}^{G} r_{\mathcal{G}_i} = r \right\}$$

with

$$d_{\mathcal{G}_i,\text{out}}(r_{\mathcal{G}_i}) = \left( n - |\mathcal{G}_i| - r_{\mathcal{G}_i} \right) \left( |\mathcal{G}_i| - r_{\mathcal{G}_i} \right)$$

- The tradeoff function clearly depends on the multiplexing rate $r_{\mathcal{G}_i}$, as well as the particular partition which is specified by $G_i$ and $|\mathcal{G}_i|$.
- 3 rate allocation schemes are examined: equal rate allocation, size proportional rate allocation, optimal rate allocation.
**Equal Rate Allocation**

\[ r_{G_i} = \frac{r}{G} \]

\[
d^{GZF}_{EqRate}(r) = \min_{i=1...G} \left\{ d^{GZF}_{G_i}(r) \right\} = (n - m + \left\lfloor G \right\rfloor - \frac{r}{G}) \left( \left\lfloor G \right\rfloor - \frac{r}{G} \right) - \frac{r}{G} \]

**Step 1:** optimized over all possible partitions with \( G \) fixed

\[
d^{GZF}_{EqRate}(G, r) = (n - m + \left\lfloor \frac{m}{G} \right\rfloor - \frac{r}{G}) \left( \left\lfloor \frac{m}{G} \right\rfloor - \frac{r}{G} \right) - \frac{r}{G} \]

**Step 2:** optimized over all possible \( G \)

\( G = 2 \)
Size Proportional Rate Allocation

\[ r_{G_i} = r \left| \mathcal{G}_i \right| / m \]

\[ d_{PropRate}^{GZF, \{p,G\}}(r) = \left| \mathcal{G}_G \right| (n - m + \left| \mathcal{G}_G \right| - \frac{r}{m} \left| \mathcal{G}_G \right|) (1 - \frac{r}{m}) \]

Step 1: optimized over all possible partitions with \( G \) fixed

\[ d_{PropRate}^{GZF, \{p,G\}}(r) = \left[ \frac{m}{G} \right] (n - m + \left[ \frac{m}{G} \right] - \frac{r}{m} \left[ \frac{m}{G} \right]) (1 - \frac{r}{m}) \]

Step 2: optimized over all possible \( G \)

\( G = 2 \)

Maximum achievable multiplexing rate is \( m \)
Equal Rate Allocation & Size Proportional Rate Allocation

$m=n=8$
$G=3$

Optimized over all possible partitions

GSIC
GZF
size proportional rate allocation
equal rate
Equal Rate Allocation & Size Proportional Rate Allocation

Optimized over all possible partitions under a given $G$
Optimal Rate Allocation

maximize

\[ d^{GZF}(r) = \min_{i=1}^{G} \left\{ d^{GZF}_{G_i}(r_{G_i}) \right\} \]

subject to

\[ \sum_{i=1}^{G} r_{G_i} = r; r_{G_i} \in [0, G_i], \forall i = 1, \ldots, G \]
Optimal Rate Allocation

$m=n=8$
$G=4$
Optimal Rate Allocation

$m = n = 8$

![Graph showing diversity-multiplexing tradeoff for different values of $G$ and $r$. The graph compares GZF and OSIC methods.]
Optimal Rate Allocation

$m = n = 8$
Remarks

• Optimal rate allocation > Size proportional rate allocation
  > Equal rate allocation

• With optimal rate allocation, GZF performs closely to GSIC and sometimes even outperforms it.

• Both GZF and GSIC can efficiently bridge the gap between the optimal performance and BLAST via decreasing G.
Proposed Scheme 1: GLST  
(Group Layered Space-Time)

GLST should have a better tradeoff than LSTBC.
Proposed Scheme 2: QoGST
(Quasi-Orthogonal Group Space-time)

- Inter-group interference can be better suppressed thanks to the orthogonal nature of STBC
- Interleaving gain can be achieved

\[ X = \left[ \tilde{A}_1 b, ..., \tilde{A}_T b \right] + \left[ \tilde{B}_1 b^*, ..., \tilde{B}_T b^* \right] \]

- Without suppressing the inter-group interference
- No interleaving gain
Design Example of Inter-group STBC

Consider a case that the bit stream is divided into $G=2$ groups and transmitted by $m=4$ transmit antennas over $T=2$ time slots.

$$X_{GLST} = \begin{bmatrix} b_{1,1} & -b_{1,2} \\ b_{1,2} & b_{1,1} \\ b_{2,1} & -b_{2,2} \\ b_{2,2} & b_{2,1} \end{bmatrix}$$

$$X_{QoGST} = \begin{bmatrix} b_{1,1} & -b_{2,1} \\ b_{1,2} & -b_{2,2} \\ b_{2,1} & b_{1,1} \\ b_{2,2} & b_{1,2} \end{bmatrix}$$
Tradeoff Comparison: LSTBC, GLST & QoGST

• LSTBC

\[ d_{LSTBC}(r) = g_m(n - m + g_m)(1 - Tr/K) \]

• GLST

\[ d_{lower\_GLST}(r) \leq d_{GLST}(r) \leq d_{upper\_GLST}(r) \]

\[ (x - Tr/G)(g_b - Tr/G) \]

\[ g_m n (1 - Tr/K) \]

• QoGST

\[ d_{lower\_QoGST}(r) \leq d_{QoGST}(r) \leq d_{upper\_QoGST}(r) \]

\[ (G_m n - m + g - Tr/G_b)(g - Tr/G_b) \]

\[ (G_m n - Tr/G_b)(g - Tr/G_b) \]
Tradeoff Comparison: LSTBC, GLST & QoGST

\(m=K=6, n=4, 2\) groups

- **QoGST:** \(g=3, G_m=G_b=2, T=2\)
- **GLST:** \(G=2, g_m=g_b=3, T=4\)
- **LSTBC:** \(G=2, g_m=g_b=3, T=4\)
Tradeoff Comparison: LSTBC, GLST & QoGST

$m=K=6$, $n=4$, 3 groups

LSTBC cannot work in this case!

- LSTBC requires $n \geq m - g_m + 1$
- GLST (QoGST) requires $Tn \geq m - g_m + 1$

QoGST: $g=2$, $G_m=G_b=2$, $T=2$

GLST: $G=2$, $g_m=g_b=2$, $T=2$
FER Comparison: LSTBC, GLST & QoGST

K=m=6, n=4
QoGST: G_m = G_b = 2, g=3
GLST: g_m = g_b = 2, G=3

K=m=6, n=4
QoGST: G_m = G_b = 2, g=3
GLST: g_m = g_b = 2, G=3

K=m=6, n=4
LSTBC: g_m = g_b = 3, G=2

K=m=4, n=2
QoGST: G_m = G_b = 2, g=2
GLST: g_m = g_b = 2, G=2
FER Comparison: LSTBC, GLST & QoGST

LSTBC cannot work in these cases!

QoGST and GLST have better diversity-multiplexing tradeoff than LSTBC!
Thank you!